

STUDENTS ENTERING ADVANCED PRECALCULUS SUMMER REVIEW WORK 2022

Because of the cumulative nature of math, you have learned that you need to have mastered concepts and procedures before you can learn new ones. The problems you will be completing will help you review material from Advanced Algebra II.

This assignment should take you approximately several hours, depending on how well you remember this material. Please do not start it until August. However, give yourself plenty of time as you may need to spend time reviewing your notes and/or examples from Advanced Algebra II.

In an organized fashion, show your work neatly on separate sheets of notebook paper. You may work with another rising Advanced Precal/Trig student, but the Honor Code applies: your work must be your own. I have included formulas at the beginning, please review them before working on the problems. Don't forget www.khanacademy.org which is a wonderful website that explains mathematical topics through video. Just put in the topic and choose the one you need.

Have this assignment completed prior to the first week of classes. You can expect some kind of evaluation on it but the main reason for doing this is to solidify the concepts you need to do well in Advanced Precalculus.

DIRECTIONS RECAP:

1. Show all work neatly, thoroughly and in pencil on separate paper.
2. Careful documentation of your work is extremely important. (ex: start by copying down the original problem; start by writing out the formula that you are using)
3. Follow directions for each section carefully.
4. Do not use your graphing calculator unless told otherwise.
5. You may work together on this assignment. In fact, I encourage you to work with one or two other students. Help each other but don't copy someone's work as that will not benefit you when you take the test.

If you have specific questions (beginning in August) regarding the assignment, you may email me at lindsayaverett@greensboroday.org and I will be happy to assist.

Classroom materials needed for Advanced Precalculus/Trigonometry: 3-ring binder with loose leaf paper, graphing paper, pencils, colored pens, graphing calculator.

Good luck and I look forward to working with you next year!

Ms. Averett

Formulas That You Are Expected To Know

Linear

Slope between two points: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Point -Slope Form: $y - y_1 = m(x - x_1)$

Slope-Intercept Form: $y = mx + b$

Standard Form: $Ax + By = C$ (A, B, and C must be integers)

Horizontal Line: $y = k$ (zero slope)

Vertical Line: $x = k$ (no slope)

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint Formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Quadratic

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant: $b^2 - 4ac$

$b^2 - 4ac > 0$ 2 real unequal roots

$b^2 - 4ac = 0$ 1 double real root

$b^2 - 4ac < 0$ 2 complex conjugate roots

Sum of Roots: $-\frac{b}{a}$

Product of Roots: $\frac{c}{a}$

Standard Form of a Parabola: $y = ax^2 + bx + c$ or $x = ay^2 + by + c$

Vertex Form of a Parabola: $y = a(x - h)^2 + k$ or $x = a(y - k)^2 + h$

$a = \frac{1}{4p}$; p is the distance from vertex to focus or p is the distance from vertex to directrix

Solving different types of equations:

Quadratic equation - factor, complete the square, quadratic formula

Quadratic-like equation - use a dummy variable

Rational equation - remember to check solutions in original equation for any restrictions on the domain

Radical equation - remember to check solutions for possible extraneous roots

Exponential and Logarithmic Functions

Exponential Function: $y = b^x$ with $b > 0, \neq 1$; Domain: \mathbb{R} Range: $\mathbb{R} > 0$

Logarithmic Function: $y = \log_b x$ with $b > 0, \neq 1$; Domain: $\mathbb{R} > 0$ Range: \mathbb{R}

Exponential/Logarithmic Identities and Properties

$$\begin{array}{lll} \log_b b = 1 & \ln e = 1 & \log_b x + \log_b y = \log_b xy \\ \log_b 1 = 0 & \ln 1 = 0 & \log_b x - \log_b y = \log_b \frac{x}{y} \\ \log_b b^x = x & \ln e^x = x & k \log_b x = \log_b x^k \\ b^{\log_b x} = x & e^{\ln x} = x & \end{array}$$

Solving exponential/logarithmic equations

$$b^x = b^y \dots \text{iff} \dots x = y \qquad \log_b x = \log_b y \dots \text{iff} \dots x = y$$

Converting from exponential to logarithmic form

$$x = b^y \Leftrightarrow y = \log_b x$$

$$\text{Change of Base Formula: } \log_b a = \frac{\log_c a}{\log_c b} \qquad \text{or } \log_b a = \frac{\log a}{\log b} \qquad \text{or } \log_b a = \frac{\ln a}{\ln b}$$

$$\text{Corollary: } \log_b a \cdot \log_a c = \log_b c$$

$$\text{Exponential Application Formulas: } \begin{array}{ll} A(t) = A_0(1 \pm r)^t & A(t) = A_0 e^{kt} \\ A = P \left(1 + \frac{r}{n} \right)^{nt} & A = Pe^{rt} \end{array}$$

$$\begin{array}{l} \text{Equations of a Circle:} \\ x^2 + y^2 = r^2; \text{ center} = (0,0) \text{ and radius} = r \\ (x-h)^2 + (y-k)^2 = r^2; \text{ center} = (h,k) \text{ and radius} = r \\ x^2 + y^2 + ax + by + c = 0; \text{ must complete the square in } x \text{ and } y \end{array}$$

$$\text{Polynomial Functions: } y = ax^n + bx^{n-1} + cx^{n-2} + \dots + mx^0$$

- **Degree** (highest exponent) determines the number of roots (both real and nonreal) and the end behavior of the function
- **Even degree:** end behavior is in the same direction (both positive if leading coefficient positive, both negative if leading coefficient negative)
- **Odd degree:** end behavior is in opposite directions (starts down/ends up if leading coefficient positive, starts up/ends down if leading coefficient negative)

Linear and Quadratic Functions

I. **Solve each equation over complex numbers using the most appropriate method (factoring, completing the square, quadratic formula, squaring both sides, dummy variable). Do NOT approximate any irrational roots (leave in radical form). Start by copying the original problem on your own paper.**

1. $4x^2 + 3 = 4x$

2. $1 = \sqrt{x+4} - \sqrt{x-1}$

3. $x - 11\sqrt{x} + 30 = 0$

4. $(x^2 - 3x)^2 - 3(x^2 - 3x) = 10$

5. $x^2 + 2x\sqrt{3} - 3 = 0$

6. $3 - 2|x - 2| < -9$

7.
$$\begin{cases} \frac{x}{y} = -2 \\ \frac{2x+y}{x-y} = -2 \end{cases}$$

8.
$$\begin{cases} y^2 - 6x^2 = 4 \\ y = 3x^2 - 2 \end{cases}$$

9. $3x^3 + 5x^2 - 26x + 8 < 0$ (syn division)

II. **State the x and y-intercepts, vertex, axis of symmetry, focus, equation of directrix. Sketch a graph. Use your calculator to approximate any irrational roots to the nearest tenth.**

1. $y = -\frac{1}{12}(x+3)^2 + 2$

2. $x = \frac{1}{4}y^2 + y - 3$

III. **Determine the equation of the following:**

1. perpendicular bisector of \overline{AB} given $A(2, -3)$ and $B(6, 7)$.

(give your equation in slope-intercept form)

2. linear function $f(x)$ with $f(-1) = -3$ and $f(-4) = 12$.

(give your equation in slope-intercept form in terms of $f(x)$)

3. quadratic equation with roots $\frac{-4 \pm 2i\sqrt{3}}{3}$.

(give your equation in $ax^2 + bx + c = 0$ form)

4. parabola with focus $(-1, -5)$ and directrix $x = 9$.

(give your equation in $y = a(x - h)^2 + k$ form)

5. quadratic function $h(x)$ with zeros -3 and 7 and *maximum value* of 6 .

(give your equation in $y = a(x - h)^2 + k$ form)

IV. Applications of linear equations and quadratic functions:

1. Find the equation of the line that contains the point $(4, -6)$ and the point of intersection of the lines $3x + 4y = 2$ and $2x - y = 5$. (leave your equation in standard form)
2. Suppose you have 160 meters of fencing to make five side-by-side rectangular enclosures. If its width is w , express the area of this figure in terms of w . What is the maximum area that you can enclose?
3. From a platform 45 meters above the ground, a ball is thrown upward with an initial speed of 40 m/s. The approximate height of the ball above the ground t seconds later is given by $h(t) = 45 + 40t - 5t^2$. You may solve this problem by hand or by using your calculator. If you use your calculator, state what keys you used to arrive at your answers.
 - A) What is the maximum height reached by this ball?
 - B) After how many seconds does the ball hit the ground?
 - C) How high is the ball after 1.25 seconds?
 - D) Draw a graph of $h(t)$. What domain and range make sense in the problem situation?
4. The table below shows the number of CDs shipped in the United States over the years 1986 - 1991. (These are net figures; returns from the retailers have been accounted for)

Year	1986	1987	1988	1989	1990	1991
CDs (million)	53	102.1	149.7	207.2	286.5	373.3

- A) Find the linear regression model for these data rounding slope and y-intercept to the nearest thousandth.
- B) What is the meaning of slope in context to this problem?
- C) Use your model to estimate the number of CDs that might have been shipped in 2006.
- D) What is the correlation coefficient and what does the correlation coefficient tell you about the accuracy of your model?

Exponential and Logarithmic Functions

I. Simplify: (no calculators)

1. $\log_8 128$

2. $(3^{-2} + 4^{-2})^{-1/2}$

3. $\log_6 6\sqrt[3]{1/36}$

4. $\ln \sqrt[5]{e^{-3}}$

5. $243^{\log_3 2}$

6. $\sqrt[10]{32} \div \sqrt[6]{4}$

II. Solve: If you must use your calculator, find the value of x to the nearest hundredth.

Remember: $y = b^x$ have restrictions on b and y

$y = \log_b x$ has restrictions on b and x

1. $27^{4-x} = \left(\frac{1}{81}\right)^{x-1}$

2. $9^{x-4} = 7.13$

3. $e^{2x+1} - 1 = 6$

4. $\log_4(x-4) + \log_4 x = \log_4 5$

5. $e^{2x} - 13e^x - 48 = 0$

6. $2\log_3 x = \log_3(x-2) + 2$

III. Applications of exponential and logarithmic functions: Calculator-active.

- Lizy wants to purchase a car in 6 years. She needs a down payment of \$6600. Determine the continuously compounded interest rate at which she would have to deposit \$3600 so that she will have the money she needs at the end of 6 years?
- A bacteria colony grows from 2,000 to 1,500,000 in 30 days. Find out how many days (nearest tenth of a day) it takes this particular bacteria to quadruple.
- The Chapman family wants to give Claire \$36,000 when she is in college. They now have \$25,000 to invest. Determine how many years (nearest tenth of a year) it will take them to achieve their goal given that they invest this amount at 6.25% compounded monthly.
- Suppose a car is presently worth \$10,150 and was bought new five years ago for \$25,000. Assuming that the car depreciates the same amount each year, find the rate of depreciation, rounding to the nearest tenth of a percent.

IV. **Sketch a graph.** Use a table if necessary. Check with your calculator.

1. $f(x) = \left(\frac{1}{3}\right)^x$

$g(x) = -\left(\frac{1}{3}\right)^x$

$h(x) = \left(\frac{1}{3}\right)^{-x}$

$j(x) = \left(\frac{1}{3}\right)^{x+3}$

2. $f(x) = e^x$

$g(x) = \ln(x)$

$h(x) = |\ln x|$

$j(x) = \ln(x-4)$

Graphing and more...

I. **Determine all key features of the functions, then sketch the graph of each.**
Be sure you label points on your coordinate plane.

1. $y = x^2(4-x)(2x+1)$

2. $y = x^3 + 5x^2 - 9x - 45$

3. $2y = \sqrt{16+x^2}$

4. $f(x) = \frac{1}{x} + 2$

5. $r(x) = \sqrt{x-10}$

6. $h(x) = 4 - |x|$

7. $j(x) = -\sqrt{-x}$

8. $f(x) = (x+3)^3 - 4$

9. $g(x) = -(x-4)^2 + 2$

10. $f(x) = -3[x+2]$

11. $f(x) = \begin{cases} 4, & \text{if } x \leq 0 \\ 4-x^2, & \text{if } 0 < x \leq 2 \\ 2x-6, & \text{if } x > 2 \end{cases}$

II. **Carry out the indicated operation of each of the following.**

1. Use synthetic division to divide $3x^5 - 5x^3 - 4x - 4$ by $x - 2$.

2. Identify all roots and describe the end behavior of $y = 2x^4 - x^3 - 7x^2 + x + 2$.

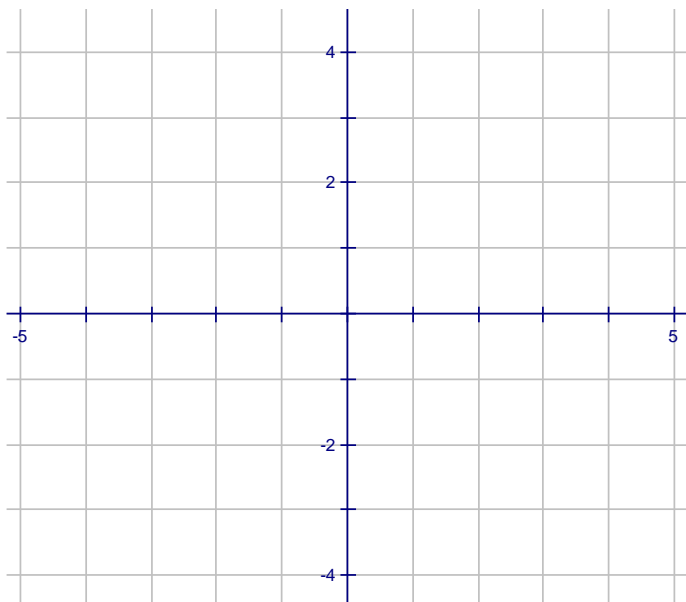
III. **Determine the domain of each of the following.**

1. $f(x) = \frac{-2x^2-6}{x^2-x-6}$

2. $h(x) = \log_2(x^2 - 2x - 3)$

3. $g(x) = \frac{\sqrt{x-4}}{|x|-5}$

IV. For the function $h(x) = \sqrt{x} + 1$, find $h^{-1}(x)$, find the domain and range of $h(x)$ and $h^{-1}(x)$, and graph the functions on the given axis.



$h^{-1}(x) =$ _____

Domain of $h(x)$: _____

Range of $h(x)$: _____

Domain of $h^{-1}(x)$: _____

Range of $h^{-1}(x)$: _____