



TONBRIDGE SCHOOL

Scholarship Examination 2021

MATHEMATICS I

Monday 26th April 2021
11.30 am

Time allowed: 1 hour 30 minutes

*Answer as many questions as you can.
Questions 1 to 5 are worth 8 marks each;
Questions 6 to 9 are worth 15 marks each.
You may attempt the questions in any order.*

*All answers must be supported by adequate explanation.
Calculators may be used in any question.*

1. A machine produces two sizes of square tiles.

Big ones have area x cm²; small ones have area y cm².

You are given that:

- 3 big tiles and 5 small tiles have a total area of 8 cm²;
- A big tile has an area which is 2 cm² greater than that of a small tile.

Use this information to write down two simultaneous equations involving x and y .

Hence find the perimeter of a big tile.

[8 marks]

2. On the Réamur temperature scale, the freezing point of water is 0 degrees Réamur and the boiling point of water is 80 degrees Réamur. On the Fahrenheit temperature scale, the freezing point of water is 32 degrees Fahrenheit and the boiling point of water is 212 degrees Fahrenheit.

- (a) What is 50 degrees Réamur in degrees Fahrenheit?
- (b) What is 68 degrees Fahrenheit in degrees Réamur?
- (c) What (negative) number corresponds to the same temperature in degrees Réamur and degrees Fahrenheit?

[8]

3. This question is about the interior and exterior angles of regular polygons. In each part, find the number of sides of the regular polygon from the information given:

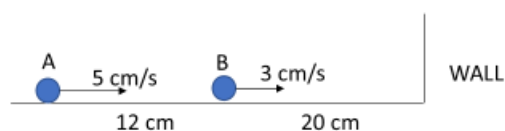
- (a) The interior angle is smaller than the exterior angle.
- (b) The interior angle is 100° more than the exterior angle.
- (c) The interior angle is 100 times the exterior angle.

[8]

4. Two very small balls are moving towards a wall, as shown in the diagram.

Ball A travels at 5 cm/s and Ball B travels at 3 cm/s.

Initially, they are 12 cm apart when Ball B is 20 cm from the wall.



- (a) Show that the balls collide when Ball B is 2 cm from the wall.
- (b) Suppose the balls start in the same positions as in the diagram, but the speeds of the balls are swapped, so that Ball A travels at 3 cm/s and Ball B at 5 cm/s. If Ball B rebounds from the wall at the same speed (5 cm/s), how far from the wall do the balls collide?

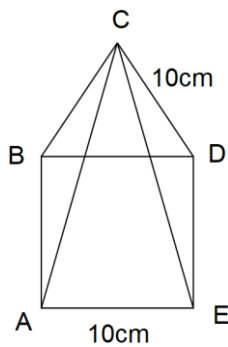
[8]

5. The kinetic energy, E , due to the rotation of a solid sphere may be estimated from the formula $E = \frac{8Mr^2}{T^2}$ where M is the mass of the sphere in kilogrammes, r its radius in metres and T the time in seconds for one complete revolution of the sphere.

- (a) Find M when $E = 550, T = 3.6, r = 1.7$.
- (b) Find T when $E = 1345, M = 9.3, r = 0.85$.
- (c) Calculate E for the earth where $M = 5.98 \times 10^{24}$ kg, $r = 6.38 \times 10^6$ m and it takes one day to complete one revolution.

[8]

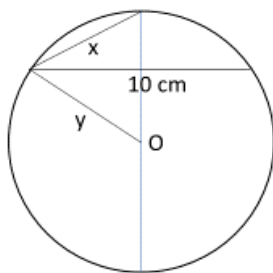
6. The diagram shows an equilateral triangle BCD with sides of length 10 cm next to a square $ABDE$ whose sides are also of length 10 cm.



- (a) Find the size of angle $\angle ACE$.
- (b) By first finding the perpendicular distance from C to AE , find the area of triangle ACE .
- (c) Use your answers to (a) and (b) to find the area of a regular dodecagon (12-sided polygon) with sides of length 10 cm.
- (d) Find the radius of the circle which passes through all the vertices (corners) of the dodecagon in (c).

[15]

7. The three lengths shown in the diagram of a circle, centre O , are connected by the formula $y = \frac{x^2}{\sqrt{4x^2 - 100}}$.

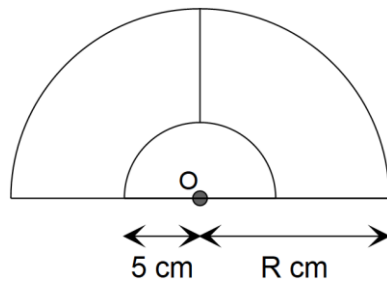


- (a) When $x = 5.5$, show that $y = 6.60$ (to 2 decimal places).
- (b) Find the values of y for $x = 6.0, 6.5, 7.0, 7.5, 8.0, 8.5$.
- (c) Choosing sensible scales, use your values in (a) and (b) to plot a graph of y against x .
- (d) Find the coordinates of the minimum point on the graph.
- (e) Draw as carefully as you can the diagram above with the values of x and y you found in (d).

[15]

TURN OVER

8. The figure shows a small semicircle, radius 5 cm, a large semicircle, radius R cm, and a line which is part of a radius of the large semicircle. Both semicircles have the same centre O .



- (a) Find the value of R if all three regions in the figure have the same area.
- (b) Find the value of R if all three regions in the figure have the same perimeter.

[15]

9. Study carefully the pattern of numbers in the table below. The numbers in Column C come from multiplying those in Columns A and B.

	Column A	Column B	Column C
Row 1	3	2	6
Row 2	5	5	25
Row 3	7	10	70
Row 4	9	17	153
Row 5			
Row 6			
Row 7			
Row n			

- (a) Write down the entries in Columns A, B and C for Rows 5, 6, 7.
- (b) For Row n , find formulae in terms of n for the entries in Columns A, B and C.
- (c) If the entry in Column A is 197, what is the entry in Column C?
- (d) If the entry in Column B is 3482, what is the entry in Column C?
- (e) If the entry in Column C is 10150, find the entries in Columns A and B.
[Hint: Factorise 10150.]
- (f) What can you say about the Row numbers for which the entries in Column C are a multiple of 3?

[15]

END OF PAPER