Honors Calculus Summer Prep

Topics from Algebra and Pre-Calculus

- The following topics represent expected knowledge for Honors Calculus
- Complete the problems on separate sheet paper.
- Access your earlier course work notes and online help when needed.
- Due the first day of your class meeting

Name _____

COMPLEX FRACTIONS

Simplify each of the following complex fractions.

1. $\dfrac{2+\dfrac{2}{x}}{3-\dfrac{3}{x^2}}$

2. $\dfrac{x-\dfrac{1}{3}}{3-\dfrac{1}{x}}$

3. $\dfrac{\dfrac{1}{x}-\dfrac{1}{5}}{\dfrac{1}{x^2}-\dfrac{1}{25}}$

4. $\dfrac{\dfrac{1}{3}+\dfrac{1}{3x}}{\dfrac{1}{x}+\dfrac{1}{3}}$

5. $\dfrac{\dfrac{2}{x}-2}{\dfrac{1-x}{x}}$

6. $\dfrac{\dfrac{1}{4x^2}-1}{2-\dfrac{1}{x}}$

7. $\dfrac{2+\dfrac{4}{x-2}}{\dfrac{2}{x-2}}$

8. $\dfrac{\dfrac{3}{x}-\dfrac{4}{x^2}}{\dfrac{2}{x^3}+\dfrac{5}{x^2}}$

FACTORING

Factor each of the following completely:

1. $3x^2+21xy-54y^2$
2. $3x^2 + 24x + 48$
3. $3m^2-18m+27$
4. $8x^3-200x$
5. $3x^3 + 78x^2 + 507x$
6. $x^6 - 16x^4$
7. $4x^3 - 8x^2 - 25x + 50$
8. $8x^3 + 27$
9. $x^4 - 1$
10. $-24x^3y^3+16x^4y^2-8x^5y^3$
11. $x^3 + 4x + x^2 + 4$
12. $2x^3 + x^2 + 8x + 4$
13. $15x^3 + 5x^2 + 3x + 1$
14. $20n^3 + 12n^2 + 25n + 15$
15. $9p^3 + 3p^2 + 15p + 5$
16. $6x^3 + 10x^2 + 3x + 5$
17. $1000 + 27a^3$
18. $s^3 - 64$
19. $y^3 + 125$
20. $3a^3 - 81x^3$
21. $(2x + 3)^3 - y^3$
22. $r^3 + 8b^3$

PROPERTIES OF LOGARITHMS

Expand each logarithm completely.

1) $\ln \dfrac{1}{\sqrt{t}}$ 2) $\log_3 11x$ 3) $\log_3 \sqrt[3]{x+1}$ 4) $\log_4 \sqrt{3x}$

5) $\log_2 \dfrac{z}{17}$ 6) $\ln \dfrac{5}{x-2}$ 7) $\log_5 \sqrt{xy}$ 8) $\ln \dfrac{x^2 y}{z^7}$

Condense each logarithm into one logarithm.

1) $\log_5 2 + \log_5 3 + \log_5 4$ 2) $\log_2 48 - \dfrac{1}{3}\log_2 27$ 3) $\dfrac{2}{3}\ln 8 - 2\ln 5$

4) $2\log_{10}\sqrt{x} + 3\log_{10} x^{1/3}$ 5) $\dfrac{1}{2}\left(\log M - \log N - \log P\right)$ 6) $5\left(\log A + \log B\right) - 2\log C$

7) $\log_8 \sqrt{80} - \log_8 \sqrt{5}$ 8) $\dfrac{1}{2}\ln 25 + \ln 2$ 9) $\log_2 5 + \log_2\left(x^2 - 1\right) - \log_2\left(x - 1\right)$

RATIONAL EXPRESSIONS

Simplify:

1) $\dfrac{x^3 - 9x}{x^2 - 7x + 12}$ 2) $\dfrac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$ 3) $\dfrac{9 - x^{-2}}{3 + x^{-1}}$

Perform the indicated operation on the rational expression

1. $\dfrac{5}{8} - \dfrac{3}{8x}$
2. $\dfrac{2}{4x+12} + \dfrac{7}{x+3}$
3. $\dfrac{7}{x+2} - \dfrac{4}{x-5}$
4. $\dfrac{3}{y} + \dfrac{2}{y^4}$
5. $\dfrac{5}{4x^2} + \dfrac{3}{2x}$
6. $\dfrac{2}{x-3} - \dfrac{1}{x+7}$
7. $\dfrac{7}{3x} - \dfrac{2}{5}$
8. $\dfrac{3}{2x+6} + \dfrac{4}{6x+18}$
9. $\dfrac{3}{x+2} + \dfrac{4}{x-7}$
10. $\dfrac{1}{y+3} + \dfrac{4}{y^2+4y+3}$
11. $\dfrac{2}{5x} - \dfrac{3}{10x^3}$
12. $\dfrac{2x+3}{5x-30} - \dfrac{3x+4}{x-6}$

RADICALS AND RATIONAL EXPONENTS

REVIEW: n^{th} Root: If $b^n = a$, then *b* is the n^{th} root of *a*. This is written $\sqrt[n]{a} = b$. *n* is called the **index** of the radical. *a* is called the **radicand**. Roots as Rational Exponents: The n^{th} root, $\sqrt[n]{a}$, can be written as an exponent $a^{\frac{1}{n}}$. $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ - Notice the placement of the *m* and *n*. The root index is the denominator and the exponent is the numerator.

Rationalize the denominator:

1) $\dfrac{2}{\sqrt{3}+\sqrt{2}}$ 2) $\dfrac{2}{1-\sqrt{5}}$ 3) $\dfrac{1}{1+\sqrt{3}-\sqrt{5}}$

Evaluate.

1) $(-27)^{1/3}$ 2) $(-16)^{1/4}$ 3) $16^{-3/4}$

Write in exponential form.

1) $\sqrt[5]{a^2}$ 2) $(\sqrt[6]{a})^5$

Write as a radical expression with a smaller index.

1) $\sqrt[4]{49}$ 2) $\sqrt[14]{x^7}$

Simplify.

1) $\sqrt[4]{x} \bullet \sqrt{x}$ 2) $\dfrac{\sqrt[3]{x^2}}{\sqrt[4]{x}}$ 3) $\sqrt[3]{2} \bullet \sqrt[3]{5}$

4) $\dfrac{\left(2a^2\right)^3}{b}$ 5) $\sqrt{9ab^3}$ 6) $\dfrac{a(2/b)^3}{3/a}$

7) $\dfrac{ab-a}{b^2-b}$ 8) $\dfrac{a^{-1}}{b^{-1}\sqrt{a}}$ 9) $\left(\dfrac{a^{2/3}}{b^{1/2}}\right)^2\left(\dfrac{b^{3/2}}{a^{1/2}}\right)$

LITERAL EQUATIONS

1. Solve for a.	2. Solve for v.	3. Solve for a.
$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$	$tv+tv=s$	$V=2(ab+bc+ca)$
4. Solve for positive r.	5. Solve for d.	6. Solve for x.
$A=2\pi r^2+2\pi rh$	$2x-2yd=y+xd$	$\frac{2x}{4\pi}+\frac{1-x}{2}=0$

LIBRARY OF FUNCTIONS AND END BEHAVIOR OF POLYNOMIALS

Describe the end behavior of the graphs of the following functions. Decide if $f(x)\to+/-\infty$ as $x\to+/-\infty$. You do NOT need to completely distribute the ones that are factored—all that matters is the term with the highest power of *x*.

	as $x\to\infty$, $f(x)\to$?	as $x\to-\infty$, $f(x)\to$?
1. $f(x)=-2x^3+6x-11$		
2. $f(x)=x^4-5x^3-x^2+2x+1$		
3. $f(x)=2(x-1)(x+3)(x-5)$		
4. $f(x)=-x^2(x+7)(x-3)$		
5. $f(x)=3x^2(x-1)^3(x+2)$		

Graph each of the following parent functions WITHOUT a calculator:

6. $f(x)=x^2$

7. $f(x)=x^3$

8. $f(x)=\sqrt{x}$

9. $f(x)=\sqrt[3]{x}$

10. $f(x)=\frac{1}{x}$

11. $f(x)=\frac{1}{x^2}$

LINEAR EQUATIONS

Determine the equations of the following lines:

1) the line through $(-1, 3)$ and $(2, -4)$;
2) the line through (0, 6) with slope -4;
3) the line through $(-1,2)$ and perpendicular to the line $2x - 3y + 5 = 0$;
4) the line through $(2, 3)$ and the midpoint of the line segment from $(-1,4)$ to $(3,2)$.

DOMAIN AND RANGE

Find the domain of the functions:

1) $f(x) = \dfrac{3x-1}{\sqrt{x^2+x-2}}$ 2) $f(x) = \dfrac{5x-3}{2x+1}$ 3) $f(x) = 7$

FUNCTIONS

Simplify: $\dfrac{f(x+h)-f(x)}{h}$**, where:**

1) $f(x) = 2x+3$ 2) $f(x) = \dfrac{1}{x+1}$ 3) $f(x) = x^2$

INVERSE FUNCTIONS

Find the inverse of the function:

1) $f(x) = 2x+3$ 2) $f(x) = \dfrac{x+2}{5x-1}$ 3) $f(x) = x^2 + 2x + 1,\ x > 0$

ASYMPTOTES

Determine the equations for the horizontal and vertical asymptotes:

1) $f(x) = \dfrac{4x-2}{2x+4}$ 2) $f(x) = \dfrac{2x-3}{x^2-7x+12}$ 3) $f(x) = \dfrac{2x^2+3x-5}{x-1}$