Summer Work for students entering Precalculus

Congratulations on your successful completion of
Algebra I, Geometry, and Algebra II!

This summer packet is designed to help you review some of the important concepts in Algebra II, in preparation for Precalculus/Trigonometry. The topics covered in this packet are: Coordinate Geometry, Graphing Linear Equations/Inequalities, Writing Linear Equations, Solving Linear Systems, Radical Expressions, Complex Numbers, and Solving Quadratic Equations.

In an organized fashion, show your work neatly on separate sheets of notebook paper. Put your final answers in the spaces provided on the answer sheet. Your answer sheet and your work will be collected on the first day of school and will be graded. You may work with another rising Precalculus student, but the Honor Code applies: your work must be your own. I have included formulas, examples, and explanations at the end of this packet to help you. Your teacher will be available by request (please schedule a time via e-mail) during the August teacher workdays to assist with any problems you may encounter.

I strongly suggest that you do not begin this packet until mid-July. You will be tested on this material within the first few days of school, so it is important that these concepts are not forgotten over the course of the summer. This work is due the second day of class.

I look forward to a productive and successful year in Precalculus!

Section I: Distance, Midpoint, Pythagorean Theorem

1. Find the distance between the points (15, -8) and (17, 2).
2. Find the distance between the points $(-11\sqrt{2}, 4)$ and $(-3\sqrt{2}, -5)$
3. Find the coordinates of the midpoint of the segment with endpoints (0, 6) and (-4, 13).
4. Find the coordinates of the midpoint of the segment with endpoints $(^1/_4, -2)$ and $(^1/_5, 4)$.
5. Given: A(-11, -9), M(6, -7). If M is the midpoint of segment AB, find the coordinates of B.
6. The endpoints of a diameter of a circle are (2, 7) and (6, 5). Find the radius of the circle.
7. The endpoints of a diameter of a circle are (2, 7) and (6, 5). Find the center of the circle.
8. The sides of a triangle are 20 cm, 99 cm, and 101 cm. Determine whether the triangle is right, acute, or obtuse.
9. The leg of a right triangle is 8; the hypotenuse is 12. Find the measure of the other leg.
10. In parallelogram ABCD, the coordinate of B is (5, -3) and the coordinate of D is (-2, -7). Find the midpoint of segment AC.
11. The length of a rectangle is 7 inches and the width is 5 inches. Find the length of the diagonal.
12. The length of the diagonal of a square is $\frac{\sqrt{6}}{4}$ ft. Find the length of the side of the square.

Section II: Graphing Linear Equations and Inequalities

Without the use of a graphing calculator, graph the following linear equations or inequalities:

1. $y = -\,^3/_5\, x + 2$
2. $y < -10 + 3x$
3. $y = -2$
4. $x > 4$
5. $4x - 3y \leq 9$
6. $2(x + 5) = 3 - 2y$
7. $3x + 18y > 2x + 17y$
8. $3x - 7 - 2y \geq x - y$

Section III: Slope & Writing Linear Equations

1. Find the slope between the points (-3, 5) and (1, 2).
2. Find the slope between the points (13, 52) and (13, -27).
3. Find the slope between the points (-6, -19) and (1, -19).
4. Determine the value of *a* that makes the slope of the line through the two given points equal to the given value of *m*… (4, -3) and (2, *a*); $m = ¼$
5. Determine the value of *a* that makes the slope of the line through the two given points equal to the given value of *m*… (-3*a*, -*a*) and (4, 2*a*); $m = ½$
6. Write linear equation in standard form with $m = -2$ and $b = -3$.
7. Find the slope and y-intercept of the line $3(y - 2x) = 4x + 2$.
8. Find the x- and y-intercepts of the line $4x - 9y = 18$.
9. Write an equation of a vertical line passing through the point (34, -65).
10. Write an equation of a horizontal line passing through the point $(-^1/_{10}, ^7/_9)$.
11. A line has an x-intercept of -5 and a y-intercept of 13. Find the equation of this line in slope intercept form.
12. Write a linear equation in slope intercept form passing through the points (2, -2) and (-4, 13).
13. Write an equation of a line in standard form passing through the points (-2, 4) and (1, -5).
14. Write an equation of a line in standard form passing through the points (0, -7) and (7, -4).
15. Write a linear equation in slope intercept form passing through (3, 10) parallel to $3y - x = 4$.
16. Write an equation of a line in standard form passing through (1, -4) perpendicular to $x + 4y = -1$.
17. Write an equation of a line passing through the point (-6, 15) perpendicular to $x = -7$.
18. Determine the value of *k* so that the line through the points (-2, 3) and (6, *k*) has y-intercept 4.
19. Determine the value of *k* so that the points (*k* – 1, 2), (*k*, 4), and (3, 11) are collinear.

Section IV: Solving Linear Systems

Solve by Elimination (also known as the "Addition Method"):

1. $4x + y = 5$
   $x - 2y = 8$

2. $4m + 5n = 3$
   $5m + 6n = 2$

Solve by Substitution:

3. $y = 2x - 3$
   $-2x + y = 5$

4. $3b - 6a = 0$
   $4a + 3b = 26$

Solve by Graphing:

5. $2y - x = 0$
   $\frac{1}{2}x - y = 0$

6. $y = 1 - x$
   $2x + y = 9$

** You should be familiar with graphing and solving systems on the calculator, also. Check your answers to this section with a calculator.

Word Problems: (Use any method)

7. Eric bought 3 rolls of film and 1 battery for $11. The next day he purchased 2 rolls of film and 3 batteries for $12. Find the cost of one roll of film and one battery.

8. There are 8 more quarters than dimes in a parking meter. Three times the number of dimes is 1 less than twice the number of quarters. Find the number of dimes and quarters.

Section V: Radical Expressions & Complex Numbers

Simplify:

1. $\sqrt{24a^3}$
2. $\sqrt{-72x^4y^5}$
3. $(2 + 3i) - (5 - 7i)$
4. $(4 - 3i)(5 + 2i)$
5. $(2i\sqrt{5})^2$
6. $\dfrac{2i+1}{3i}$
7. $\dfrac{2}{2+\sqrt{5}}$
8. $\dfrac{4+5i}{1+i}$

Section VI: Solving Quadratic Equations

Solve by factoring:

1. $5y = 10y^2$
2. $n^2 - 4n + 3 = 0$
3. $3x^2 + 8x = 3$
4. $6y^2 + 2y - 4 = 0$
5. $2w^3 - 4w^2 - 16w = 0$

Solve by using the quadratic formula:

6. $x^2 + 6x + 4 = 0$
7. $2x^2 + 8x + 5 = 0$
8. $5t^2 = -12t - 8$
9. $h^2 + 7 = 4h$
10. $3x(x - 4) = 5(2 - x)$

Formulas, Examples, and Explanations

This portion of the packet is designed to assist you with the problems. Here you will find formulas, and *some* examples & explanations. You will also want to look in your Algebra II notes from the previous year for help.

Section I:

To find the distance between two points (x_1, y_1) and (x_2, y_2) use the distance formula:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Example: Distance between (5, 4) and (-3, 0) $= \sqrt{(-3-5)^2 + (0-4)^2}$

$= \sqrt{80} = 4\sqrt{5}$

To find the midpoint between two points (x_1, y_1) and (x_2, y_2) use the midpoint formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The Pythagorean Theorem describes the relationship between the lengths of the sides in a right triangle, where *a* and *b* are the legs, and *c* is the hypotenuse:

$a^2 + b^2 = c^2$

Sections II & III:

Formulas & Concepts:

- Slope between two points: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Zero slope ($^0/_{\#}$)… a horizontal line (y = ?)
- Undefined Slope ($^{\#}/_0$)… a vertical line (x = ?)
- Parallel lines have equal slopes
- Perpendicular lines have opposite reciprocal slopes (example: $m_1 = {}^3/_2$, $m_2 = {}^{-2}/_3$; slopes are $\perp$)
- To find the x-intercept, plug in 0 for y and solve for x.
- To find the y-intercept, plug in 0 for x and solve for y.

Forms of a linear equation:

- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Slope-Intercept Form: $y = mx + b$
  (*m* is slope; *b* is y-intercept)
- Standard From: $Ax + By = C$
  (where A, B, & C must be integers; A > 0)

Examples:

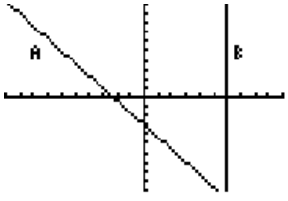
1. A) Graph the line $4x + 3y = -9$.
   B) Graph the line $x = 6$.

A) Solve for y to put the equation into slope-intercept form…

$y = -{}^4/_3 x - 3$

Notice that $-{}^4/_3$ is the slope and -3 is the y-intercept. Starting at the y-intercept of -3, you must "rise over run" $-{}^4/_3$: move down 4 and right 3, then make your next point. Do this a few times in both directions, then sketch the line.

B) The line x = 6 has undefined slope, so we must sketch a vertical line passing through x = 6.



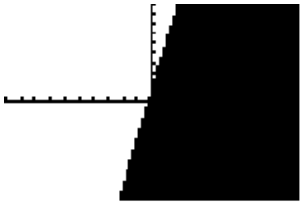
2. Graph in the inequality $5x - y > -1$

Solve for y to put the line in slope-intercept form. Remember when you multiply or divide by a negative, you must flip the inequality…

$y < 5x + 1$

Remember that a < or > indicates a dotted line, whereas ≤ or ≥ indicates a solid line. Graph a dotted line, starting from the y-intercept of +1. Since the slope is 5, you will rise 5 and run 1.

Lastly, "less than" means to shade below the line.



Examples (continued):

3. Write a linear equation in standard form passing through the points (4, 1) and (0, -4).

- *Find the slope using the slope formula… $m = 5/4$.*
- *Use point-slope form… choose either of the two given points.*

  $y - 1 = 5/4\,(x - 4)$
- *Put the equation into standard form, eliminating all the fractions, and making A positive.*

  $y - 1 = 5/4\,x - 5$

  $y = 5/4\,x - 4$ …slope-intercept form

  $-5/4\,x + y = -4$

  5x – 4y = 16 … standard form
- *You can check your answer by plugging in both of the given coordinates for x and y.*

4. Write a linear equation in standard form with an x-intercept of 8, perpendicular to the line $x - 5y = 19$

- *If the x-intercept is 8, then we have a coordinate (8, 0).*
- *Perpendicular lines have opposite reciprocal slopes, so we should find the slope of $x - 5y = 19$.*
- *Do this by putting the equation into slope intercept form…* $y = -1/5\,x - 19/5$
- *Since this line has a $-1/5$ slope, our line will have a +5 slope.*
- *Use the point (8,0) and the slope $m = -1/5$ in point-slope form…*

  $y - 0 = -1/5\,(x - 8)$
- *Put the equation into standard form*

  $y = -1/5\,x + 8/5$

  $1/5\,x + y = 8/5$

  x + 5y = 8

Section IV:

When we solve a system, we are finding the coordinate in which two lines intersect. We could also say that we are finding the ordered pair (x, y) that makes both equations true. There are four methods for solving systems with which you should be familiar: Graphing, Elimination (or "addition"), Substitution, and using the TI-83 Calculator.

Examples:

1. Solve by Elimination:

$$2x - y = 6$$
$$3x + 5y = 22$$

First, eliminate a variable by multiplying either equation through by a constant… multiplying the top equation by +5 makes the most sense, in order to eliminate the y's…

$10x - 5y = 30$

$3x + 5y = 22$

Now add both equations to eliminate the y's, and solve for x…

$10x - 5y = 30$

$+\ \underline{3x + 5y = 22}$

$13x = 52$

$x = 4$

Now plug in 4 for x in either of the original equations to solve for y…

$2(4) - y = 6$… $y = 2$

Solution: (4, 2)

2. Solve by Substitution:

$$2k - 3m = 4$$
$$m = 2/3\,k - 4/3$$

Notice that the second equation is already solved for m; simply substitute $2/3\,k - 4/3$ for m into the first equation…

$2k - 3(2/3\,k - 4/3) = 4$

Now solve for k…

$2k - 2k + 4 = 4$

$4 = 4$

In this problem, the variable k cancelled, and since 4 = 4 is a <u>true</u> statement, any x and any y will make this system true. We write a solution that indicates all (x,y)'s on the line are solutions.

Solution: {(x,y)| 2k – 3m = 4}

(Footnote: A <u>false</u> statement indicates no solution.)

3. Solve by Graphing Calculator:

$$x - 8y = 24$$
$$y = -\tfrac{3}{4}\,x + 12$$

To type an equation into the calculator, you must solve for y.

$x - 8y = 24$…

$y = 1/8\,x - 3$

Type the equations into Y1 & Y2, and graph (Zoom Standard will reset your x- & y-axes.)

Notice that the lines intersect, but not on the screen. You must adjust your X-Max by choosing Window.

To find the intersection, go to 2nd Calc, then intersect. Press enter 3 times (1st curve, 2nd curve, guess) and your calculator will find the intersection.

Solution: (17.143, -0.857)

Section V:

- A radical expression is not simplified if it contains factors that are perfect squares.

  Example: $\sqrt{90a^3b^6} = \sqrt{9\cdot 10\cdot a^2\cdot a\cdot b^6} = 3ab^3\sqrt{10a}$

- The imaginary number i allows us to take the square roots of negative numbers.
- Facts about imaginary numbers: $i^2 = -1$ (and therefore) $\sqrt{-1} = i$

Examples: *1)* $\sqrt{-128x}+\sqrt{-18x} = \sqrt{-1\cdot 64\cdot 2\cdot x}+\sqrt{-1\cdot 9\cdot 2\cdot x} = 8i\sqrt{2x}+3i\sqrt{2x} = 11i\sqrt{2x}$

2) $(10-5i)(-2+3i)$… Foil Method… $= -20+30i+10i-15i^2 = -5+40i$ (+15)

- Recall that √'s or i's cannot appear in the denominator.

Examples: *1)* $\dfrac{3}{2i\sqrt{5}}\cdot\dfrac{i\sqrt{5}}{i\sqrt{5}} = \dfrac{3i\sqrt{5}}{2i^2\sqrt{25}} = \dfrac{3i\sqrt{5}}{-2(5)} = \dfrac{-3i\sqrt{5}}{10}$

2) $\dfrac{2-i\sqrt{2}}{6+i\sqrt{2}}\cdot\dfrac{6-i\sqrt{2}}{6-i\sqrt{2}} = \dfrac{12-2i\sqrt{2}-6i\sqrt{2}+2i^2}{36-6i\sqrt{2}+6i\sqrt{2}-2i^2} = \dfrac{12-8i\sqrt{2}-2}{36-2i^2} = \dfrac{10-8i\sqrt{2}}{38} = \dfrac{5-4i\sqrt{2}}{19}$

↑ "*conjugate*"

Section VI:

- Quadratic Equations can be solved many ways. The two reviewed in this packet are:
  1. Solving by factoring 2. The Quadratic Formula

Examples : *1) Solve by factoring:* $6x^2 - 11x = 10$

$6x^2 - 11x - 10 = 0$

$(3x + 2)(2x - 5) = 0$

$3x + 2 = 0$ or $2x - 5 = 0$

$x = -\frac{2}{3}$ or $x = \frac{5}{2}$

2) Solve by factoring: $12x = 44x^2$

$12x - 44x^2 = 0$

$4x(3 - 11x) = 0$

$4x = 0$ or $3 - 11x = 0$

$x = 0$ or $x = \frac{3}{11}$

3) Solve using the Quadratic Formula: $3x^2 - 8x + 14 = 0$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-8)\pm\sqrt{(-8)^2-4\cdot 3\cdot 14}}{2(3)} = \frac{8\pm\sqrt{-104}}{6} = \frac{8\pm 2i\sqrt{26}}{6} = \frac{4\pm i\sqrt{26}}{3}$$

Summer Work Answer Sheet. Write answers here, attach work on separate sheets of paper:

Section I:

1. _____
2. _____
3. _____
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Section II: Use graph paper and attach

Section III:

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Section IV:

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Section V:

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Section VI:

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