Things to Know for Calculus

TRIGONOMETRY

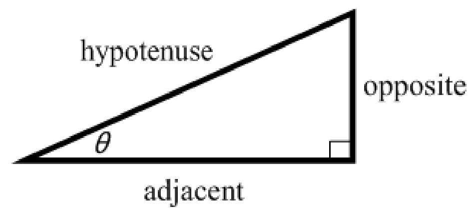
Trig Functions

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ | $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ | $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

Reciprocal Functions

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\text{hyp}}{\text{opp}}$ | $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{\text{hyp}}{\text{adj}}$ | $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\text{adj}}{\text{opp}}$

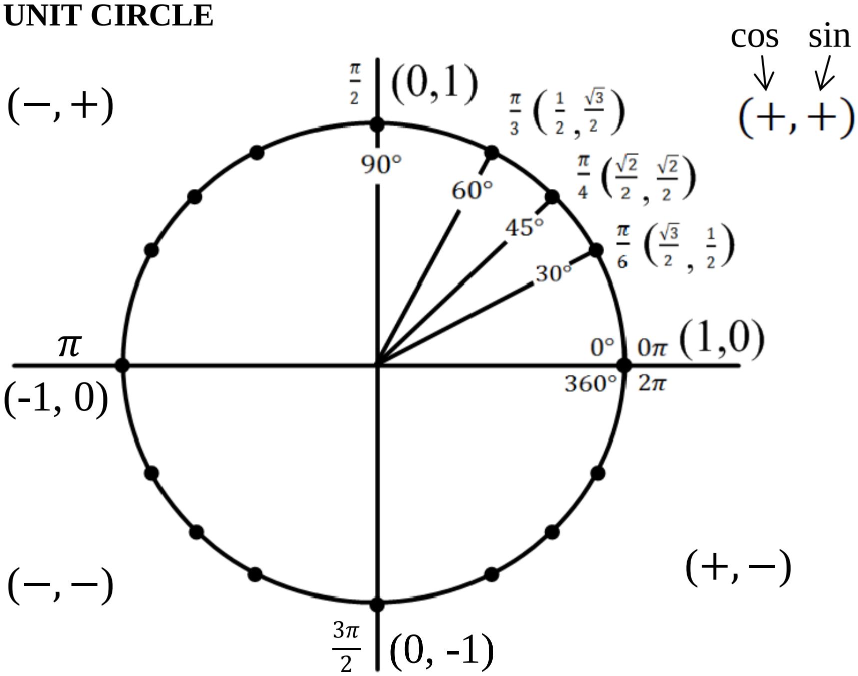
SOH-CAH-TOA



TEST ONLY USES RADIANS!

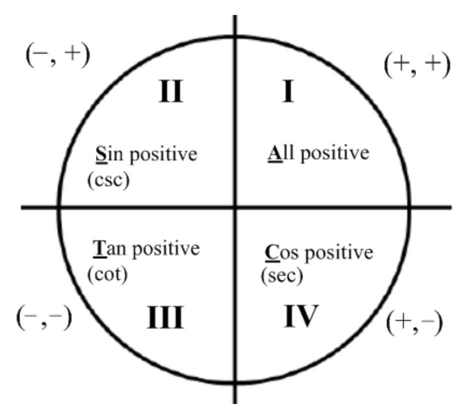
Must know trig values of special angles $0\pi, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ using Unit Circle or Special Right Triangles.

UNIT CIRCLE



To help remember the signs in each quadrant

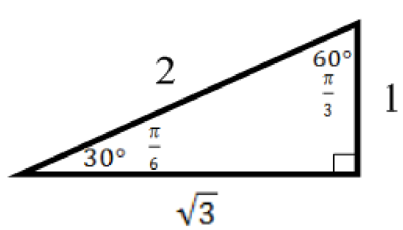
All **S**tudents **T**ake **C**alculus



SPECIAL RIGHT TRIANGLES

$30° - 60° - 90°$ Triangles

Which are $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$ Triangles

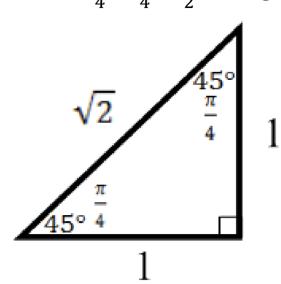


Find $\tan\left(\frac{\pi}{6}\right)$

$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$ simplify to $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$45° - 45° - 90°$ Triangles

Which are $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$ Triangles

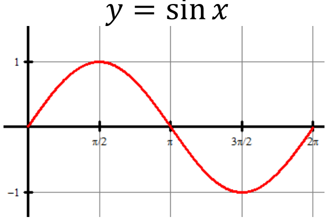


Find $\sin\left(\frac{\pi}{4}\right)$

$\sin\left(\frac{\pi}{4}\right) = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$ simplify to $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

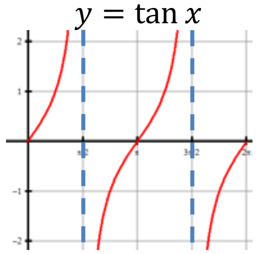
Graphs of trig functions

$y = \sin x$



$y = \cos x$

$y = \tan x$



Inverse Trig Function

$\sin^{-1}\theta$ is the same as $\arcsin\theta$

$\sin^{-1}\theta = \left(\frac{\sqrt{3}}{2}\right)$ means what angle has a sine value of $\frac{\sqrt{3}}{2}$

that means $\theta = \frac{\pi}{3} \pm 2\pi n$ *or* $\frac{2\pi}{3} \pm 2\pi n$

Since θ has infinite answers then it isn't a function. Bummer. To make it a function we define inverses like:

sin/csc and tan/cot use quadrant I and IV for inverses
cos/sec use quadrant I and II for inverses

So… $\theta = \frac{\pi}{3}$ because it is in the first quadrant

Trig Identities

There are a bunch, but you really only need to know Pythagorean Identity. $\mathbf{\sin^2 x + \cos^2 x = 1}$

Subtract $\sin^2 x$ to get $\cos^2 x = 1 - \sin^2 x$ or subtract $\cos^2 x$ to get $\sin^2 x = 1 - \cos^2 x$

Divide by $\sin^2 x$ to get $1 + \cot^2 x = \csc^2 x$ or divide by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$

GEOMETRY

FORMULAS

AREA

Triangle $= \frac{1}{2}bh$

Circle $= \pi r^2$

Trapezoid $= \frac{1}{2}(b_1 + b_2)h$

CIRCUMFERENCE

Circle $= 2\pi r$

SURFACE AREA

Sphere $= 4\pi r^2$

LATERAL AREA

Cylinder $= 2\pi rh$

VOLUME

Sphere $= \frac{4}{3}\pi r^3$

Cylinder $= \pi r^2 h$

Cone $= \frac{1}{3}\pi r^2 h$

Prism $= Bh$

Pyramid $= \frac{1}{3}Bh$

B is the area of the base

DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ALGEBRA

Linear Functions

Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept Form

(slope-intercept Form)

$$y = mx + b$$

Point Slope Form

$$y - y_1 = m(x - x_1)$$

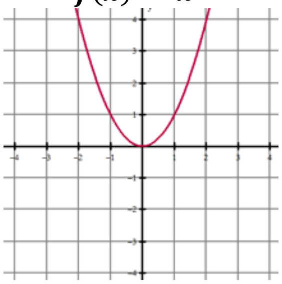
Parallel Lines
Have the same slope

Perpendicular Lines
Have the opposite reciprocal slopes

Functions

Quadratic Function

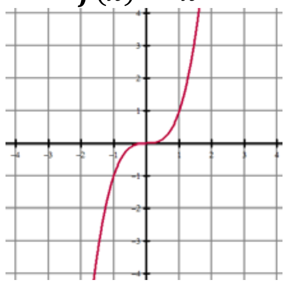
$f(x) = x^2$



$y = a(x-h)^2 + k$

Cubic Function

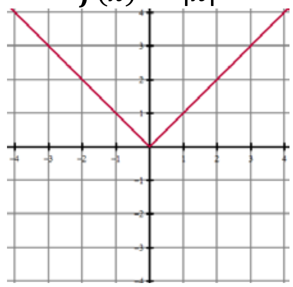
$f(x) = x^3$



$y = a(x-h)^3 + k$

Absolute Value

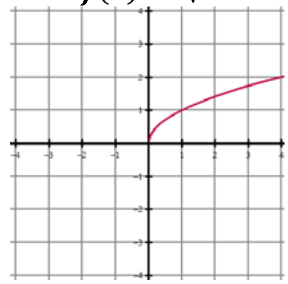
$f(x) = |x|$



$y = a|x-h| + k$

Square Root Function

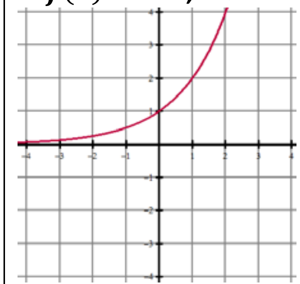
$f(x) = \sqrt{x}$



$y = a\sqrt{x-h} + k$

Exponential Function

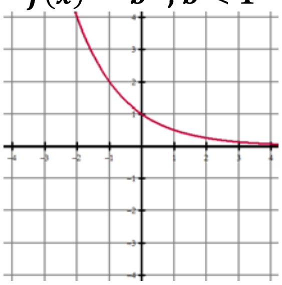
$f(x) = b^x \, , b > 1$



$y = a \cdot b^{(x-h)} + k$

Exponential Function

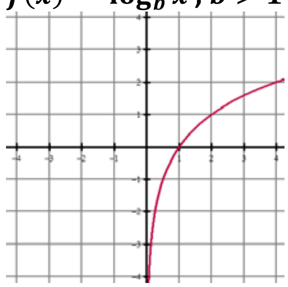
$f(x) = b^x \, , b < 1$



$y = a \cdot b^{(x-h)} + k$

Logarithmic Function

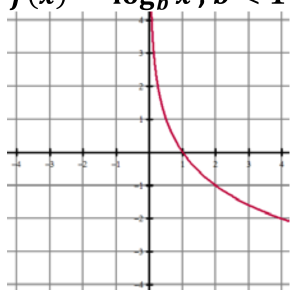
$f(x) = \log_b x \, , b > 1$



$y = a \log_b(x-h) + k$

Logarithmic Function

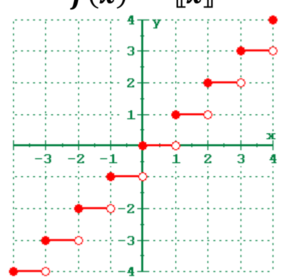
$f(x) = \log_b x \, , b < 1$



$y = a \log_b(x-h) + k$

Greatest Integer

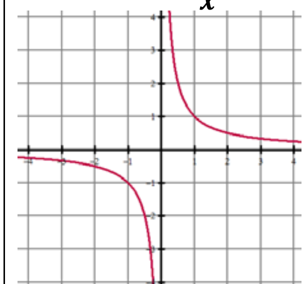
$f(x) = [\![x]\!]$



$y = a[\![x-h]\!] + k$

Rational Function

$f(x) = \frac{1}{x}$



$y = \frac{a}{x-h} + k$

Translations

All functions move the same way!

Given the parent function $y = x^2$

Move up 4	Move down 3	Move left 2	Move right 1	Move left 2 and down 3
$y = x^2 + 4$	$y = x^2 - 3$	$y = (x+2)^2$	$y = (x-1)^2$	$y = (x+2)^2 - 3$

To flip (reflect) the function vertically $y = -x^2$
To flip (reflect) the function horizontally $y = (-x)^2$

So $f(x) = -\sqrt{x-3} + 1$ is a square root function reflected vertically, shifted right 3 and up 1

Notation

Notice open parenthesis ( ) versus closed [ ]

Inequality		Interval
$-3 < x \le 5$	$\longleftrightarrow$	$(-3,5]$
$-3 \le x \le 5$	$\longleftrightarrow$	$[-3,5]$
$-3 < x < 5$	$\longleftrightarrow$	$(-3,5)$
$-3 \le x < 5$	$\longleftrightarrow$	$[-3,5)$

Infinity is always open parenthesis

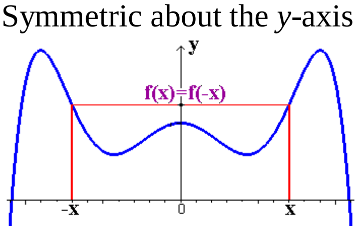
Inequality		Interval
$x < 3$	$\longleftrightarrow$	$(-\infty, 3)$
$x \le 3$ or $x > 5$	$\longleftrightarrow$	$(-\infty, 3](5, \infty)$
$x \ne 3$	$\longleftrightarrow$	$(-\infty, 3)(3, \infty)$
all Real numbers	$\longleftrightarrow$	$(-\infty, \infty)$

Even and Odd Functions

EVEN

$f(-x) = f(x)$

Symmetric about the y-axis

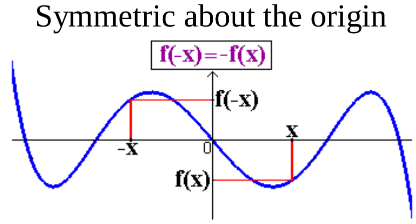


Even Function

ODD

$f(-x) = -f(x)$

Symmetric about the origin



Odd Function

Domain and Range
Domain = all possible x values
Range = all possible y values

Algebraically
You can't divide by zero
You can't square root a negative

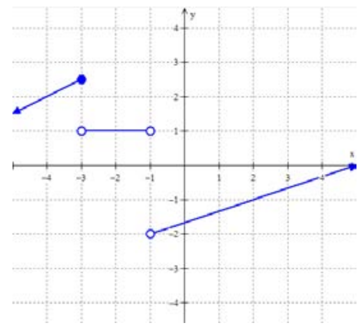
$y = \sqrt{2x+5}$

D: $[-\frac{5}{2}, \infty)$

$$y = \frac{x^2-1}{x^2+7x+12}$$

D: $(-\infty,-4)(-4,-3)(-3,\infty)$

Graphically
Just look at it



D: $(-\infty,-1)(-1,5]$

R: $(-\infty, 2.5]$

Finding zeros

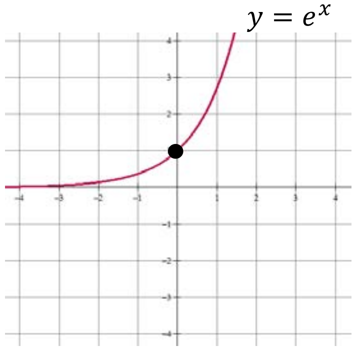
Must be able to factor and use the quadratic formula: $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Special products
Sum of cubes: $a^3 + b^3 = (a+b)(a^2-ab+b^2)$
Difference of cubes: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

Exponential and Logarithmic Properties
The exponential function b^x of base b is one-to-one which means it has an inverse which is called the logaritmic function of base b or logarithm of base b which is denoted $\log_b x$ which reads "the logarithm of base b of x" or "log base b of x". So…

BASE e

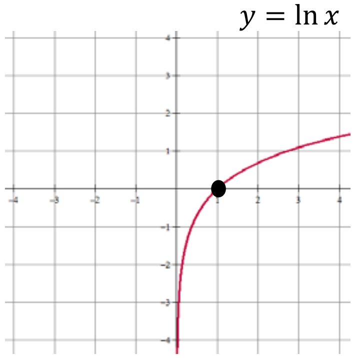
$y = e^x$



KNOW!
$e^0 = 1$

Horizontal asymptote at
$y = 0$

$y = \ln x$



KNOW!
$\ln 1 = 0$

Vertical asymptote at
$x = 0$

$$y = \log_b x \longleftrightarrow x = b^y$$

Exponential		Logarithmic
$b^x b^y = b^{x+y}$	Product Rule	$\log_b xy = \log_b x + \log_b y$
$\frac{b^x}{b^y} = b^{x-y}$	Quotient Rule	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
$(b^x)^y = b^{xy}$	Power Rule	$\log_b x^y = y\log_b x$
$b^{-x} = \frac{1}{b^x}$		$\log_b \left(\frac{1}{x}\right) = -\log_b x$
$b^0 = 1$		$\log_b 1 = 0$
$b^1 = b$		$\log_b b = 1$
	Change of Base	$\log_b x = \frac{\log_c x}{\log_c b}$
	Natural Log	$\log_e x = \ln e$
	Common Log	$\log_{10} x = \log x$