GRADE 8 COURSE OUTLINE

CONTENT AREA	CCSS STANDARDS & FORMATIVE ASSESSMENT LESSONS	# OF DAYS
8.0: Introduction: Mathematical Investigations		5
8.1 Work with radicals and integer exponents 8.EE.1, 2, 3, & 4	Estimating Length Using Scientific Notation	10
8.2 Understand the connections between proportional relationships, lines, and linear equations 8.EE.5, 6	Lines and Linear Equations	10
8.3 Analyze and solve linear equations and pairs of simultaneous linear equations 8.EE.7 & 8	Solving Linear Equations in One Variable Classifying Solutions to Systems of Linear Equations	10
8.4 Investigate patterns of association in bivariate data 8.SP 1, 2, 3, & 4		10
8.5 Critical Areas 1: 8.EE; 8.SP Critical Areas 1: Formulating and reasoning about expressions and equations, including modeling an association in bivarate data with a linear equation, and solving linear equations and systems of linear equations	Solving a Real-Life Problem: Baseball Jerseys	11
8.6 Define, evaluate, and compare functions 8.F.1, 2, 3 Use functions to model relationships between quantities 8.F.4, 5	Interpreting Distance-time Graphs	15
8.7 Critical Areas 1 & 2: 8.EE; 8.SP; 8.F Critical Areas 2: Grasping the concept of a function and using functions to describe quantitative relationships	Modeling Situations with Linear Equations	15
8.8 Know that there are numbers that are not rational, and approximate them by rational numbers 8.NS.1, 2	Repeating Decimals	10
8.9 Understand congruence and similarity using physical models, transparencies, or geometry software 8.G.1, 2, 3, 4, & 5		10
8.10 Understand and apply the Pythagorean theorem 8.G.6, 7, 8	The Pythagorean Theorem: Square Areas	10
8.11 Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres 8.G.9	Making Matchsticks	10
8.12 Critical Area 1, 2, 3: 8:EE; 8.F; 8.G Critical Areas 3: Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean theorem		18
	TOTAL:	144

MIDDLE GRADES OVERVIEW

Green indicates a Smarter Balance Assessment Consortium *major* cluster, and yellow indicates an *additional or supporting* cluster.

GRADE 6	GRADE 7	GRADE 8
6.RP Understand ratio concepts and use ratio reasoning to solve problems.	7.RP Analyze proportional relationships and use them to solve real-world and mathematical problems.	8.EE Understand the connections between proportional relationships, lines, and linear equations.
6.EE Represent and analyze quantitative relationships between dependent and independent variables.		8.F Define, evaluate, and compare functions. 8.F Use functions to model relationships between quantities.
6.EE Apply and extend previous understandings of arithmetic to algebraic expressions.	7.EE Use properties of operations to generate equivalent expressions.	8.EE Work with radicals and integer exponents.
6.EE Reason about and solve one-variable equations and inequalities.	7.EE Solve real-life and mathematical problems using numerical and algebraic equations.	8.EE Analyze and solve linear equations and pairs of simultaneous linear equations.
6. NS Apply and extend previous understandings of numbers to the system of rational numbers. 6.NS Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	7. NS Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	8.NS Know that there are numbers that are not rational, and approximate them by rational numbers.
6.NS Compute fluently with multi-digit numbers and find common factors and multiples.		
6.G Solve real-world and mathematical problems involving area, surface area, and volume.	7.G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.G Draw, construct, and describe geometrical figures and describe the relationships between them.	8.G Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
		8.G Understand congruence and similarity using physical models, transparencies, or geometry software. 8.G Understand and apply the Pythagorean theorem.
6.SP Develop understanding of statistical variability. 6.SP Summarize and describe distributions.	7.SP Use random sampling to draw inferences about a population. 7.SP Draw informal comparative inferences about two populations. 7.SP Investigate chance processes and develop, use, and evaluate probability models.	8.SP Investigate patterns of association in bivariate data.
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision.	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

CCSS TOOLS: PROGRESSIONS

Source: http://ime.math.arizona.edu/progressions/

GRADE 6	GRADE 7	GRADE 8
6.RP Understand ratio concepts and use ratio reasoning to solve problems.	7.RP Analyze proportional relationships and use them to solve real-world and mathematical problems.	8.EE Understand the connections between proportional relationships, lines, and linear equations.
6.EE Represent and analyze quantitative relationships between dependent and independent variables.		8.F Define, evaluate, and compare functions. 8.F Use functions to model relationships between quantities.
6.EE Apply and extend previous understandings of arithmetic to algebraic expressions. 6.EE Reason about and solve one-variable equations and inequalities.	7.EE Use properties of operations to generate equivalent expressions. 7.EE Solve real-life and mathematical problems using numerical and algebraic equations.	8.EE Work with radicals and integer exponents. 8.EE Analyze and solve linear equations and pairs of simultaneous linear equations.
6.NS Apply and extend previous understandings of numbers to the system of rational numbers. 6.NS Apply and extend previous understandings of multiplication and division to divide fractions by fractions. 6.NS Compute fluently with multi-digit numbers and find common factors and multiples.	7.NS Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	8. NS Know that there are numbers that are not rational, and approximate them by rational numbers.
6.G Solve real-world and mathematical problems involving area, surface area, and volume.	7.G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 8.G Draw, construct, and describe geometrical figures and describe the relationships between them.	8.G Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. 8.G Understand congruence and similarity using physical models, transparencies, or geometry software. 8.G Understand and apply the Pythagorean theorem.
6.SP Develop understanding of statistical variability. 6.SP Summarize and describe distributions.	7.SP Use random sampling to draw inferences about a population. 7.SP Draw informal comparative inferences about two populations. 7.SP Investigate chance processes and develop, use, and evaluate probability models.	8.SP Investigate patterns of association in bivariate data.

8.0 MATHEMATICAL INVESTIGATIONS
5 DAYS

This introductory unit is designed to provide students with an opportunity to engage in worthwhile mathematical investigations; it has a number of related purposes. The most important of these is to let students draw heavily on the eight Mathematical Practices and establish their importance to this entire course. Through working with the mathematical practices, students will come to understanding the importance of seeing, doing, re-constructing, and supposing, in learning mathematics, and hopefully also realize that mathematics is not just facts to be memorized.

Examples of the kinds of mathematical investigations that students will work on during this unit can be found at Henri Piciotto's Mathematics Education Page (http://www.mathedpage.org/).

One of these investigations is called "Area on Graph Paper"; it explores the least and greatest perimeters that can be made with polyominoes of a fixed area. This calls on students to make and test hypotheses, be systematic, and draw conclusions. It will encourage students to deploy MP.1 (*Make sense of problems and persevere in solving them*) and MP.7 (*Look for and make use of structure*). (http://www.MathEdPage.org/new-algebra/new-algebra.html#graph)

The McNuggets Problem, another type of mathematical investigation, which can also be found on Henri's website is particularly valuable in an introductory unit such as this. Students are invited to consider the number of chicken nuggets that might be bought if a fast food store sells them in quantities of 6, 9, and 20. Students are asked to determine how many nuggets can and cannot be bought. Students are also invited to determine what is the largest number of nuggets that cannot be bought, and then prove that every number greater than this largest number can be bought. (http://www.mathedpage.org/early-math/early.html)

Other interesting investigations can be found at Henri's Website. For example, in his paper *Operation Sense, Tool-Based Pedagogy Curricular Breath: A Proposal* he describes investigations such as *Perimeter Patterns* and *Angles* which can be enacted using Pattern Blocks (http://www.mathedpage.org/early-math/early.html#Fun-Math)

Another purpose of this introductory unit is to establish the norms for doing mathematics, and set the scene for the rest of the course. Teachers will be encouraged to enact these investigations in a way that enables students to take responsibility for their own learning and act as instructional resources for each other. The teacher will provide feedback that moves the learning forward and engineer effective discussion so as to facilitate learning. The intent of each mathematical investigation will be made explicit so as to provide a purpose for students.

The genius of this type of investigation is that, in a heterogeneous class of students, it is highly likely that every student will be able to do something, but also highly likely that no student will be able to do everything.

8.1 NUMERICAL EXPRESSIONS
10 DAYS

Our conversations with students who are learning about scientific notation have been revealing. For example, we asked students, “What does 10^0 mean?” Many responded, “It means that you don’t move.” This type of answer was prevalent among 8th-grade students and revealed that many students had a limited conceptualization of powers of ten in terms of moving left or right or not moving at all, without any understanding that when they moved (the decimal place) to the left they were dividing by 10. Many students had no idea that 10^0 represents the quantity 1. This evidence illustrated to us that although some students can remember rules or shortcuts, simply memorizing these rules does not enable students to actually learn the mathematics. Rules and shortcuts can even inhibit understanding, since they are of little use to students when they need to re-construct mathematical meaning or make connections to other mathematical concepts.

The mathematics specified in 8.EE.1,2,3 is widely used in the rest of the mathematics curriculum, so it is critical that students develop a flexible understanding of it. If a teacher pays particular attention to the mathematical practices when teaching this mathematical content, students are more likely to develop conceptual understanding. In this instance the following mathematical practices are particularly relevant:

MP.6 Attend to precision;
MP.7 Look for and make use of structure;
MP.8. Look for and express regularity in repeated reasoning.

Expressions and Equations 8.EE
Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = q$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

FORMATIVE ASSESSMENT LESSONS

Estimating Length Using Scientific Notation

This lesson asks students to translate back and forth between numbers written in decimal and scientific notation, and to order such numbers given either representation. It also asks students to estimate the length of several objects and to compare their relative size—that is, to estimate how many times bigger one is than another. Students are given a set of cards with length measurements (in meters). Each measurement (except for two blank cards that students must fill in themselves) appears on two cards, once as a decimal and once in scientific notation. They are also given another set of cards with drawings of various objects of a wide range of lengths. They need to match the two sets. There is an additional set of cards with multipliers and arrows for students to make multiplicative comparisons among the lengths of the different objects. Students show their matches and comparisons in posters, and they explain and critique all the posters.

In order to do the required tasks, students need to know how to manipulate numbers in decimal and scientific notation and they need to make reasonable estimates and comparisons about lengths of objects.

Teaching and Assessment Resources

Anita Wah and Henri Picciotto. *Algebra: Themes, Tools, Concepts (ATTC).* Mountain View, California: Creative Publications, 1994. (Textbook available free for non-commercial use from www.mathedpage.org)

See Chapter 7: Products and Powers

Section 7.9: Powers and Large Numbers, pp. 272–274

Section 7.10: Using Scientific Notation, pp. 275–276

Section 7.11: Using Large Numbers, pp. 277–278

Novice Tasks	Apprentice Tasks	Expert Tasks
N03: Short Tasks—Expressions and Equations	A02: 100 People	
	A03: A Million Dollars	
	A14: How Old Are They?	

8.2 PROPORTIONAL RELATIONSHIPS, LINES, AND EQUATIONS
10 DAYS

While working with students we learned that they **acquire** many ideas about linear functions that no teachers would ever teach. For example, we learned that many students believe that a linear equation in two unknowns (say, $3x + 3y = 9$) is *not a linear equation at all* until it is written in slope-intercept form ($y = -x + 3$). Or, as many students say, $3x + 3y = 9$ is not a linear equation until you solve for *y*. We also learned that many students believe that a linear equation in two unknowns (say, $y = -x + \frac{3}{4}$) is *not* a linear equation because the value of *b* is a fraction. Somehow many students come to the incorrect conclusion that *b* must be a whole number.

Expressions and Equations 8.EE
Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

These misconceptions are common, and many students believe this is what their teachers have actually taught them. We know that teachers don't teach these misconceptions.. But we also know that particular instructional moves lead students to grasp the wrong end of the stick.

For example, when teachers focus on *solving for y* (writing a linear equation in slope-intercept form), many students become convinced that an equation is not an equation until it has been processed in this way. Also when students are asked to work only with linear equations where the parameter *b* is a whole number, they make the mistake of over-generalizing that *b must* be a whole number.

We also observed a popular teaching sequence that is used to introduce students to linear equations in two unknowns and teach students how to graph them. This teaching sequence involves teaching students to solve for *y*, identifying $y = mx + b$, and then instructing students that, when graphing a specific equation, *b* tells you *where to begin* and *m* tells you *how to move.* This might seem confusing to someone unfamiliar with this teaching sequence, but this is how it plays out in the classroom. With their linear equation, say $y = \frac{1}{2}x + 3$, in slope-intercept form students are ready to graph. They are told that *b* is the *y*-intercept and that *b* tells you where to begin—so students are shown that, in this case, the place to begin is (0,3). Students are then told that *m* is the slope and this tells you how to move. In the case of $y = \frac{1}{2}x + 3$, you begin drawing the line at (0,3) and then $m = \frac{1}{2}$ tells you that you must move 1 unit to the right and 2 units up. Students are told, if you keep moving in this way you will always get back to the line.

Thus, by starting at *b* and moving in a way that is given by *m*, the student manages to graph the line. It is not surprising that students who are taught with this teaching sequence come away with these damaging misperceptions.

The consequences of this are far-reaching, and the problem can lead to a narrowing of the curriculum. For example, we found that students who had been taught in this way were stymied when we asked them to provide a couple of ordered pairs that would satisfy a given linear equation in two unknowns. It appeared that students had no way of thinking about linear equations in two unknowns as mathematical objects in their own right. So rather than develop the idea of a linear equation as something about which students could reason, the teaching sequence caused students to rely too heavily on known procedures and become unable to deviate from them, and unable, in the final analysis, to engage in mathematical reasoning in any but the most shallow conceptual framework.

To help students develop the conceptual understanding that they will need to reason their way toward correct conceptions, it is important that the teaching sequence introduces linear equations in two unknowns as mathematical objects. Thus, students will learn what a linear equation in two unknowns is, what it means to solve a linear equation in two unknowns, and that linear equations can be written in multiple forms and represented in multiple ways. With a firm understanding of what a linear equation is, students can engage in the mathematical practices and develop a variety of ways of graphing them. The focus on linear equation in this course must be broad so as to address these common misconceptions.

Formative Assessment Lessons

Lines and Linear Equations

This lesson asks for students to interpret and create mathematical models of linear relationships in two contexts. In the first context, two runners race each other. They run at constant speeds. The slower runner starts closer to the finish line but nonetheless is overtaken by the faster runner. The second context is about water flowing out of a container and into a container directly below.

(NOTE: This second context is problematic because the physics of the situation makes this a non-linear problem in real life. The water would flow faster at the beginning and slower as time goes on. However students are asked to *assume* that the height of the water will change linearly with time in both containers.)

For the racing context, students need to interpret a graph showing distance vs. time for the two runners. They need to understand the meaning of the slope of the lines (speed), the point of intersection (time and distance when the lead changes), and intercepts (distance at the beginning, time it takes to finish). They are asked to write down equations describing the runners' graphs. They also need to produce a new graph where the distance is measured from the finish line instead of the starting point of the faster runner.

For the water flowing context, students need to match graphs for each of the containers and descriptions of the containers with initial amounts of water and specified rates of flow. Some graphs or drawings are missing numerical information, and when this happens, students need to be able to supply the missing numbers.

Students are asked to work collaboratively, to analyze other groups' work, and to discuss any discrepancies in methods or results, arguing critically and clearly.

Teaching and Assessment Resources

Anita Wah and Henri Picciotto. *Algebra: Themes, Tools, Concepts (ATTC).* Mountain View, California: Creative Publications, 1994. (Textbook available free for non-commercial use from www.mathedpage.org)

See Chapter 4: Interpreting Graphs
Section 4.1: A 100-Mile Trip, pp. 124–126
Section 4.2: Points, Graphs, and Equations, pp. 127–128
Section 4.5: Lines Through the Origin, pp. 134–136
Section 4.6: In the Lab, pp. 137–139
Section 4.9: Rules of the Road, pp. 148–150
Section 4.10: Up in the Air, pp. 151–153
Section 4.11: Horizontal and Vertical Lines, pp. 154–156

Novice Tasks	Apprentice Tasks	Expert Tasks
	A06: Bike Ride	
	A16: Journey	
	A24: Shelves	

8.3 LINEAR EQUATIONS & SYSTEMS OF LINEAR EQUATIONS
10 DAYS

If students are to learn about equations in a way that will help them succeed in high school and beyond, they must have the opportunity to develop a robust and rounded understanding of linear equations in one unknown. For example, students will need to know what a linear equation in one unknown is; what it means to solve any such equation; what it means for two linear equations in one unknown to be equivalent; how to classify equations as linear or non-linear; how to classify linear equations in terms of the number of unknowns; and how to classify linear equations according to the number of solutions. Clearly, the students' focus on linear equations in one unknown must go well beyond learning the steps that are needed to solve them. Their work around solving linear equations in one unknown must be grounded upon some deep conceptual learning.

Expressions and Equations 8.EE
Analyze and solve linear equations in one unknown.

7. Solve linear equations in one variable.
7a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
7b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8. Analyze and solve pairs of simultaneous linear equations.
8a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
8b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
8c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

In the preceding section, we discussed the likely consequences of deploying an overly narrow teaching sequence when addressing linear equations in two unknowns. Another consequence is the impact that this teaching sequence will have on students when they begin to analyze and solve pairs of linear equations in two unknowns simultaneously. Basically, if students have not been given the opportunity to develop a flexible understanding of linear equations in two unknowns, they are likely to fail when they are asked to deal with them in pairs. This is because in *CCSS* sections 8.EE.8a, 8b, and 8c, no new mathematical object is referenced. Instead students are to re-engage with the mathematical object that was introduced in 8.EE.5 and 6. So if students can not think flexibly about the infinite set of solutions that satisfy a single linear equation in two unknowns, it is unlikely that they are going to be able to think about the points of intersection that satisfy both equations simultaneously.

It is important to note that although 8.EE.8b calls on students to learn how to solve systems of two linear equations in two unknowns algebraically, 8.EE.8b of the *CCSS* does not specify that students must learn to solve such systems by the two methods that are commonly known as solving by elimination and solving by substitution. It does specify that students should solve simple cases by inspection, and solving by inspections should be considered to be a viable algebraic method of solution. It is also important to note that the *CCSS* calls upon students to work with systems of linear equations in two unknowns in greater depth in a subsequent course. For example, in the *Cluster Reasoning with Equations and Inequalities* (REI) of the High School Algebra standards students are asked to solve systems of linear equations in two unknowns exactly (algebraically) and approximately (e.g., with graphs)—but the *CCSS* does not specify that students use specific algebraic methods.

FORMATIVE ASSESSMENT LESSONS

Solving Linear Equations in One Variable

In this lesson, students need to solve linear equations in one variable. They need to classify many different such equations as "always true", "sometimes true" or "never true" according to whether the equations are satisfied by all numbers, exactly one number, or no numbers. In each case, they need to explain their answers. In particular, they need to find the unique solution for every "sometimes true" equation and find at least one value of x (the variable) that does not satisfy the equation. They are not asked to prove that the "sometimes true" equations have one unique solution. They simply need to find a solution (which happens to be unique, of course) and a value that does not satisfy the equation.

Students need to be able to add monomials (usually referred to as collecting like terms), add, subtract, multiply or divide the same quantity (including negative numbers) from both sides of an equation, and use the distributive property. They need to explain their work, analyze given sample student work, and collaborate in small groups.

(Note: In these Course Outlines, we replace the commonly used expression "collect like terms" with "adding monomials" because the latter is more easily understood by English Language Learners.)

Classifying Solutions to Systems of Linear Equations

This is a lesson about linear equations and systems of two linear equations (in two variables). Given a table of values, an equation, or a graph of a linear relationship in two variables, students need to produce the other two representations. They need to understand that the graph of an equation consists of all the points in the Cartesian plane satisfying the equation. They need to decide which tables of values agree with a given equation or graph, be able to graph a line given an equation or table, and find a linear equation given a graph or table of values. They also need to determine when a system of two linear equations has no simultaneous solutions, one unique solution, or infinitely many solutions. They should understand why these are the only possibilities. They should be able to tell how many solutions there are both from the equations of the lines and from their graphs. They need to interpret each case in terms of the slopes of the two lines and whether or not two equations are equivalent (meaning their graph is the same line). They must know that two different parallel lines have equal slopes and never meet, and that two non-parallel lines meet at a unique point. When there is a unique solution, they need to be able to find it, graphically and algebraically.

Students grouped in pairs are given cards containing partial information about a linear relationship. Each card must be completed so it will have a table of values, an equation, and a graph. After the cards are completed and put in a poster, students are given several new cards saying "no common solutions," "infinitely many common solutions," or "one common solution when x = ___ , y = ___." Students need to connect several different pairs of cards of linear relationships in their posters with these new cards according as the system of equations formed by these pairs has zero, one, or infinitely many simultaneous solutions. When there is a unique solution, students must complete the new card by finding it.

Teaching and Assessment Resources

Anita Wah and Henri Picciotto. *Algebra: Themes, Tools, Concepts (ATTC).* Mountain View, California: Creative Publications, 1994. (Textbook available free for non-commercial use from www.mathedpage.org)

See Chapter 6: Making Comparisons

Section 6.3: Solving Linear equations, pp. 211–214

Section 6.4: Equations and Identities, pp. 215–216

Section 6.5: Graphical solutions, pp. 218–219

Section 6.6: Solving Techniques: Addition and Subtraction, pp. 221–223

Section 6.7: How Much More Than? How Many Times as Much, pp. 224–226

Section 6.8: Solving Techniques: Multiplication and Division, pp. 227–229

For an approach to solving linear equations in one unknown that does not use Algebra Tiles, see the following:

Kunihiko Kodaira et al. *Japanese Grade 7 Mathematics*. The University of Chicago School of Mathematics Project, 1992. (ISBN 0-936745-53-3)

See Chapter 4: Equations, pp. 76–94

Novice Tasks	Apprentice Tasks	Expert Tasks
	A09: Buying Chips and Candy	E04: Hot under the Collar

8.4 BIVARIATE DATA
10 DAYS

Displaying and exploring bivariate data is a powerful tool for modeling real-world relationships with math. Special attention should be given to developing the idea of slope in the context of a problem. Students often misunderstand that the slope is the rate of change of the dependent variable *per* unit increase of the independent variable. It is also important to emphasize the difference between *association* and *causation*. Students often assume that because two variables share a strong association that one must cause the other. This misconception cannot be corrected enough. Informal lines of best fit are often mistakenly calculated as the line that goes through the most points. Emphasize, instead, that the best fit line is the one that minimizes the sum of the distances of *all* points to itself.

Statistics and Probability 8.SP
Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

TEACHING AND ASSESSMENT RESOURCES FOR BIVARIATE DATA

Anita Wah and Henri Picciotto. *Algebra: Themes, Tools, Concepts (ATTC).* Mountain View, California: Creative Publications, 1994. (Textbook available free for non-commercial use from www.mathedpage.org)

See Chapter 12: Mathematical Modeling

Section 12.1: The U.S. Population 1890–1990, pp. 426–428

Section 12.2: The Median–Median Line, pp. 429–431

Novice Tasks	Apprentice Tasks	Expert Tasks
	A23: Scatter Diagram	

8.5 CRITICAL AREA 1
11 DAYS

This section of the course is designed so that students can pull together into a more coherent whole what they have learned about proportional relationships, linear equations in one and two unknowns, systems of linear equations, and bivariate data. It is an opportunity for students to make a connection between static linear equations on a graph and real-world statistical models. The line of best fit connects neatly to a linear equation and provides a model for a data distribution. This model can be used to build understanding and fluency with linear equations because it will lend meaning to *slope* and *y-intercept*.

Critical Area 1
Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing it fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.
Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

TEACHING AND ASSESSMENT RESOURCES

The Mathematics Department at the Phillips Exeter Academy provides set of mathematics problems for it mathematics courses. While this course has not been aligned to any of the Phillips Exeter Academy courses, the problems that can be found on the website below are an excellent resource: http://www.exeter.edu/documents/math1all.pdf.

http://map.mathshell.org/materials/index.php

http://www.MathEdPage.org/

http://illustrativemathematics.org/

8.6 FUNCTIONS
15 DAYS

It is a strength of the *CCSS* that it specifies a study of both linear functions (8.F.3) and linear equations in two unknowns (8.EE.5, 6) in the standards for Grade 8. Clearly, the *CCSS* is requiring students to study both. This signals that the *CCSS* is rightfully treating linear functions and linear equations in two unknowns as two distinct mathematical objects. There is no doubt that since there is significant overlap in what we ask students to do with linear functions and linear equations in two unknowns, some meshing of them will occur in their teaching. Still, it is not really fair to students to treat them as if they are synonyms. This might be jarring for some, but let us briefly share some of the benefits that students stand to gain if we treat them as different.

Students, for example, might understand that the solution to a linear equation in two unknowns is an infinite set of ordered pairs, which is simply the line that represents these ordered pairs. Students might also understand that a function rule in the form of $y = mx + b$, where x denotes the input and y denotes the output, generates a set of ordered pairs. Given the idea of a function (8.F1) students will come to understand that not all straight lines can represent a linear function. For example, the line that is represented by the linear equation $x = 3$ cannot represent a linear function because this rule does not assign each input to exactly one output.

Also, when students start to study high school mathematics they will realize that although it does make sense to talk about writing linear equations in two unknowns in various equivalent forms (standard form, point-slope form, and slope-intercept form), it does not make sense to talk about linear functions in the same way.

Functions 8.F
Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change*.
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line*.

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

[1]Function notation is not required in Grade 8

Formative Assessment Lessons

Solving a Real-Life Problem: Baseball Jerseys

This is a lesson about comparing options for buying jerseys at two companies. One company charges a fixed price per jersey. The other company charges a smaller fixed price per jersey plus an initial set-up fee. Therefore, for both companies, the price is a linear function of the number of jerseys ordered. Students need to decide which company to use for a specified number of jerseys (i.e., which company will cost less). They also need to find the range over which each company offers a better deal than the other, and the break-even point. Solutions can involve making a table or chart with several data points, solving a linear inequality (or solving a linear equation to find the break-even point, and then doing some reasoning or computations to find what company is cheaper over each side of this point), or graphing the total cost as a function of the number of jerseys. Interpreting this graph readily yields the break-even point and the range over which each company is cheaper. Students are also asked to find two possible pricing functions that would yield the same prescribed total cost for a package of jerseys, and they are asked how many jerseys would be in the package under each pricing function. They are also asked to consider how reasonable their pricing functions are.

In order to solve the tasks assigned, students need to be able to model the problem mathematically, interpret their models (tabular or graphical linear data), and solve linear equations or inequalities. Students are also asked to explain and critique each other's solutions, as well as four sample solutions that are provided for discussion.

Teaching and Assessment Resources

Novice Tasks	Apprentice Tasks	Expert Tasks
N04: Functions	A17: Linear Graphs	
	A19: Meal Out	

http://map.mathshell.org/materials/index.php

http://www.MathEdPage.org/

http://illustrativemathematics.org/

8.7 CRITICAL AREAS 1 AND 2
15 DAYS

This section provides students with the opportunity to solve problems that cut across the first five units of this course and deploy all eight mathematical practices. Thus section should be thought of as the opportunity for students to integrate and forge connections among the various aspects of mathematics.

> Critical Area 2
> Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

Critical Area 2 brings the general concept of a function and the particular concept of a linear function into the curricular mix. Here students are given the opportunity make connections across proportional relationships, linear equations in one and two unknown(s), systems of linear equations, statistics, the concept of a function, examples of linear functions, and examples of non-linear functions.

> Critical Area 1
> Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing it fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.
> Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

We would not want students to leave this section of the course with the idea that any quantitative relationship can be represented by a straight line or that any quantitative relationships is a proportional relationships. Therefore it is important that students be called upon to classify relationships as either linear or nonlinear or as either proportional or non-proportional.

FORMATIVE ASSESSMENT LESSONS

Modeling Situations with Linear Equations

This lesson involves modeling and interpreting situations involving four variable quantities, and fixing two at a time to see how the other variables depend on each other. This dependence is linear. Mathematically, the situations are identical, but they have different contexts. To give an idea, one of the contexts deals with the profit, p, made by selling a number of candles, n, at a unit price, u, when there is a fixed cost, k. Students need to graph p as a function of n if u and k are fixed. They also need to solve for each of the four variables in terms of the other three.

Students need to manipulate symbolic expressions to solve equations, make graphs, and find specific values. They need to interpret the meaning of expressions and values, such as the meaning of the x-intercept of *the* p vs. n function above, or the meaning of the expression $u \cdot n$.

Interpreting Distance–Time Graphs

In this lesson students are asked to interpret distance–time graphs, and match them with possible scenarios about a person's displacement from a fixed point (home) along a straight path (road). They also need to match these to different tables of values for time and distance from home. Students need to understand that the steepness or slope of the graph at a point is the speed at that moment, so that larger slopes correspond to faster movement. Also, they need to understand that a negative slope indicates movement toward home. The graphs are for the most part piecewise linear (there are two exceptions). For these graphs, students need to understand that each linear segment corresponds to a time interval over which the speed is constant. One of the graphs has a vertical line segment. Students need to realize that this graph cannot reflect any real situation, since a person cannot be at more than one place at a given moment.

TEACHING AND ASSESSMENT RESOURCES

The Mathematics Department at the Phillips Exeter Academy provides set of mathematics problems for it mathematics courses. While this course has not been aligned to any of the Phillips Exeter Academy courses, the problems that can be found on the website below are an excellent resource: http://www.exeter.edu/documents/math1all.pdf.

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8.8 RATIONAL AND IRRATIONAL NUMBERS
10 DAYS

Many students believe that $\frac{22}{7}$ is an irrational number, and this betrays a basic lack of understanding about rational and irrational numbers. It also evidences that students have no clear way of thinking about or evaluating simple mathematical statements. In answering the question, *Is* $\frac{22}{7}$ *irrational?*, many students resort to recall rather than thinking about the question (that is, they think of pi when they look at its fractional approximation and they recall that pi is irrational). When learning about rational and irrational numbers, students must be engaged in transforming numbers from one representation to another and in classifying numbers. Students should also fully understand the concept of having a rational approximation for an irrational number.

The Number System 8.NS
Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi 2$). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Formative Assessment Lessons

Repeating Decimals

In this lesson, students need to translate back and forth between fractions and their decimal representations. Emphasis is placed on repeating decimals. Students also need to understand what happens when these decimals are multiplied by powers of 10, and select the appropriate powers to set up simple linear equations in order to obtain the fraction with the same value as a given decimal.

Note: this method of setting up a simple equation that eliminates the infinitely repeating decimal cannot be formally justified at this stage, since a true justification requires working with infinite series. Still, the subtraction needs to be justified. This should probably be made clear, even if only in passing, while assuring students that the method does indeed work and can be justified.

Some of the work needs to be done without a calculator, while in other parts students are allowed the use of calculators. In small groups, students also need to match cards from three lists (and make a poster with the matches they find): one has fractions, one has decimals, and one has linear equations that would be useful to translate from the decimal to the fraction. Sometimes the third matching card of a triple is missing, and students need to create the missing card. Students are asked to explain their work to others and to critique each other's arguments and results.

TEACHING AND ASSESSMENT RESOURCES

Anita Wah and Henri Picciotto. *Algebra: Themes, Tools, Concepts (ATTC).* Mountain View, California: Creative Publications, 1994. (Textbook available free for non-commercial use from www.mathedpage.org)

See Chapter 11: Interpreting Ratios

Section 11.2: Decimals and Fractions, pp. 401–402
Section 11.3: Using Scientific Notation, pp. 403–405
Section 11.4: Using Large Numbers, pp. 406–407

8.9 TRANSFORMATIONS
10 DAYS

This content area calls for a practical hands-on approach, where students are occupied rotating, reflecting, translating, and dilating images. Shortcuts or rules are counterproductive because students need the experience to build their conceptual understanding.

Geometry 8.G
Understand congruence and similarity using physical models, transparencies, or geometry software.
1. Verify experimentally the properties of rotations, reflections, and translations:
1a. Lines are taken to lines, and line segments to line segments of the same length.
1b. Angles are taken to angles of the same measure.
1c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Many teachers use the term flips, slides, and turns to stand for reflections, translations, and rotations respectively. When working with English Language Learners (ELLs) such shorthand is usually counterproductive, making it actually hard for ELLs to learn the math.

TEACHING AND ASSESSMENT RESOURCES

Henri Piccotto. *Geometry Labs.* 1999. (*Geometry Labs* can be downloaded from www.mathedpage.org for non-commercial use. Henri Piccotto grants the teacher who downloads Geometry labs the right to reproduce materials for any non-commercial use.)

1. Angles

Lab 1.6: The Exterior Angle Theorem, pp. 11–12

Novice Tasks	Apprentice Tasks	Expert Tasks
	A04: Aaron's Designs	

8.10 PYTHAGOREAN THEOREM
10 DAYS

Many students believe that the Pythagorean theorem is simply $a^2 + b^2 = c^2$. Few understand that it has anything to do with area and fewer still seems to understand that it has to do with the area of the squares formed by the sides of a right triangle. To prevent students from skirting around the essence of this important theorem, teaching and learning must begin with a right triangle with a square on each side. This can easily be done on a geoboard or dot paper. When teaching and learning begins with area, students have the opportunity to understand the true nature of the Pythagorean theorem and can then apply it to determine information about lengths or distance.

Geometry 8.G
Understand and apply the Pythagorean theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Effective teaching of the Pythagorean theorem does not begin with computing with roots or exponents or with $a^2 + b^2 = c^2$. Effective teaching of the Pythagorean theorem begins with the theorem and its converse. Once these are established, students can deepen their understanding of them by using them to solve a mixed bag of problems that include finding the distance between two points on the coordinate plane, finding missing sides of a right triangle, and solving various other problems involving triangles. Generally, it is not effective to teach students how to solve various problem-types and then give them time to practice these methods. There is nothing wrong with practice as long as students are practicing how to think and how to formulate an approach to a problem.

Interestingly enough, the Pythagorean theorem can be used as a context for deepening the work specified in 8.EE 1 and 2 and consolidating the simple polynomial arithmetic work that began in Grades 6 and 7. For example, asking students to tackle the following problem may provide an opportunity for them to deepen their understanding of the Pythagorean theorem and their basic skill with polynomial arithmetic.

> 6. The hypotenuse of a right triangle is twice as long as the shortest side, whose length is m. In terms of m, what is the length of the intermediate side?
>
> (From Phillip Exeter Academy Mathematics 1 materials Page 88, available at http://www.exeter.edu/documents/math1all.pdf)

Formative Assessment Lessons

The Pythagorean Theorem: Square Areas

In this lesson, students explore and prove the Pythagorean theorem by considering the area of a square inscribed and tilted inside another square. Both squares have vertices on a lattice (grid paper), with consecutive horizontal and vertical dots separated evenly by intervals of 1 unit.

Students can compute the area of the inscribed square by different methods: using the area of right triangles, subtracting from the large square, dissecting the inscribed square, or rearranging congruent pieces. Pairs of students are subsequently asked to analyze all possible areas of lattice squares with a given tilt. For this, they need to collect and organize their data, find patterns, and share the results with the rest of the class. As a whole class, the results from the different pairs are collected to conclude that the areas are all sums of two squares of whole numbers. Students are asked to consider what areas can or cannot result from tilted lattice squares of any tilt (a question whose solution requires more sophistication than students can be expected to handle, but this is explicitly noted in the teacher materials with a brief discussion of the matter). Finally, they are led to a proof of the Pythagorean theorem, and hence to the length of the hypotenuse of a right triangle, by a method of rearranging the four congruent right triangles between the large and the inscribed square.

Students are given sample student work, which is not very detailed, and asked to make sense of the reasoning behind these sample methods, by providing the missing details. They need to collaborate and critique each other's work.

Teaching and Assessment Resources

Henri Piccotto. *Geometry Labs.* 1999. (*Geometry Labs* can be downloaded from www.mathedpage.org for non-commercial use. Henri Piccotto grants the teacher who downloads Geometry labs the right to reproduce materials for any non-commercial use.)

See Chapter 9: Distance and Square Root

Lab 9.2: The Pythagorean Theorem, pp. 123–124
Assessment Resources for Pythagorean Theorem 8.G

Novice Tasks	Apprentice Tasks	Expert Tasks
	A15: Jane's TV	

8.11 VOLUME OF CONES, CYLINDERS, AND SPHERES 10 DAYS

It is helpful for students to be able to reconstruct the formula for the volumes of cones and cylinders. Students should be given the opportunity to learn that regardless of the shape of the base of a cylinder, the formula for its volume is the area of its base multiplied by its perpendicular height. Similarly, regardless of the shape of the base of a cone, its volume is one-third of its area of its base multiplied by its perpendicular height.

> Geometry 8.G
> Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
> 9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems

Similarly, students should be given the opportunity to derive the formulas for the surface areas of various three-dimensional figures from their nets. For example, it is important that students can see that the surface area of a cylinder is found from adding the area of a rectangle to the area of two circles. It is also important for students to be able to derive the length of the rectangle from information such as the radius of the base of the cylinder.

FORMATIVE ASSESSMENT LESSONS

Making Matchsticks

In this lesson, students need to estimate how many matchsticks can be made from a pine tree. The dimensions of a matchstick are given in inches, and a sketch of the tree is provided with measurements (height and diameter at the base) in feet. Students are also provided with a sheet containing area and volume formulas for several 2-D and 3-D geometric objects. Students need to model the tree as a geometric object, use appropriate formulas, make and justify their estimates and calculations, and round their answers to a reasonable degree of accuracy.

They need to pay attention to units and how volume in particular is computed after a change of units, because in order to obtain an answer, they need to compare the volume of a matchstick to the volume of the tree in the same units.

Students need to analyze sample solutions and evaluate their own and each other's work. They need to work collaboratively and communicate with each other clearly. They need to be able to explain their solutions thoroughly, explicitly noting their estimates and justifying their formulas and computations.

TEACHING AND ASSESSMENT RESOURCES

http://map.mathshell.org/materials/index.php

http://www.MathEdPage.org/

http://illustrativemathematics.org/

8.12 CRITICAL AREAS 1, 2, AND 3
18 DAYS

This section lets students solve problems that cut across this entire course and deploy all eight mathematical practices. In this section students have the opportunity to integrate and forge connections among the various aspects of the course.

> Critical Area 3
> Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Students can be asked to graph the volume of liquid in a cylinder as a function of its height, and then analyze the parameter of the function in terms of the cylinder of water. Student might also be asked to say whether the relationship represents a proportional relationship and to explain why it does or does not.

> Critical Area 2
> Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

> Critical Area 1
> Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing it fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

TEACHING AND ASSESSMENT RESOURCES

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