ADVANCED ALGEBRA II SUMMER REVIEW WORK
2022-2023

Because of the cumulative nature of math, you have learned that you need to have mastered concepts and procedures before you can learn new ones. The problems you will be completing will help you review material from Advanced Algebra I. These problems were chosen because it is necessary that you not only understand how to do them but also handle them with ease and confidence.

This assignment should take you approximately <u>3 - 5 hours</u>, depending on how well you remember the material. It would behoove you not to start it until August. However, give yourself plenty of time as you may need to spend time reviewing your notes and/or examples from Advanced Algebra I, many of which are included at the beginning of this packet.

This packet will be graded. It is due the first full day of classes. A twenty-five percent deduction will be taken for each day it is late. You can expect that we will also briefly review these concepts and then explore them in greater depth during our first two and the fourth units of the semester. If you did not remember how to work certain problems, we highly suggest you **<u>get help</u>** with them before you turn in this packet!

Extra help will be available before school starts. I will send you specific times in early-August.

<u>DIRECTIONS</u>:

1. Show all work neatly, thoroughly and in pencil on separate paper.
2. Careful documentation of your work is extremely important. When appropriate, start by copying down the original problem, then write out the formula that you are using, etc.
3. Follow directions for each section carefully.
4. Do not use your graphing calculator unless the directions say it is permissible.
5. You may work together on this assignment. In fact, we encourage you to work with one or two other students. Help each other but don't copy someone's work as that will not benefit you when you take any evaluation.
6. You do ***<u>not</u>*** have to turn in this original packet (except for page 8) when submitting in your work in August.
7. Remember that you also have online review work at your disposal like Khan Academy, etc.

Mrs. Morris

REVIEW OF BASIC CONCEPTS AND PROCEDURES

I. <u>ORDER OF OPERATIONS</u> "PEMDAS"

1. Simplify the expression within each grouping symbol, working outward from the innermost grouping.
2. Simplify powers.
3. Perform multiplications and divisions in order from left to right.
4. Perform additions and subtractions in order from left to right.

<u>Examples</u>: Simplify:

1) $3(4)^2+(5+2^4)4\div 2-20\div 2\cdot 3=$

$3\cdot 16+(5+16)4\div 2-20\div 2\cdot 3=$ **exponents**

$3\cdot 16+21\cdot 4\div 2-20\div 2\cdot 3=$ **parenthesis**

$48+(84\div 2)-(10\cdot 3)=$ **multiply & divide left to right**

$48+42-30=$ **clear parentheses**

60 **add & subtract left to right**

2) $\left[3\cdot 6-12\div 2\right]\div 6-2=$

$\left[18-6\right]\div 6-2=$

$12\div 6-2=$

$2-2\quad=$

0

3) $-\frac{7a}{4}\div\left[\frac{1}{2}\left(-\frac{a}{2}+\frac{a}{3}\right)\right]=$

$-\frac{7a}{4}\div\left[\frac{1}{2}\left(\frac{-3a+2a}{6}\right)\right]=$

$-\frac{7a}{4}\div\left[\frac{1}{2}\left(\frac{-a}{6}\right)\right]=$

$-\frac{7a}{4}\div\left(\frac{-a}{12}\right)=$

$-\frac{7a}{4}\cdot\frac{12}{a}=$

-21

4) $3|7-9|-|-8-2|-|3-1|^3=$

$3|-2|-|-10|-|-2|^3=$ **simplify | |**

$3(2)-(10)-(2)^3=$ **evaluate ( )**

$6-10-8=$ **+ and -**

-12

II. ADDING and SUBTRACTING POLYNOMIALS

We add and subtract polynomials by combining like terms and writing our sum/difference in **standard form** (decreasing order in terms of one variable). In subtracting polynomials we have to remember that if a minus sign precedes an expression in parentheses, then the sign of every term within the parentheses is changed when we remove the parentheses:

$-(b+c-d)=-b-c+d$ ***Property of Opposite of a Sum***

Examples: Simplify

1) $\left(2x+x^3-6x^2+4\right)-\left(5x^2-7x+x^3\right)$

$2x+x^3-6x^2+4-5x^2+7x-x^3$

$-11x^2+9x+4$

2) $3\{2a+4(5-4a)\}-6\{3a-(2a-5)\}$

$3\{2a+20-16a\}-6\{3a-2a+5\}$

$3\{-14a+20\}-6\{a+5\}$

$-42a+60-6a-30$

$-48a+30$

III. SETS of NUMBERS

Review the types of numbers that make up the real number system

N – Natural numbers - $\{counting\ numbers\}$ or $\{1,2,3,4,5,...\}$

W – Whole Numbers - $\{natural\ numbers\ and\ zero\}$ or $\{0,1,2,3,4,5,...\}$

Z (J) – Integers - $\{whole\ numbers\ and\ their\ opposites\}$ or $\{...,-3,-2,-1,0,1,2,3,...\}$

Q – Rational numbers - $\{any\ terminating\ or\ repeating\ decimal\}$

$\left\{any\ number\ can\ be\ represented\ in\ the\ form\ \frac{a}{b},\ where\ a\ and\ b\ are\ integers\ and\ b\neq 0\right\}$

Examples: $\frac{1}{2},\ -\frac{3}{7},\ 46=\frac{46}{1},\ 0.17=\frac{17}{100},\ 0.66666...=\frac{2}{3}$

Reminder: $\frac{3}{0}=\text{undefined}$ $\frac{0}{0}=\text{indeterminate}$

I – Irrational numbers -

$\{any\ non-terminating\ and\ non-repeating\ decimal-\sqrt{p}\ where\ p\ is\ not\ a\ perfect\ square\}$

Examples: $\sqrt{2},\ \sqrt{3},\ \sqrt{5},\ \sqrt[3]{2},\ \pi,\ \frac{3}{5\pi^2}$

$\Re$ **- Real Numbers -** $\{all\ rational\ and\ irrational\ numbers\}$

Examples: Graph each of the following on a number line:

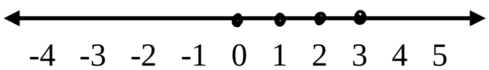
1) $\{-3 \le W < 4\}$

2) $\{\Re > -4\}$

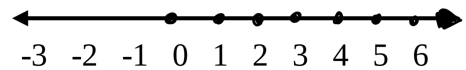
3) $\{non-negative\ Z\ greater\ than\ -2\}$

4) $\{negative\ integers\ that\ are\ multiples\ 3\}$

Answers:

1)


2)


3)


4)


IV. Sets and Intervals

A **set** is a collection of objects, and these objects are called the **elements** of the set. If S is a set, the notation $a \in S$ means that a is an element of S, and $b \notin S$ means that b is not an element of S. For example, if Z represents the set of integers, then $-3 \in Z$ but $\pi \notin Z$. Some sets can be described by listing their elements within **braces**. For instance, the set A that consists of all positive integers less than 7 can be written as

$$A = \{1, 2, 3, 4, 5, 6\}$$

If S and T are sets, then their **union** $S \cup T$ is the set that consists of ***all*** elements that are in S ***or*** T (or in both). The **intersection** of S and T is the set $S \cap T$ consisting of all elements that are in both S ***and*** T. In other words, $S \cup T$ means to take **all** elements and $S \cap T$ is the **common** elements in S and T.

The **empty set**, denoted by $\varnothing$ or $\{\ \}$, is the set that contains no elements.

Examples: Let $S = \{1, 2, 3, 4, 5\}$, $T = \{4, 5, 6, 7\}$, and $V = \{6, 7, 8\}$, then

$S \cup T = \{1, 2, 3, 4, 5, 6, 7\}$ ***"all"***

$S \cap T = \{4, 5\}$ ***"common to both"***

$S \cap V = \varnothing$ ***"no elements common to both"***

Back to Sets of Numbers: $Q \cup I = \Re$; $Q \cap I = \varnothing$; $N \subset W \subset Z \subset Q \subset \Re$;

V. PROPERTIES OF REAL NUMBERS

Commutative Property of Addition	$a + b = b + a$	When you add or multiple
Commutative Property of Multiplication	$ab = ba$	two real numbers, ***order*** doesn't matter.
Associative Property of Addition	$(a+b)+c = a+(b+c)$	Grouping doesn't matter
Associative Property of Multiplication	$(ab)c = a(bc)$	when you add or multiply three real numbers.
Identity Properties	$a + 0 = a$	Identity element for addition
	$a \cdot 1 = a$	Identity element for mult.
Inverse Properties	$a + -a = 0$	Property of additive inverses
	$a \cdot \frac{1}{a} = 1$	Property of multiplicative inverses
Distributive Property	$a(b+c) = ab + ac$	The Distributive Property is crucial because it describes the way addition & multiplication interact with each other.

VI. PROPERTIES of EQUALITY

For all real numbers a, b, and c:

Reflexive Property: $a = a$

Symmetric Property: If $a = b,$ then $b = a$

Transitive Property: If $a = b$ and $b = c,$ then $a = c$

Addition Property: If $a = b,$ then $a + c = b + c$

Multiplication Property: If $a = b,$ then $ac = bc$

Definition of Subtraction: For all real numbers a and b, $a - b = a + -b$.

Definition of Division: For all real numbers a and b with $b \neq 0$, $a \div b = \frac{a}{b} = a \cdot \frac{1}{b}$

VII. PROPERTES of EXPONENTS

When multiplying like bases, add their exponents. $RULE: \; x^m \cdot x^n = x^{m+n}$

When dividing like bases, subtract their exponents. $RULE: \; \frac{x^m}{x^n} = x^{m-n}$ or $\frac{x^m}{x^n} = \frac{1}{x^{n-m}}$

(if you want the exponent to be positive)

When raising a power to a power, multiply the exponents. $RULE: \; \left(x^m\right)^n = x^{mn}$

Examples: Simplify

1) $\left(2a^3bc^4\right)\left(-4a^4b^3c^5\right)$

$-8a^7b^4c^9$

2) $\left(-5a^{-2}c^3\right)^3$

$-125a^{-6}c^9$

$\frac{-125c^9}{a^6}$

3) $\frac{-24x^5y^2z^9}{36x^3y^7z^{-4}}$

$\frac{-2x^2z^{13}}{3y^5}$

4) $(-a)\left(ab^2\right)^2\left(a^2b\right)^3$

$(-a)\left(a^2b^4\right)\left(a^6b^3\right)$

$-a^9b^7$

VIII. MULTIPLYING POLYNOMIALS

To obtain the product of two polynomials, multiply each term of one of the polynomials by each term of the other and then add all like terms. In other words, we use distributive property several times.

Examples: Simplify

1) $\left(x^2 - 2x + 3\right)\left(2x^2 + 4x - 1\right)$ ***Distribute***

$2x^4 + 4x^3 - x^2 - 4x^3 - 8x^2 + 2x + 6x^2 + 12x - 3$

$2x^4 - 3x^2 + 14x - 3$

2) $(3a + 2)(5a - 6)$ ***"FOIL"***

$15a^2 - 18a + 10a - 12$

$15a^2 - 8a - 12$

3) $\left(4x^3 - 2y\right)\left(x^3 - 5y\right)$ ***"FOIL"***

$4x^6 - 22x^3y^2 + 10y^2$

4) $\left(3c^2 + 4b^4d\right)\left(5c^2 - 2b^4d\right)$ ***"FOIL"***

$15c^4 + 14b^4c^2d - 8b^8d^2$

(find O, representing "outer" product, and I, representing "inner" product, for middle term in one step)

Special Product Formulas

1. $(a+b)(a-b)=a^2-b^2$ ***Product of the Sum & Difference = Difference of Squares (DOTS)***

2. $(a+b)^2=a^2+2ab+b^2$
   $(a-b)^2=a^2-2ab+b^2$ ***Binomial Squared = Perfect Trinomial Square (PTS)***

3. $(a+b)(a^2-ab+b^2)=a^3+b^3$ ***Binomial times Special Trinomial = Sum of Cubes (SOC)***
   $(a-b)(a^2+ab+b^2)=a^3-b^3$ ***Binomial times Special Trinomial = Difference of Cubes (DOC)***

Examples: Simplify

5) $(3s-4t)(3s+4t)$
   $9s^2-16t^2$

6) $(4x^3+2y^5)^2$
   $16x^6+16x^3y^5+4y^{10}$

7) $(x+2y)(x^2-2y+4y^2)$
   x^3+8y^3

8) $(3y-4z)(9y^2+12yz+16z^2)$
   $27y^3-64z^3$

9) $(6m^2n-5t^3)(6m^2n+5t^3)$
   $36m^4n^2-25t^6$

10) $(5a^2b^7-3c^4d)^2$
   $25a^4b^{14}-30a^2b^7c^4d+9c^8d^2$

IX. FACTORING POLYNOMIALS

To factor a polynomial, you express it as the product of polynomials that are members of a specified factor set. A factorization of a polynomial is complete when each of the factors is either a monomial or a polynomial whose greatest monomial factor is 1.

The following factor patterns occur frequently:

GCF	greatest common factor:	$ab^2c^4+a^2b^4c^3=ab^2c^3(c+ab^2)$
GBF	greatest binomial factor:	$a(b+c)-d(b+c)=(b+c)(a-d)$
DOTS	difference of two squares:	$a^2-b^2=(a+b)(a-b)$
DOC	difference of cubes:	$a^3-b^3=(a-b)(a^2+ab+b^2)$
SOC	sum of cubes:	$a^3+b^3=(a+b)(a^2-ab+b^2)$
PTS	perfect trinomial square:	$a^2+2ab+b^2=(a+b)^2$
		$a^2-2ab+b^2=(a-b)^2$
GT	general trinomial:	$48-2x-x^2=-1(x^2+2x-48)=-1(x+8)(x-6)$
2 by 2	four terms:	

$$x^2+zx-xy-zy=x^2-xy+zx-zy=x(x-y)+z(x-y)=(x-y)(x+z)$$

Remember to always look for a possible GCF first!!!

X. SOLVING EQUATIONS

To solve an equation means to find a value for the variable that makes the equation true. Whatever you do to one side of the equation, you must also do to the other side. Solving equations with rational coefficients is easier if you "clear" the denominators first by multiplying both sides by the LCD.

Examples: Solve each equation:

1) $\frac{2}{3}(x-4)=\frac{3}{5}(2+x)$

$15\left[\frac{2}{3}(x-4)\right]=15\left[\frac{3}{5}(2+x)\right]$	**LCD is** 15
$5[2(x-4)]=3[3(2+x)]$	**simplify**
$10(x-4)=9(2+x)$	**multiply**
$10x-40=18+9x$	**distribute**
$x=58$	**solve**
$\{58\}$	

2) $3(x-5)-(2-x)+6=2\left[-3+4\left(2+\frac{1}{2}x\right)\right]$

$3x-15-2+x+6=2[-3+8+2x]$	**distribute**
$4x-11=2[5+2x]$	**combine terms**
$4x-11=10+4x$	**distribute**
$-11=-10$	**solve; FALSE!**
$\varnothing$	

XI. FINDING EQUATIONS of LINES

To find the equation of a line, you need a ***point*** and ***slope.***

- An old favorite: **slope intercept form:** $y = mx + b$ where m is the slope of the line and b is where the line crosses the y–axis (also called the y-intercept). (x,y) represents any point on the given line.
- Clearing any fractions in the slope intercept form and bringing x (whose coefficient > 0) and y together on the same side with the constant on the other is the **standard form of a linear equation:** $Ax + By = C$
- The **point slope form** is considered the easiest because it is the formula for finding the slope between two points but has been slightly manipulated: $y - y_1 = m(x - x_1)$ where m is the slope of the line and (x_1,y_1) represent a point on the line.
- The equation of any **horizontal** is $y = \#$ and this line has 0 slope; a **vertical** line is represented by $x = \#$ and this line has ***undefined slope.***
- **Parallel** lines have the same slope; **perpendicular** lines' slopes are opposite reciprocals of each other.

Examples:

1) Find the equation of the line through (3,2) and (2,1).

Slope = (2 – 1)/(3 – 2) = 1. Either $y - 2 = 1(x - 3)$ or $y - 1 = 1(x - 2)$ would be the equation of the line. They are equivalent equations.

2) Find the equation of the line through (6,1) with slope = -2/3.

$y - 1 = (-2/3)(x - 6)$

3) Find the equation of the line through (-3,5) and perpendicular to the line $y = (-2/7)x$.

The perpendicular slope is 7/2 so the line is $y - 5 = (7/2)(x + 3)$

NOW IT'S YOUR TURN!

Remember to show all work and answers on your own paper. **DO NOT use a calculator for any of this.**

I. Replace each _____ with one of the words *ALL, SOME,* or *NO* to make a true statement.

1. ______ real numbers are irrational numbers.
2. ______ natural numbers are integers.
3. ______ whole numbers are natural numbers.
4. ______ real numbers are rational numbers or irrational numbers.
5. ______ rational numbers are negative integers.

II. Given $A=\{2,3,4,5,6,7,8\}$, $B=\{0,2,4,6,8\}$, $C=\{7,8,9,10\}$. **Find**

1. $A\cap(B\cup C)$
2. $(A\cup B)\cap C$
3. $(A\cap B)\cap(C\cup A)$

Find the indicated set if $A=\{x\,/\,x\geq -4\}$, $B=\{x\,/\,x<6\}$, $C=\{x\,/\,-7\leq x<2\}$ **Show the answer on a number line.**

4. $A\cap B$
5. $B\cup C$
6. $A\cap C$

III. State the property or definition of real numbers being used:

1. $(a-b)(a+b)=(a+b)(a-b)$
2. $x\div w=\dfrac{x}{w}$
3. $\dfrac{5}{2}[(2+a)+-a]=\dfrac{5}{2}[2+(a+-a)]$
4. If $3x-2y=5w\div 4z$, then $5w\div 4z=3x-2y$
5. $\left(-\dfrac{d}{f}\right)\left(-\dfrac{f}{d}\right)=1$
6. $(x+a)(x+b)=(x+a)x+(x+a)b$
7. If $3x^2+x=2x^2+6$, then $3x^2+x-5=2x^2+6-5$
8. $2(A-4B)=2(A+-4B)$

IV. Factor *Completely*:

1. $3x^6-48x^2$
2. $5a^2-16ab+12b^2$
3. $c^3-d^3-c^2d+cd^2$
4. $3x^2y^4+81x^2y$
5. $40xy-16x^2-25y^2$
6. $27cd^2-12d^3+27c^2d$
7. $-x^2-11x-24$
8. $2p^2+15qr+10pq+3pr$
9. $27j^3-k^3$
10. $3x^3+6x^2+3x$
11. $45a^2b^4+18a^4b^3-81a^3b^4$
12. $2xz-21wy-14yz+3wz$
13. $b^2c^3+8bc^4+12c^5$
14. $20b^4-45c^2$
15. $8a^3b^2+2a^2b^3-3ab^4$

V. Simplify: Be sure that you copy the original problem on your paper!

1. $\frac{1}{2}\left[12 \div 2(6)\right] \div \left[12 \cdot 3 \div 6 + 3\right]$
2. $\left(39 - 15 + 2^6 \div 2^3\right) \div 2^4 \cdot 2^2$
3. $-6\left[y - 2(9 - 3y)\right] + 4\left[-y - 8(y + 5)\right]$
4. $4|3-8| - 3|-7-5| - 2|-9-(-2)|$
5. $\left(9x^2 - 7xy + y^2\right) - \left(-3x^2 - 4xy - y^2\right) - 2\left(5x^2 - xy + 2y^2\right)$
6. $\frac{3}{2}\left\{\frac{4|2(4)-15|}{6-3^3}\right\} \div \left(\frac{1}{2}\right)^{-2}$
7. $\left(-\frac{2}{3}p^2q^6\right)\left(-6p^3q^{-4}\right)\left(-\frac{1}{4}p^{-1}q^3\right)$
8. $a^{x+2} \cdot a^3 \cdot a^{2x-4}$
9. $\frac{\left(3r^4s^2\right)^3\left(-r^{-1}s\right)^4}{\left(-9r^{-3}s^{-2}\right)^2}$
10. $\left(-3x^{2-n}\right)^2 \cdot 3^{-1} \cdot \left(x^{n-1}\right)^3$
11. $(3cd)^2\left(-5c^3d^2\right)^3 - \left(4c^2d\right)^2\left(-2c^2d^2\right)^2$
12. $\frac{a^{n-1}b^{2n}}{a^{n+1}\left(b^2\right)^{n-1}}$
13. $2yz^2\left(3y^2 - 4z\right)\left(5y^2 - 2z\right)$
14. $\left(3a^2 - 2b^3c\right)\left(3a^2 + 2b^3c\right)$
15. $(4a - 3c)\left(3a^2 - 5ac + 2c^2\right)$
16. $(3x + 2y)\left(9x^2 - 6xy + 4y^2\right)$
17. $-2n\left(6m^2 - 5np^3\right)^2$
18. $\left(4a^3 - 3b^2c\right)^3$
19. $(4x - 3y)\left(16x^2 + 12xy + 9y^2\right)$
20. $\left(9x^2 + 16\right)(3x + 4)(3x - 4)$
21. $\frac{6 - 6x^2}{3x + 4} \div \frac{x^2 + x - 2}{9x^2 + 24x + 16} \cdot \left(18x^2 - 32\right)^{-1}$
22. $\frac{a + 3b}{a^2 - 7ab + 12b^2} - \frac{a - 3b}{a^2 - ab - 12b^2}$

VI. Solve each equation:

1. $5\left[12 - 3(2 - a) - 2a\right] = -2(a - 1)$
2. $3(5a - 1) - 5(2 + 3a) = 7$
3. $\frac{1}{5}(x - 3) - \frac{2}{3}\left(x - \frac{5}{2}\right) = \frac{7}{15}$
4. $(4x - 3)(2x + 1) = 63$
5. $(4a - 3)^2 + 11a = (a + 2)(a - 2) + 33$
6. $\frac{16}{x^2 - 1} + 3 = \frac{8}{x - 1}$

VII. Find an equation in standard form for the line described below. ($ax + by = c;\ \ a, b, c \in Z$ **)**

1. The line goes through (4,-2) and (-4,10).
2. The line has $x-$ intercept -4 and is parallel to the graph of $2x - 5y = 7$.
3. The line goes through $(-5, 4)$ and is perpendicular to the line containing (-2,-5) and (-2,3).

I pledge that the work on this paper is my own and I abided by the directions given at the beginning of the assignment.

____

I worked with a tutor: ___________ **I did NOT work with a tutor: __________**

Attention: Parents of Algebra II and Advanced Algebra II Students

From: Trish Morris, Interim Math Department Chair

Date: April 29, 2022

Re: Purchase of Graphing Calculator

The learning of mathematics is much more powerful than it was in the time before technology. Graphing calculators and Computer Algebra Systems (CAS) allow students to deal with real life problems because of the ease in handling data that was once considered too complicated and cumbersome to work with.

The Upper School Math Department recommends any of the Texas Instruments calculators: TI-83+ (TI-83 plus), the TI-83+ Silver Edition, the TI-84+ or TI-84+ Silver Edition. These are enhanced versions of the TI-83, the model that we have used for many years. These upgrades carry more memory but are not necessarily any faster than the TI-83. However, if your student currently owns a TI-83, there is no need to replace it. Any of these calculators will serve your student throughout high school and well into college.

The availability of powerful calculators has significantly expanded the types of problems and the range of topics that can be studied in a mathematics course. With the aid of this technology, students can concentrate on exploring, understanding, and applying mathematics without becoming bogged down in calculations or tedious plotting of points. Although technology has the power to enhance the study of mathematics, technology does not drive these courses. Technology is a tool that supports and extends but does not dominate the teaching and learning of mathematics.

Parents and students often question why we use expensive calculators when all of our students have high-powered laptop computers. While it is true that we could purchase software that would enable the laptops to do much of what we do with the graphing calculators, the College Board does not yet allow computers to be used on any of its exams. All the calculators mentioned above are on the list of models approved for use on the SAT, ACT and on the AP Calculus, AP Statistics and all AP science exams. Students must be able to use these calculators with facility that can only come from repeated use.

These calculators can be purchased at many local stores including Office Depot, Staples, Target and Wal-Mart. Frequently these stores will have a sale on calculators in early August, and we have seen them as low as $79 (after a mail-in rebate). Our GDS bookstore also carries a few TI-84+'s in stock at a price of $130. You can also go on Amazon and find used calculators that are less expensive but also dependable.

We recognize that these graphing calculators are expensive, and we greatly appreciate your support for your student's study of mathematics in this way. If you have any questions, please contact me, Trish Morris.