Summer Work for students entering Intro to Calculus ADV

This summer packet is designed to help you review some of the important concepts in Algebra II and Precalculus, in preparation for Intro to Calculus ADV. The topics covered in this packet are: Solving Equations of All Types (linear, quadratic, rational, radical, exponential, and logarithmic), Writing Linear and Quadratic Equations, and Graphing Linear and Quadratic Functions.

READ CAREFULLY:

In an organized fashion, show your work neatly on separate sheets of notebook paper. Put your final answers in the spaces provided on the answer sheet. Your answer sheet and your work will be collected on the second day of class and will be graded. You may work with another rising Intro to Calculus ADV student, but the Honor Code applies: your work must be your own. Use past notes to assist you. Your teacher will be available by request (please schedule a time via e-mail) during the August teacher workdays to help with any problems you may encounter.

I strongly suggest that you do not begin this packet until mid-July. You will be **tested on this material** within the first few days of school, so it is important that these concepts are not forgotten over the course of the summer. This packet is due the Monday after we return to school.

Make a plan! *5 problems per day for 10 days* OR *10 problems per day for 5 days* OR *1 section per day in the 3 days leading up to school.* You must understand the concepts when you walk in my door in August, so use it as a tool to prepare for senior year!

I look forward to a productive and successful year in Intro to Calculus ADV.

Answer Sheet for Intro to Calculus Summer Work – Attach your work to this page

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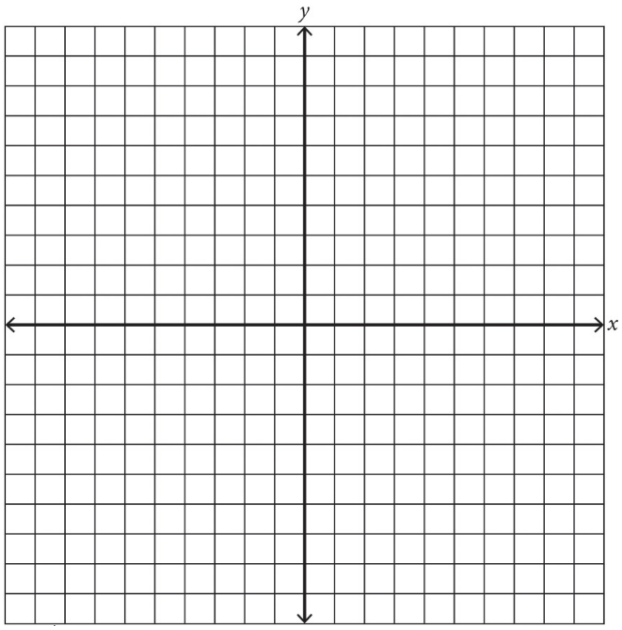

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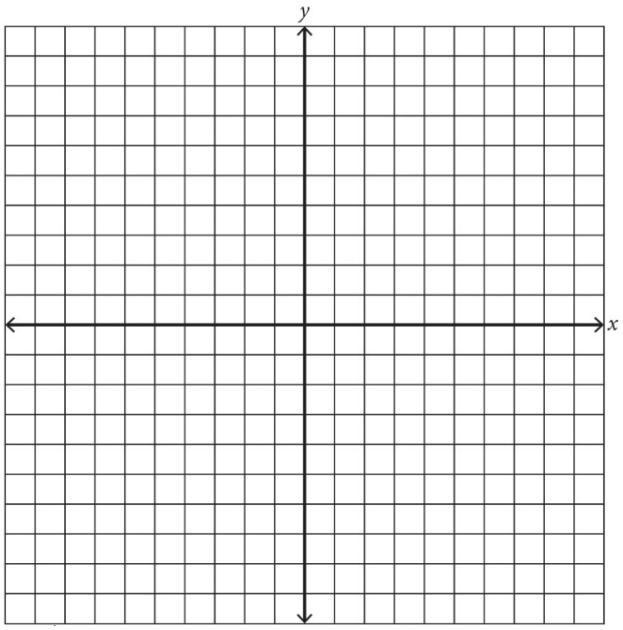

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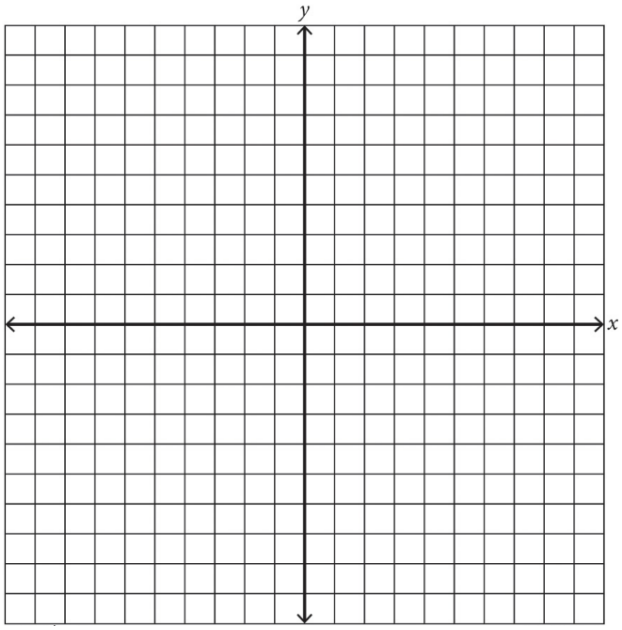

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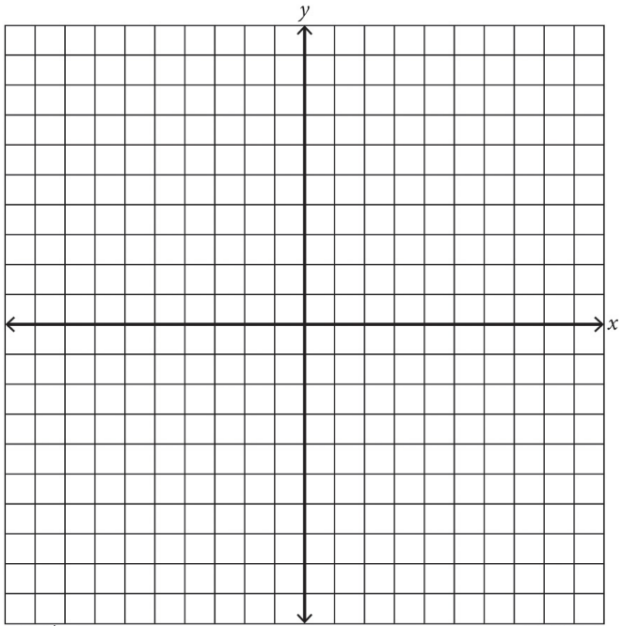

Section I: Solving Equations of All Types

1. $-\frac{s}{4}+7\left(\frac{s}{14}+\frac{s}{21}\right)=13-\frac{s}{2}$
2. $2-\frac{3}{4}\left(\frac{2}{3}k+4\right)=8k-\frac{1}{2}\left(\frac{4}{5}-k\right)$
3. $3b^2 + 14b - 24 = 0$
4. $\frac{16}{a}-1=a+5$
5. $x(x - 3) = 2x(5 - 8x) - 4$
6. $(s - 1)(s - 3) = 24$

7. $9m^2 + 16 = -24m$
8. $28q = 6q^2$
9. $x^3 + 3x^2 - 25x - 75 = 0$
10. $\frac{3}{k-3}=\frac{k}{k-3}-2$
11. $\frac{2y+3}{4y-1}=\frac{3y-2}{3y+2}$
12. $\sqrt{3x+2}-\sqrt{2x+7}=0$
13. $\sqrt[3]{7b}-\sqrt[3]{5b+2}=0$
14. $x+8=\sqrt{x^2+16}$
15. $\sqrt{y-3}+5=y$
16. $25^x = 5^{2x-1}$
17. $16^{a+2} = 32^{a-1}$
18. $6^x = 19$ *
19. $4^{2x-1} = 15$ *
20. $\log_2 8 = w$ *
21. $\log_5 18 = x + 3$ *
22. $\log x + \log 3 = \log 15$
23. $\log 20 - \log a = 2$
24. $\log m^2 = 4$
25. $\ln (x - 4) = 5$ *
26. $3 \ln x + \ln 2 = \ln 2000$

You may use a calculator on the asterisked problems

Section II: Slope, Intercepts, & Writing Linear Equations

27. Find the slope *m* between the points (-3, 5) and (1, 2).
28. Consider the coordinates (4, -3) and (2, *a*). Determine the value of *a* so that the slope *m* between these two coordinates is ¼.
29. Write a linear equation in standard form with $m = -2$ and $b = -3$. Graph this line on graph paper.
30. Find the slope and y-intercept of the line $3(y - 2x) = 4x + 2$. Graph this line on graph paper.
31. Find the x- and y-intercepts of the line $4x - 9y = 18$.
32. Write an equation of a vertical line passing through the point (34, -65).
33. Write an equation of a horizontal line passing through the point (12, -2).
34. A line has an x-intercept of -5 and a y-intercept of 13. Find its equation in slope intercept form.
35. Write a linear equation in slope intercept form passing through the points (2, -2) and (-4, 13).
36. Write an equation of a line in standard form passing through the points (-2, 4) and (1, -5).
37. Write an equation of a line in standard form passing through the points (13, -4) and (1, 1).
38. Write a linear equation in slope intercept form passing through (3, 10) parallel to $3y - x = 4$. Graph both lines on graph paper.
39. Write an equation of a line in standard form passing through (1, -4) perpendicular to $x + 4y = -1$. Graph both lines on graph paper.

Section III: Quadratic Equations

40. Put this standard form quadratic equation into vertex form by completing the square: $x^2 + 4x - 6 = 0$
41. Put this vertex form quadratic equation into standard form: $2(x - 3)^2 - 7 = 0$.
42. Put this standard form quadratic equation into vertex form by completing the square: $3x^2 - 18x - 2 = 0$
43. Put this vertex form quadratic equation into standard form: $y = ½ (x + 4)^2 - 7$.
44. Write a quadratic equation in standard form with roots 3 and –10 and a y-intercept of +30.
45. Write an equation of a parabola in vertex form where f(-5) = -11, having a vertex of (-7, -13).
46. Write an equation of a parabola in standard form having zeros 2 and 3, and a y-intercept of 12.
47. Find the vertex, axis of symmetry, direction of opening, and the zeros of $f(x) = -2x^2 + 4x$. Sketch a graph of the parabola.
48. Find the vertex, axis of symmetry, direction of opening, and the zeros of $g(x) = 3x^2 - 24x + 45$. Sketch a graph of the parabola.
49. Find the vertex, axis of symmetry, direction of opening, and the zeros of $h(x) = x^2 + 2x - 7$. Sketch a graph of the parabola.
50. Find the vertex, axis of symmetry, direction of opening, and the zeros of $f(x) = -½(x + 2)^2 + 4$. Sketch a graph of the parabola.

We will be doing a lot of graphing leading up to calculus concepts.
It is imperative that functions can be visualized or quickly sketched, and knowing how to identify key points such as intercepts, max/min, asymptotes and orientation.
Hints are contained on the following pages, please only use them after attempting a problem *and* revisiting it if it can't be solved.

Hints, Formulas, and Explanations

This portion of the packet is designed to assist you with the problems. Here you will find formulas, and *some* examples & explanations. You will also want to look in your Algebra II notes from the previous year for help.

Section I: Hints & Suggestions

1. Distribute, combine like terms, isolate the variable
2. Distribute, combine like terms, isolate the variable
3. Factor or use quadratic formula
4. Multiply through by *a*, set = 0; then factor or use quadratic formula
5. Set equation equal to 0; factor if possible or use quadratic formula
6. Same as #5
7. Same as #5
8. Same as #5
9. Factor by grouping
10. Multiply through by $k - 3$, then solve. (What can *k* not equal?)
11. Cross multiply and set = 0; then use quadratic formula
12. Subtract $\sqrt{2x+7}$; square both sides... remember to check the answer(s)
13. Same as 12, but cube both sides... remember to check the answer(s)
14. Square both sides... remember to check the answer(s)
15. Subtract 5 and square both sides... remember to check the answer(s)
16. Rewrite 25 as 5^2
17. Rewrite 16 and 32 as bases of 2
18. Take *log* of both sides; move *x* in front of *log 6*
19. Take *log* of both sides; move $(2x - 1)$ in front of *log 4*
20. Rewrite as exponential equation
21. Use #20 hint, then use #19 hint
22. 1st Property of logs: *log A + log B = log (AB);* logs cancel on both sides
23. 2nd Property of logs: *log A – log B = log (A/B);* then use #20 hint
24. Same as #20; remember *log* has a base of *10* unless otherwise noted
25. Same as #20; remember *ln* has a base of *e*
26. The 3 becomes the exponent of x, and use the 1st Property of logs (from #22)

Section II: Linear Formulas & Concepts

- Slope between two points: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Zero slope ($^0/_{\#}$)… a horizontal line (*Example: y = 3 is a horizontal line passing through 3*)
- Undefined Slope ($^{\#}/_0$)… a vertical line (*Example: x = -2 is a vertical line passing through -2*)
- Parallel lines have equal slopes (*Example: $y = {}^1\!/_3 x - 4$ and $y = {}^1\!/_3 x + 9$*)
- Perpendicular lines have opposite reciprocal slopes (*Example: $y = -3x - 2$ and $y = {}^1\!/_3 x + 1$*)
- To find the x-intercept, plug in 0 for y and solve for x.
- To find the y-intercept, plug in 0 for x and solve for y.

Forms of a linear equation:

- Point-Slope Form: $y - y_1 = m(x - x_1)$ ...*m* is slope; (x_1, y_1) is a coordinate on the line
- Slope-Intercept Form: $y = mx + b$ ...*m* is slope; *b* is y-intercept
- Standard From: $Ax + By = C$ ...A, B, & C must be integers, and A must be positive

Section III: Quadratic Formulas & Concepts

Standard Form of a Quadratic Equation: $ax^2 + bx + c = 0$

- Solutions (a.k.a. Zeros, Roots, X-intercepts) can be found by factoring, or using the quadratic formula
- Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Vertex: The x-value of the vertex can be found using $-b/2a$; the y-value can be found by plugging in the x-value
- Axis of Symmetry: $x = -b/2a$
- The y-intercept is *c*
- *a* determines the direction and width of parabola opening... +*a*: opens up -*a*: opens down
- To change standard form into vertex form: *Complete the Square...*

  Example: $2x^2 - 8x + 5 = 0$

  $2x^2 - 8x = -5$ ...move the constant to the other side

  $2(x^2 - 4x\ \ ) = -5$ ...factor out the 2

  $2(x^2 - 4x + 2^2) = -5 + 2(2^2)$ ...take half of 4, square it, and add to both sides

  $2(x - 2)^2 = 3$ ...factor & simplify

  $2(x - 2)^2 - 3 = 0$

Vertex Form of a Quadratic Equation: $a(x - h)^2 + k = 0$

- Vertex is (*h*, *k*)
- Axis of Symmetry: $x = h$
- To change vertex form into standard form: FOIL the $x - h$, distribute *a*, add *k*
- *a* determines the direction and width of parabola opening... +*a*: opens up -*a*: opens down
- Solutions (a.k.a. Zeros, Roots, X-intercepts) can be found by simply solving for *x*