Name_____ Date_____

Pre-Calculus Summer Work

This packet is meant to provide an opportunity for the incoming Pre-Calculus students to review concepts from their previous courses including Algebra I, Geometry, and Algebra II.

Certain concepts that you have been taught over the previous years are assumed to be mastered. If you do not have these skills, you will find that you consistently get problems incorrect next year.

This summer packet is intended for you to brush up and possibly relearn these topics. Rather than give you a textbook to remind you of the formulas and techniques necessary to solve the following problems, we have listed a few websites that have full instructions on the techniques. If and when you are unsure of how to attempt these problems, use these websites. This will be the focus throughout next year as we examine the mathematical relationships between topics numerically, algebraically, and graphically. Success on your assessments throughout next year will greatly depend on whether you can effectively link these skills.

Unless specifically noted, all work in this packet should be done without a calculator and all final answers should be exact values. This packet is to be completed by the first day back to school in the fall. You may be tested on this material in the first few weeks of class.

Wait until at least mid-summer to begin this packet. If you do a few concepts a day, the whole packet will take you about a week to complete. You will be held responsible for understanding these concepts and your teacher will check for completion and assess your understanding.

Good Luck!

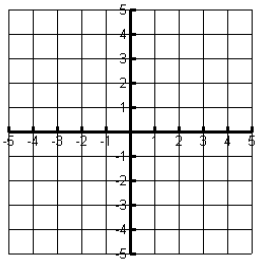
Sincerely,
Your Amazing Pre-Calculus Teachers

Pre-Calculus Resources In alphabetical order:

Cool Math	coolmath.com
Just Math Tutorials	patrickjmt.com
Khan Academy	khanacademy.org
Math by Fives	mathbyfives.com
Math TV	mathtv.com
Paul's Online Math Notes	tutorial.math.lamar.edu
Purple Math	purplemath.com
Wolfram Alpha	wolframalpha.com
Youtube	youtube.com

Graphs of Algebra 2: Parent Functions and key characteristics

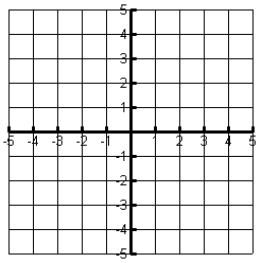
Linear: $y = x$



$y = mx + b$

$Ax + By = C$

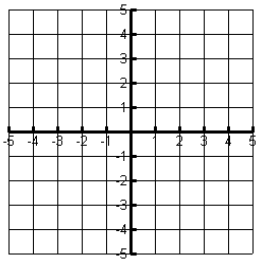
Quadratic: $y = x^2$



Translated: $y = a(x - h)^2 + k$

Vertex: (h, k)

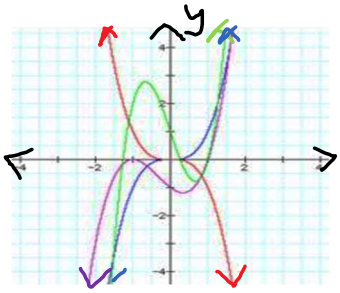
Cubic: $y = x^3$



Translated: $y = a(x - h)^3 + k$

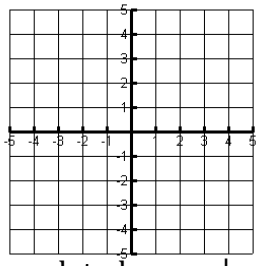
Point of Inflection: (h, k)

Polynomials: Not one 'parent' function. Here are key ideas:



Domain and Interval of Continuity: $(-\infty, \infty)$
Leading Term: Contains leading coefficient
(positive: $\lim_{x\to\infty} f(x) = \infty$ and negative: $\lim_{x\to\infty} f(x) = -\infty$
Degree: even – $\lim_{x\to\pm\infty} f(x)$ are equal, odd – $\lim_{x\to\pm\infty} f(x)$ are opposites. Also indicates the maximum number of curves (at most one less than degree)
Standard form: Identify the leading term and use factoring, synthetic division or the calculator to determine the x-intercepts/multiplicity.
Factored form: Determine the leading term, x-intercepts/multiplicity to graph.

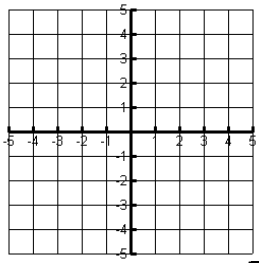
Absolute Value: $y = |x|$



Translated: $y = a|x - h| + k$

Vertex: (h, k)

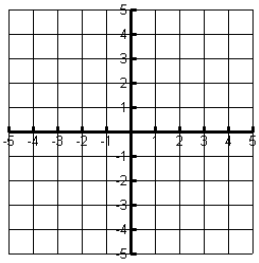
Square Root: $y = \sqrt{x}$



Translated: $y = a\sqrt{x - h} + k$

Starting point: (h, k)

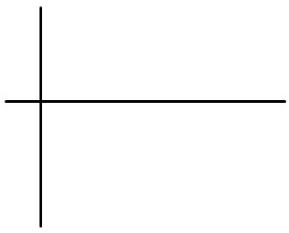
Exponential Growth: $y = b^x$



Growth: $b > 1$

Translated $y = ab^{x-h} + k$
Horizontal Asymptote: $y = k$

Sine: $y = \sin(x)$

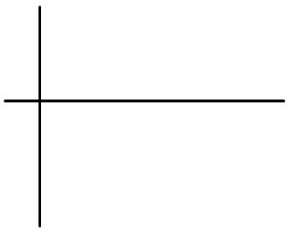


Translated: $y = A\sin(Bx) + D$

Amplitude: $|A|$

Period: $\frac{2\pi}{B}$

Cosine: $y = \cos(x)$

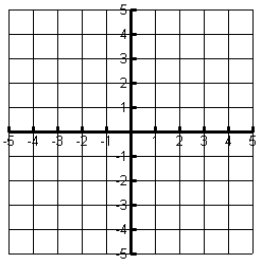


Translated: $y = A\cos(Bx) + D$

Amplitude: $|A|$

Period: $\frac{2\pi}{B}$

Exponential Decay: $y = b^x$



Decay: $0 < b < 1$

Translated $y = ab^{x-h} + k$

Horizontal Asymptote: $y = k$

1. Simplify the following expressions completely.

a. $2\left(\frac{2}{3}\cdot\frac{4}{8}\right)$ b. $\frac{5}{3}\div\frac{10}{7}$ c. $\dfrac{\frac{1}{4}-\frac{2}{3}}{\frac{3}{4}-\frac{5}{3}}$

2. Simplify the following expressions completely, if possible. Final answers should contain no negative exponents. Assume all variables represent positive values.

a. $\left(\frac{x^3y^5}{x^4y^9}\right)^2$ b. $\left(x^2y^{1/3}z^{-2}\right)^3$

3. Simplify the following expressions completely, if possible. Assume all variables represent positive values.

a. $\sqrt{32}$ b. $\sqrt{2}+3\sqrt{2}+4\sqrt{2}$ c. $\sqrt[4]{16x^8y^5}$

4. Rationalize the denominator of the following expressions. Then, simplify.

a. $\frac{7}{\sqrt{14}}$ b. $-\frac{1}{2\sqrt{5}}$ c. $\frac{2}{1-\sqrt{3}}$

5. Factor the following expressions completely, if possible.

a. x^2-100 b. x^2+8 c. $9x^6-4$

d. $5x^3-3x^2-45x+27$ e. $3x^2-10x+8$ f. $6x^2-13x-5$

g. x^6-3x^3-70 h. $6x^2-11x-7$ i. $18x^2-50$

6. Express the following inequalities in proper interval notation.

a. $-1 < x < 5$

b. $-2\pi \leq x < \frac{\pi}{2}$

c. $x \leq -\frac{7}{2}$

d. $x \geq -27.3$

e. $x < 5$ or $x \geq 9$

f. $x = 0$ or $2 < x \leq 10$

7. Solve the following equations for x. Give only exact values of all real solutions.

a. $x - 3\left(\frac{1}{3}x - (2 - x)\right) = 7$

b. $x^2 - 28 = -3x$

c. $18x^4 - 6x^2 = -3x^3$

d. $2x^2 + 4x = 5$

e. $x(2x - 13) = -6$

f. $x^2 - \sqrt{5} = 0$

g. $2|x + 1| = 16$

h. $x^4 - 2x^2 - 15 = 0$

i. $\sqrt{x - 2} = x - 2$

j. $(x - 3)^2 + 5 = 0$

8. Solve by completing the square. Give only exact values.

a. $x^2 - 4x - 7 = 0$

b. $2x^2 + 8x - 16 = 0$

9. Solve the following equations using your graphing calculator. Round your solutions to the nearest thousandth.

$x^4 - 2x^3 - 5x^2 - x - 7 = 0$

10. Given the points $(2, 3)$ and $(1, 5)$, find:

a. The distance between them.

b. The midpoint of the segment formed by them.

c. The equation of the line that passes through them in point-slope form.

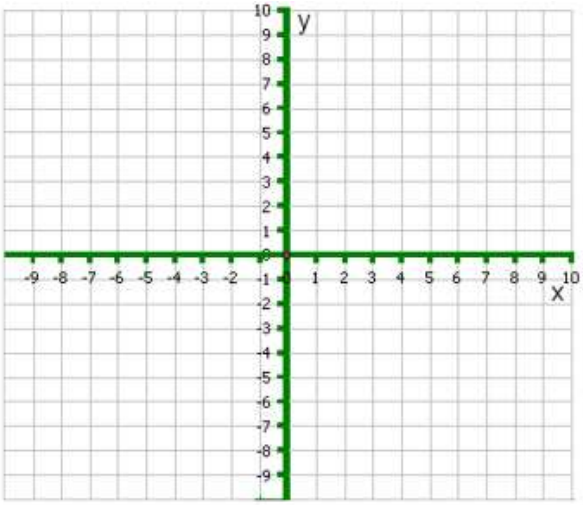
11. Given the function $f(x) = -2\sqrt{x + 4} + 2$, find:

a. The x- and y-intercepts for the function.

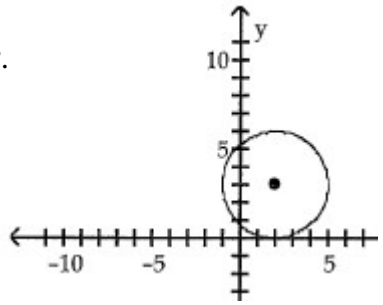
b. The domain and range of the function.

c. The exact value of $f(-2)$ and $f(3x - 4)$.

d. List the transformation required to graph the function, then draw a sketch of the graph.



12. Write the standard form for the equation of the circle below.

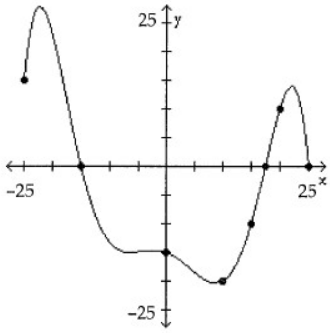


13. Find the center and radius of the circle $x^2 + y^2 + 14x + 12y + 21 = 0$ by completing the square.

14. Given $f(x) = 4x - 5$ and $g(x) = x^2 + 1$, find:

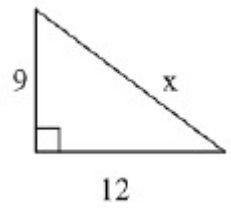
a. $(f + g)(x)$ b. $(g - f)(x)$ c. $(f \cdot g)(x)$ d. $f(g(0))$

15. Given the graph of $f(x)$ below, determine what values of x $f(x) > 0$.

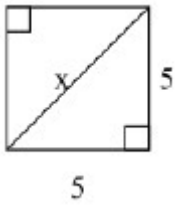


16. For each of the triangles, find the exact value of x.

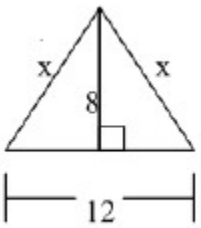
a.



b.

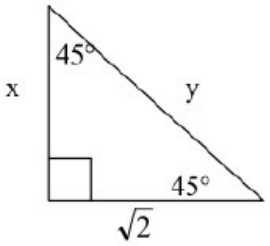


c.

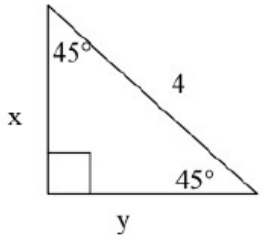


17. For each of the triangles, find the exact value of x and y.

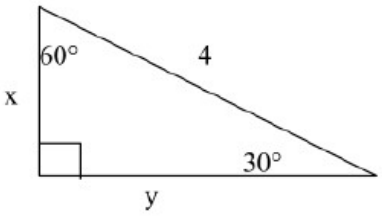
a.



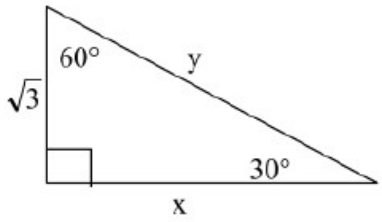
b.



c.



d.



18. Find the exact value of each of the following functions.

a. $\sin 60°$

b. $\tan \frac{2\pi}{3}$

c. $\cos(-240°)$

d. $\tan 225°$

e. $\cos \frac{7\pi}{4}$

f. $\sin(-\pi)$

19. **CALCULATOR:** A college student earned $7300 during summer vacation working as a waiter in a popular restaurant. The student invested part of the money at 9% and the rest at 7%. If the student received a total of $585 in interest at the end of the year, how much was invested at 9%?

20. **CALCULATOR:** You throw a ball into the air from a height of 4.2 feet with an initial velocity of 24 feet per second. Use the vertical motion model, $h(t) = -16t^2 + v_0 t + h_0$, where v_o is the initial velocity in feet/second and h_o is the initial height in feet, to calculate the maximum height of the ball. Round your answer to the nearest thousandth if necessary.

21. **CALCULATOR:** Using the model information from the previous problem, how long will it take for the ball to reach the ground? Round your answer to the nearest thousandth if necessary.

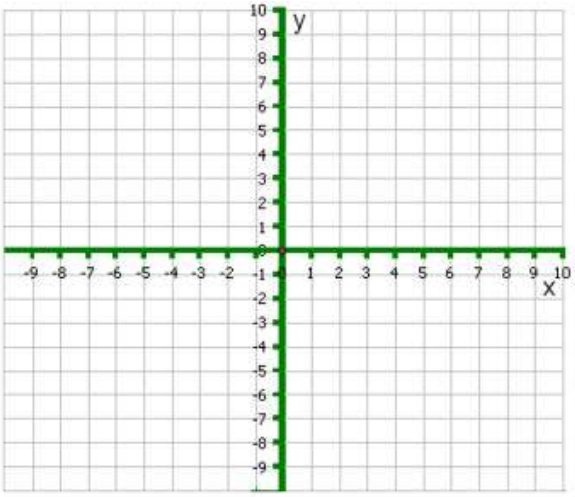
22. Answer the following about the graphs of $f(x)$. Then sketch the graph using the zeros, multiplicity, and end behavior.

a. $f(x) = 2x^4 - 7x^3 + x^2 + 16x - 12$

x-intercepts (state multiplicity):

y-intercept:

End behavior:

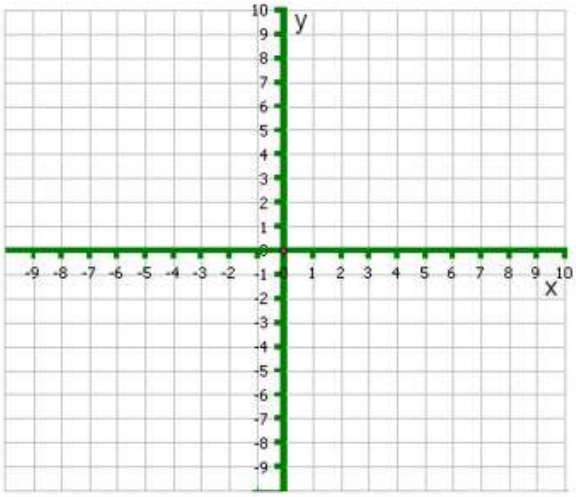


b. $f(x) = 3x^2(x-5)^3(x+4)^4$

x-intercepts (state multiplicity):

y-intercept:

End behavior:



23. Evaluate the following expressions.

a. $(-8)^{\frac{1}{3}}$ b. $\sqrt[10]{25^2}$ c. $9^{3/2}$ d. $16^{1/4}$ e. $8^{\frac{5}{3}}$

24. Factor the polynomial using the given factor and synthetic division. Then, state all zeros.

a. $x^3 + 7x^2 + 4x - 12; x + 6$

b. $x^4 - x^3 - 7x^2 + 5x + 10; (x + 1)$

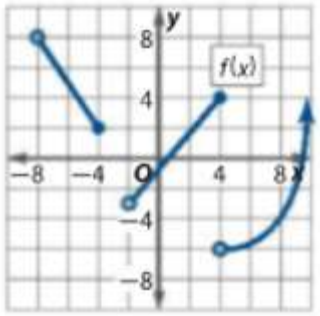
25. Divide using synthetic division.

a. $(y^3 + y^2 - 10) \div (y + 3)$

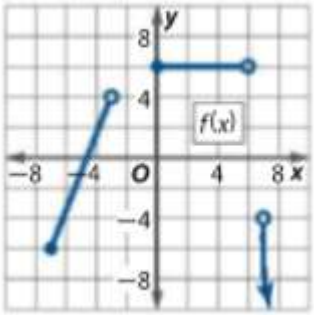
b. $(n^4 - n^3 - 10n^2 + 4n + 24) \div (n + 2)$

26. For the following functions, state the domain, range, interval of continuity and intervals of increasing, decreasing, and constant.

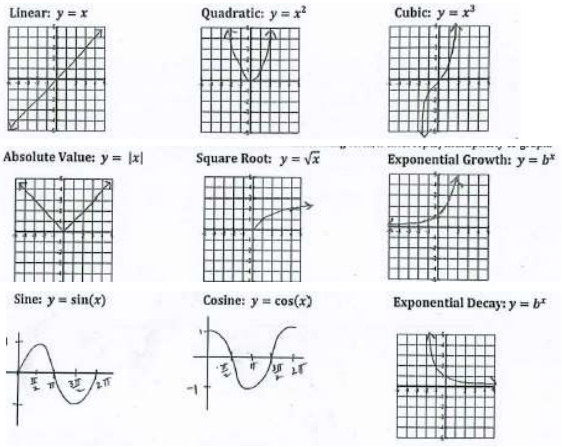
a.



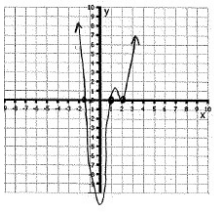
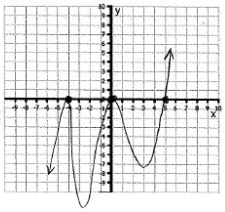
b.



Answers:



1. a. $\frac{2}{3}$ b. $\frac{7}{6}$ c. $\frac{5}{11}$
2. a. $\frac{1}{x^2y^8}$ b. $\frac{x^6y}{z^6}$
3. a. $4\sqrt{2}$ b. $8\sqrt{2}$ c. $2x^2y^4\sqrt{y}$
4. a. $\frac{\sqrt{14}}{2}$ b. $-\frac{\sqrt{5}}{10}$ c. $-1-\sqrt{3}$
5. a. $(x+10)(x-10)$
   b. not factorable (prime)
   c. $(3x^3-2)(3x^3+2)$
   d. $(5x-3)(x+3)(x-3)$
   e. $(3x-4)(x-2)$
   f. $(3x+1)(2x-5)$
   g. $(x^3-10)(x^3+7)$
   h. $(3x-7)(2x+1)$
   i. $2(3x+5)(3x-5)$
6. a. $(-1,5)$
   b. $\left[-2\pi,\frac{\pi}{2}\right)$
   c. $\left(-\infty,-\frac{7}{2}\right]$
   d. $[-27.3,\infty)$
   e. $(-\infty,5)\cup[9,\infty)$
   f. $[0]\cup(2,10]$
7. a. $x=-\frac{7}{3}$ b. $x=-7,4$ c. $x=0,-\frac{2}{3},\frac{1}{2}$
   d. $x=-1\pm\frac{\sqrt{56}}{4}$ e. $x=\frac{1}{2},6$ f. $x=\pm\sqrt[4]{5}$
   g. $x=7,-9$ h. $x=\pm\sqrt{5}$ i. $x=3,2$
   j. *no real solutions*.
8. a. $x=2\pm\sqrt{11}$ b. $x=-2\pm2\sqrt{3}$
9. $x=-1.770,3.610$
10. a. $d=\sqrt{5}$ b. $\left(\frac{3}{2},4\right)$ c. $y-3=-2(x-2)$
11. a. xint: $x=-3$ *or* $(-3,0)$
    yint: $y=-2$ *or* $(-2,0)$
    b. D: $[-4,\infty)$ R: $(-\infty,2]$
    c. $f(-2)=-2\sqrt{2}+2, f(3x-4)=-2\sqrt{3x}+2$
    d. reflext over xaxis, vertical stretch by 2, left 4, up 2
12. $(x-2)^2+(y-3)^2=9$
13. Center: $(-7,-6)$ radius: $r=8$
14. a. x^2+4x-4 b. x^2-4x+6
    c. $4x^3-5x^2+4x-5$ d. -1
15. $[-25,-15)\cup(-17.5,25)$
16. *a.* $x=15$ b. $x=5\sqrt{2}$ c. $x=10$
17. a. $x=\sqrt{2}, y=2$ b. $x=2\sqrt{2}, y=2\sqrt{2}$
    c. $x=2, y=2\sqrt{3}$ d. $x=3, y=2\sqrt{3}$
18. a. $\frac{\sqrt{3}}{2}$ b. $-\sqrt{3}$ c. $\frac{-1}{2}$ d. 1 e. $\frac{\sqrt{2}}{2}$ f. 0
19. \$3700
20. 13.2 ft.
21. 1.658 seconds
22. a. xint: $x=1\ mult.1, x=2\ mult.2, x=\frac{-3}{2}\ mult\ 1$
    yint: $y=-12$
    EB: $\lim_{x\to\pm\infty} f(x)=\infty$
    b. xint: $x=0\ mult.2, x=5\ mult.3, x=-4\ mult\ 4$
    yint: $y=0$
    EB: $\lim_{x\to-\infty} f(x)=-\infty, \lim_{x\to\infty} f(x)=\infty$

23. a. -2 b. $\sqrt[5]{25}$ c. 27 d. 2 e. 32
24. a. $x=-2,1,-6$ b. $x=\pm\sqrt{5},2,-1$
25. a. $y^2-2y+6-\frac{28}{y+3}$ b. $n^3-3n^2-4n+12$
26. a. D: $(-8,-4]\cup(-2,\infty)$ R: $(-6,\infty)$
    Continuity: $(-8,-4)\cup(-2,4)\cup(4,\infty)$
    Inc: $(-2,4)\cup(4,\infty)$ Dec: $(-8,-4)$
    Constant: None
    b. D: $(-7,-3]\cup[0,6)\cup(7,\infty)$ R: $(-\infty,4)\cup[6]$
    Continuity: $(-7,-3)\cup(0,6)\cup(7,\infty)$
    Inc: $(-7,-3)$ Dec: $(7,\infty)$
    Constant: $(0,6)$