Name________________________

Summer Work – Preparation for Algebra II

This assignment is to be completed after August 1 for best results.
This work is due the first week of school, although you will be given class time to ask questions and put the finishing touches on your work. You will be able to ask questions in class, but it should be complete. You will earn your first grade on this work.

1. **COMPLETE WORK NEATLY ON LINED PAPER!**
2. **Know this material**. We will expand on it.
3. You may work together and get help on this assignment. **Do not just copy someone's work.**
4. Even answers are included to help you check your progress.
5. Put all final (odd) answers on the provided answer sheet at the end of the packet.

<u>Topics included:</u>

I. Order of Operations (no calculator)
II. Number Sets
III. Solving Equations
IV. Solving Inequalities
V. Solving Combined Inequalities
VI. Adding and Subtracting Polynomials
VII. Multiplication of Polynomials
VIII. Properties of Exponents

****Please read the attached letter at the end of this packet and share it with your parents.****

Order of Operations/Arithmetic

When expressions have more than one operation, we have to follow rules for the order of operations:

1. First do all operations that lie inside **P**arentheses.
2. Next, do any work with **E**xponents.
3. Working from left to right, do all **M**ultiplication and **D**ivision.
4. Finally, working from left to right, do all **A**ddition and **S**ubtraction.

Simplify inside absolute value bars and then determine the sign. **Do not distribute negative or factors across absolute value.**

Example 1

$3(4)^2 + (5+2^4)4 \div 2 - 20 \div 2 \cdot 3 =$

$3 \cdot 16 + (5+16)4 \div 2 - 20 \div 2 \cdot 3 =$ *exponents*

$3 \cdot 16 + 21 \cdot 4 \div 2 - 20 \div 2 \cdot 3 =$ *parenthesis*

$[3 \cdot 16] + [21 \cdot 4 \div 2] - [20 \div 2 \cdot 3] =$ *mult. & div. left to right*

$48 + [84 \div 2] - [10 \cdot 3] =$ *(+ & − separate groups)*

$48 + 42 - 30 =$

60

Example 2

$3|7-9| - |-8-2| - |3-1|^3 =$

$3|-2| - |-10| - |-2|^3 =$ *simplify inside* $|\ |$

$3(2) - (10) - (2)^3 =$ *evaluate each abs.value*

$6 - 10 - 8 =$ *follow order of op.*

-12

I. Simplify. Do NOT use a calculator! (Remember, copy each problem on lined paper and show steps.)

1. $14 + (3 \times 12) \div 3^2 - 3$
2. $(-2)^4(-1)^5(-3)^3$
3. $3(5)^2 + (5+2^2)4 \div 2$
4. $2\left\{\dfrac{6\left[16-2(4)\right]}{6+3(2)}\right\} - 8$
5. $-\left[52 + (-18)\right] + -|20-61|$
6. $-32 \div \dfrac{1}{6} \div 3 \div (-4)$
7. $4|3-8| - 4|-9-2|$
8. $3(5) + 4 \times 5 \div 2 - 2 \div 2$
9. $\dfrac{1}{2} + \dfrac{3}{8} - \dfrac{2}{3}$
10. $\dfrac{(-9)(8) - (6)^2 \div 4}{(2-11)}$

II. Our Number System

Sets of Numbers

Real Numbers: $\Re$ $\ldots,-5,\ldots,-\sqrt{3},\ldots,0,\ldots,1,\ldots\pi,\ldots,\frac{16}{3},\ldots$

Rational Numbers: **Q** $\ldots-\frac{11}{4},\ldots,-2,\ldots,0,\ldots,\frac{1}{3},\ldots 7,\ldots$

Integers: **J (or Z)** $\ldots-4,\ldots,0,\ldots,2,\ldots$

Whole Numbers: **W** $0,1,2,\ldots$

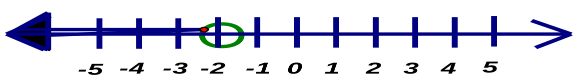
Natural or Counting Numbers: **N**
$1,2,3,\ldots$

Examples:

1) $N > -3$

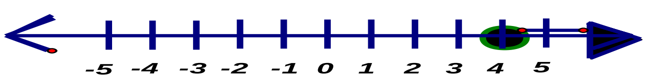
{Natural Numbers} do not contain zero. Place points (dots) over each number. Shade arrow.

2) $R < -2$



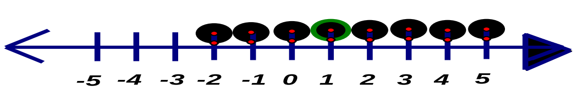
Shade this one because {Real Numbers} includes all of the fractions and decimals. Make an open circle on -2 because it is not included. Shade to the left.

3) $R \geq 4$



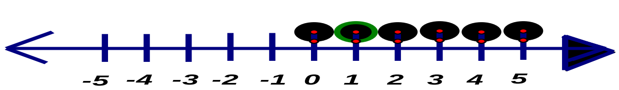
Shade this one because {Real Numbers} includes all of the fractions and decimals. Make closed circle on 4 because it is included. Shade to the right.

3) $Z \geq -2$



{Integers} contain negative numbers. Include -2! Place points (dots) over each number.

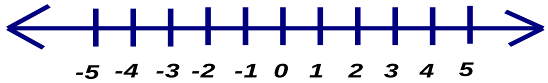
4) $W > -9$



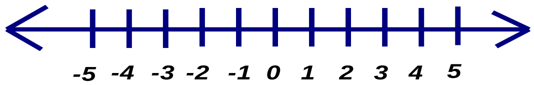
{Whole Numbers} contain 0.

II. Graph each set of numbers on a number line.

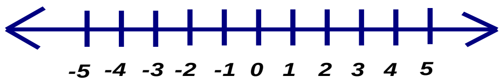
1) $R > -2$



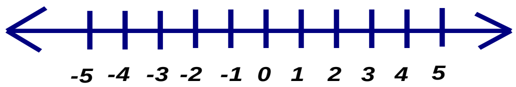
2) $N > -2$



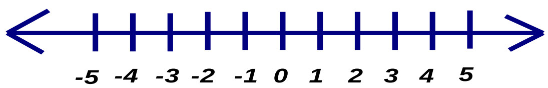
3) $Z > -2$



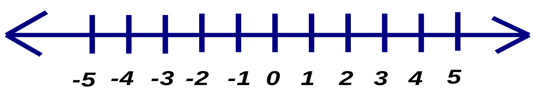
4) $W > -2$



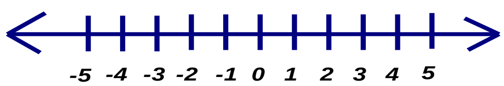
5) $Z \geq 2$



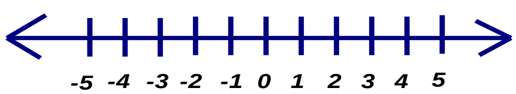
6) $R \leq 3$



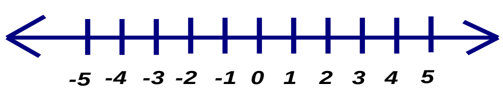
7) $W \geq 4$



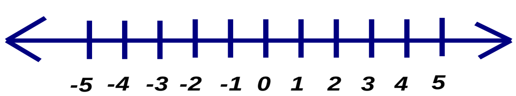
8) $Z \geq -4$



9) $Z < \frac{3}{2}$



10) $R > -\frac{1}{2}$



III. Solving Equations

To solve an equation means to find a value for the variable that makes the equation true. Whatever you do to one side of the equation, you must also do to the other side. Solving fractional equations is easier if you "clear" the fractions. Multiply everything on both sides of the equation by the LCD. Remember to distribute!

Example 1

$$\frac{2}{3}(x-4)=\frac{3}{5}(2+x)$$
$$15\left[\frac{2}{3}(x-4)\right]=15\left[\frac{3}{5}(2+x)\right] \textit{LCD is } 15$$
$$5\left[2(x-4)\right]=3\left[3(2+x)\right] \textit{ simplify}$$
$$10(x-4)=9(2+x) \textit{distribute}$$
$$10x-40=18+9x \textit{ solve}$$
$$x=58 \quad \{58\}$$

Check:

$$\frac{2}{3}(58-4) = \frac{3}{5}(2+58)$$
$$\frac{2}{3}(54) = \frac{3}{5}(60)$$
$$36=36$$

Example 2

⇐There is no solution to this equation.

Start distributing from the inside.

$$3(x-5)-(2-x)+6=2\left[-3+4\left(2+\frac{1}{2}x\right)\right]$$
$$3x-15-2+x+6=2\left[-3+8+2x\right] \textit{distribute}$$
$$4x-11=2\left[5+2x\right] \textit{ simplify}$$
$$4x-11=10+4x$$
$$-11=10 \textit{ false!}$$
$$\varnothing$$

(If an equation has the same value on both sides, it is an identity. The solution is all $\Re$)

III. Solve. Check solutions in odd problems.

1. $0.3(2x-3)=0.2x+0.9$
2. $5\left[12-3(2-y)-2y\right]=2(1-y)$
3. $\frac{x^2}{6}-\frac{x}{2}-\frac{2}{3}=0$ (remember factoring?)
4. $4(g+1)=10-2(3-2g)$
5. $\frac{2a+1}{24}=\frac{a}{12}$
6. $\frac{1}{5}(2x-3)-\frac{2}{3}\left(x-\frac{1}{2}\right)=\frac{8}{15}$
7. $\frac{2y}{3}-\frac{y+3}{6}=2$
8. $5-(8-3x)=(4x+7)-x$

IV. Solving Inequalities

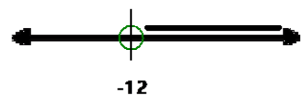
An inequality is an open sentence using one of the symbols $<$, $\leq$, $>$, or $\geq$. The steps for solving an inequality are the same as those for solving equations ***with the exception*** of when you are

- dividing by a negative number
- multiplying by a negative number

When you do this, you need to remember to reverse the inequality symbol.

EXAMPLE 1: Solve $4r - 5 < 5r + 7$

$$4r - 5 < 5r + 7$$
$$-r - 5 < 7$$
$$-r < 12$$
$$\{r > -12\}$$



Open circle, shade to the right and include the right arrow.

EXAMPLE 2: Solve $-5(x+3) < -5x + 1$

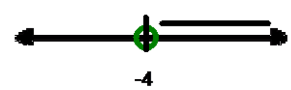
$$-5(x+3) < -5x + 1$$
$$-5x - 15 < -5x + 1$$
$$-15 < 1$$

The whole number line as well as the arrows are shaded.

This is a true statement so the answer is the set of real numbers: $\Re$

EXAMPLE 3: Solve $4\left(r - \frac{1}{2}\right) - 3 \leq 5(r-1) + 4$

$$4\left(r - \frac{1}{2}\right) - 3 \leq 5(r-1) + 4$$
$$4r - 2 - 3 \leq 5r - 5 + 4$$
$$4r - 5 \leq 5r - 1$$
$$-r - 5 \leq -1$$
$$-r \leq 4$$
$$r \geq -4$$



Closed circle, shade to the right, including the right arrow.

IV. Solve the following inequalities. Graph your solution set on a number line.

1. $-\frac{2}{5}r < 10$

2. $3(x-2) > 3x + 2$

3. $-10y < -2(5y-3)$

4. $5b + 3 > 5(b+1) - b$

5. $\frac{2}{3}[2-(3-x)] \leq \frac{1}{2}(x-3)$

6. $4[2-3(x-1)] \geq 11(2-x)$

7. $5 - 2c \leq 11$

8. $\frac{3}{4} < 6 - \frac{1}{2}a$

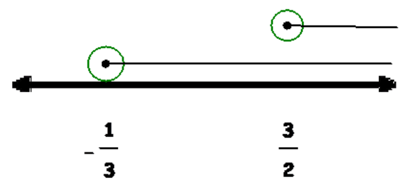
V. <u>Solving Combined Inequalities</u>

Combined inequalities are two or more inequalities that have the word "AND" or "OR" between them.

If the word "AND" is between them, the problem is considered a conjunction and you find the solution set as you would find the ***intersection*** between two or more sets of elements; you look for the numbers that are common to both (or all) inequalities.

EXAMPLE: $3a+2<5a-1$ and $2a+1>-a$

$3a+2<5a-1$	and	$2a+1>-a$
$-2a<-3$		$3a>-1$
$a>\frac{3}{2}$	and	$a>-\frac{1}{3}$



Since the numbers that are common to both inequalities are greater than $\frac{3}{2}$

the solution is $\left\{a>\frac{3}{2}\right\}$

If the word "OR" is between two or more inequalities, the problem is considered a disjunction and you find the solution set as you would the ***union*** between two or more sets of elements; you look for the numbers that solve every inequality.

EXAMPLE: $\frac{y}{3}-7<-9$ or $\frac{y+6}{2}>5$

$\frac{y}{3}-7<-9$	or	$\frac{y+6}{2}>5$
$y-21<-27$		$y+6>10$
$y<-6$		$y>4$

Since the numbers that are in both inequalities are less than -6 or greater than 4

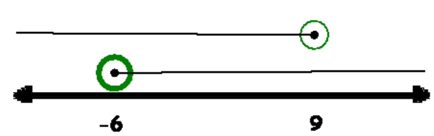
the solution is $\{y<-6 \text{ or } y>4\}$

EXAMPLE: A "chained inequality" such as $-3\le 3(c+5)<42$ is treated as an "AND" statement.

$-3\le 3(c+5)<42$

$-1\le c+5<14$

$-6\le c<9$



The solution is where the lines "overlap" which is

$\{-6\le c<9\}$

V. Solve the following combined inequalities and graph the solution set.

1. $w+2<3 \text{ or } 2-w<5$
4. $-2\le 1-t\le 4$
7. $6\le 9-\frac{1}{2}x\le 10$
2. $5-3k\ge 8 \text{ or } -4\ge 5-3k$
5. $3z+1>7 \text{ and } 2(z-1)<-4$
8. $-6x>0 \text{ and } 8-7x<-20$
3. $1>k+3>-2$
6. $5x-2\ge 8 \text{ or } 4-x>5$

VI. Adding and Subtracting Polynomials

Make sure you know the meaning of the following words. Half of the battle in mastering Algebra is understanding the vocabulary!

1) **constant** – a number (example: 10 or -7)
2) **variable** – a letter used to represent a constant (example: x or a)
3) **monomial (sometimes referred to as a "term")** – product of a constant and/or variables (example: $-2x^3yz^5$ or $5ag^3m^2$)
4) **coefficient** – the number that precedes a term; if none is given, it is assumed to be 1
5) **like monomials** – terms that have the same variables with the same exponents (example: $-x^2y^9z$ and $6x^2y^9z$)
6) **polynomials** – sum or difference of two or more monomials (terms)

binomial: $2ab+b^2$ trinomial: $x^2-2xy+y^2$ other: $5-2x+3y+4x^2y^2$

EXAMPLES:

Simplify

1. $-3x^2+5-x+4x^2-9$

$-3x^2+5-x+4x^2-9=$
$(-3x^2+4x^2)-x+(5-9)=$
x^2-x-4

2. $4xz^4-2x^2z^2-5x^3z+x^2z^2$

$4xz^4-2x^2z^2-5x^3z+x^2z^2=$
$-5x^3z+\left(-2x^2z^2+x^2z^2\right)+4xz^4=$
$-5x^3z+(-x^2z^2)+4xz^4$

Add the polynomials:

3. $8r-7$, and $-3r-4$

$(8r-7)\ +\ (-3r-4)=$
$(8r+(-3r))+(-7+(-4))=$
$5r-11$

Simplify (Do not clear fractions here! It is not an equation.)

4. $\frac{-1}{2}\left(4a^2-3a-6\right)-\frac{2}{3}\left(9-12a^2+3a\right)$

$(-2a^2+\frac{3}{2}a+3)-\left(6-8a^2-2a\right)=$
$-2a^2+\frac{3}{2}a+3-6+8a^2+2a=$
$6a^2+\frac{7}{2}a-3$

Subtract the second polynomial from the first:

5. $6v+4,\quad -2v-5$

$\left(6v+4\right)\underline{-(-2v-5)}=8v+9$

6. $3p^2-4pq+7q^2,\quad 8p^2-12q^2$

$\left(3p^2-4pq+7q^2\right)-(8p^2-12q^2)=-5p^2+-4pq+19q^2$

VI. Simplify and show all work:

1. $\left(2xy-3xy^2-y^3\right)-\left(xy-xy^2\right)$
2. $4z+(-10y)+(-8x)-12y+17x-13z$
3. $-(7w+2w^2)-(19w^2-5w^3)+5(2w-3w^3)$
4. $2\left(m+n\right)-\left[-\left(m+n\right)\right]+5\left(m-2n\right)$
5. $-4\left[2x-3\left(6x-3y+2\right)\right]+2\left[-9x-3-5\left(2x-4y\right)\right]$
6. $x^2y-3xy^2+4y^3-2x^2y-xy^2+y^3$
7. $2(x+2z)+3(-x-3z)-5(2x-z)$
8. $-12a+19a^2-6a^3-a^2-14a+2a^3+2a$
9. $-5\left[2\left(1+4n\right)-5n\right]-3\left[2n-6\left(n-1\right)\right]$
10. $-5a-\left(-\frac{1}{2}b\right)+3\frac{1}{2}a-\frac{1}{2}b$

VII. Properties of Exponents

When multiplying like bases, add their exponents. **RULE:** $x^m \times x^n = x^{m+n}$

EXAMPLES: Simplify:

1) $5a^4 \times 2a^4 = 10a^8$

2) $(7kl^2)(-l^2) = -7kl^4$

3) $\left(\frac{1}{4}s^2\right)(3s)\left(\frac{4}{3}s^3\right) = \left(\frac{1}{4}\times 3\times\frac{4}{3}\right)s^{2+1+3} = 1s^6 = s^6$

4) $5x^3 - (x)(-3x^2) = 5x^3 + 3x^3$

When raising a power to a power, multiply the exponents. **RULE:** $(x^m)^n = x^{mn}$

EXAMPLES: Simplify:

5) $5a^2(2a)^4 = 5a^2(2^4 \times a^4) = 5a^2(16a^4) = 80a^6$

6) $-5(c^2)^3 = -5c^6$

7) $(-5c^2)^3 = (-5)^3(c^2)^3 = -125c^6$

8) $(-r)(rs^2)^2(r^2s)^3 = (-r)(r^2s^4)(r^6s^3) = -r^9s^7$

9) $(-3m^2n)^2(5mn^2)(-2mn) = (9m^4n^2)(5mn^2)(-2mn) = -90m^6n^5$

Memorizing the perfect squares from 1 through 15, the powers of 2 from 2^1 to 2^{10}, and 3^1 to 3^5 would make this work easier for you!!!

VII. Simplify and show all work on a separate sheet of paper.

1. $(3z^3)(2z^2)$

2. $(-2pq^6)(8p^2q)\left(\frac{1}{2}p^2q^2\right)$

3. $a^x \times a^3 \times a^{2x-6}$ Think of the exponent rule!

4. $(-3c)(-2b^2c) - (3c)(bc)(-4b)$

5. $(-3r^2s^2)^3 + (-2r^3s^3)^2$

6. $(5cd)(-3d^2)(-2c^3d^2)$

7. $\frac{20x^2y}{3} \times \frac{12x^3y^5}{5}$

8. $(2^{x+1})(2^{x+4})$ Think of the exponent rule!

9. $(3r^4s)^3(-2r^3s)(-rs^2)^3$

10. $(3cd)(2c^3d^2)^3 - (2c^2d)^3(c^2d^2)^2$

VIII. <u>Multiplying Polynomials</u>

When multiplying a monomial by a polynomial, distribute the monomial to every term in the polynomial and use your rules for exponents.

EXAMPLE: $-3x^2y(-2x^4y^2+3xy^3+4) =$

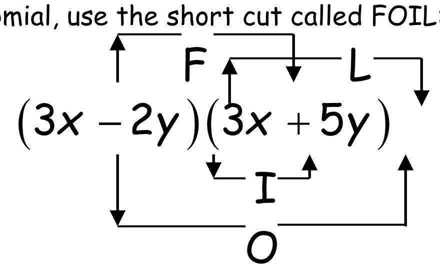
$$(-3x^2y)(-2x^4y^2)+(-3x^2y)(3xy^3)+(-3x^2y)(4)=$$
$$6x^6y^3+(-9x^3y^4)+(-12x^2y) \text{ or}$$
$$6x^6y^3-9x^3y^4-12x^2y$$

When multiplying a polynomial by a polynomial, distribute each term in the first polynomial to every term in the second one.

EXAMPLE: $(3a+2)(2a^2-2a+1) =$

$$(3a)(2a^2)+(3a)(-2a)+(3a)(1)+(2)(2a^2)+(2)(-2a)+(2)(1)=$$
$$6a^3-6a^2+3a+4a^2-4a+2=$$
$$6a^3-2a^2-a+2$$

When multiplying a binomial by a binomial, use the short cut called FOIL: First – Outside – Inside - Last



$(3x-2y)(3x+5y)$

First: $(3x)(3x) = 9x^2$

Outside: $(3x)(5y) = 15xy$

Inside: $(-2y)(3x) = -6xy$

Last : $(-2y)(5y) = -10y^2$

or

$9x^2+9xy-10y^2$

VIII. Simplify showing all work on a separate sheet of paper.

1. $-3t^3(2t^3+5t^2-3t-1)$

2. $3k^2(k-2)-k^2(4k-1)$

3. $(3a-2)(a-3)$

4. $(x^3+5)(x^3-5)$

5. $(2x+7y)^2$

6. $-2x^2y(5x^4-2x^2y^2+3y^4)$

7. $(2w-5)(2w^2-3w-2)$

8. $(2ab^2-4c)(3ab^2-2c)$

9. $(5a-3)(5a+3)$

10. $(x-y)^3$

Even Answers

I.

2) 432

4) 0

6) 16

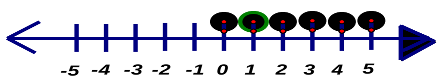
8) 24

10) 9

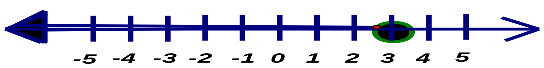
II

2) N > -2

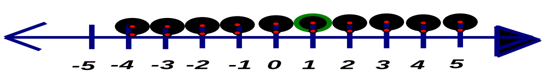
4) $W > -2$



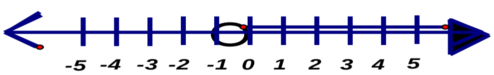
6) $\Re \le 3$



8) Z ≥ -4



10) $\Re > -\frac{1}{2}$



III.

2) $\{-4\}$

4) $\{\Re\}$ (all real numbers)

6) $\{-3\}$

8) $\varnothing$

IV.

2) $\varnothing$

4) $b > 2$

6) $-2 \ge x$

8) $\frac{21}{2} > a$

V.

2) $k \le -1$ or $k \ge 3$

4) $-3 < t \le 3$

6) $x < -1$ or $x \ge 2$

8) $\varnothing$

VI.

2) $9x - 22y - 9z$

4) $8m - 7n$

6) $-x^2y - 4xy^2 + 5y^3$

8) $-4a^3 + 18a^2 - 24a$

10) $-\frac{3}{2}a$

VII.

2) $-8p^5q^9$

4) $18b^2c^2$

6) $30c^4d^5$

8) 2^{2x+5}

10) $119c^{10}d^7$

VIII.

2) $-k^3 - 5k^2$

4) $x^6 - 25$

6) $-10x^6y + 4x^4y^3 - 6x^2y^5$

8) $6a^2b^4 - 16acb^2 + 8c^2$

10) $x^3 - 3x^2y + 3xy^2 - y^3$

Answer Sheet: **Write your answers for odd questions here. All accompanying work should be shown on separate paper.**

Section I

1.________________________

3.________________________

5.________________________

7.________________________

9.________________________

Section II

1.

3.

5.

7.

9.

Section III

1.________________________

3.________________________

5.________________________

7.________________________

Section IV

1.______________________

3.______________________

5.______________________

7.______________________

Section V

1. ______________________

3.______________________

5.______________________

7.______________________

Section VI

1. ______________________

3. ______________________

5.______________________

7.______________________

9.______________________

(Continued on next page)

Section VII

1. ________________________

3.________________________

5.________________________

7.________________________

9.________________________

Section VIII

1. ________________________

3.________________________

5.________________________

7.________________________

9.________________________

Greensboro Day School

Attention: Parents of Algebra II and Advanced Algebra II Students

From: Trish Morris, Interim Math Department Chair

Date: April 29, 2022

Re: Purchase of Graphing Calculator

The learning of mathematics is much more powerful than it was in the time before technology. Graphing calculators and Computer Algebra Systems (CAS) allow students to deal with real life problems because of the ease in handling data that was once considered too complicated and cumbersome to work with.

The Upper School Math Department recommends any of the Texas Instruments calculators: TI-83+ (TI-83 plus), the TI-83+ Silver Edition, the TI-84+ or TI-84+ Silver Edition. These are enhanced versions of the TI-83, the model that we have used for many years. These upgrades carry more memory but are not necessarily any faster than the TI-83. However, if your student currently owns a TI-83, there is no need to replace it. Any of these calculators will serve your student throughout high school and well into college.

The availability of powerful calculators has significantly expanded the types of problems and the range of topics that can be studied in a mathematics course. With the aid of this technology, students can concentrate on exploring, understanding, and applying mathematics without becoming bogged down in calculations or tedious plotting of points. Although technology has the power to enhance the study of mathematics, technology does not drive these courses. Technology is a tool that supports and extends but does not dominate the teaching and learning of mathematics.

Parents and students often question why we use expensive calculators when all of our students have high-powered laptop computers. While it is true that we could purchase software that would enable the laptops to do much of what we do with the graphing calculators, the College Board does not yet allow computers to be used on any of its exams. All the calculators mentioned above are on the list of models approved for use on the SAT, ACT and on the AP Calculus, AP Statistics and all AP science exams. Students must be able to use these calculators with facility that can only come from repeated use. These calculators can be purchased at many local stores including Office Depot, Staples, Target and Wal-Mart. Frequently these stores will have a sale on calculators in early August, and we have seen them as low as $79 (after a mail-in rebate). Our GDS bookstore also carries a few TI-84+'s in stock at a price of $130. You can also go on Amazon and find used calculators that are less expensive but also dependable.

We recognize that these graphing calculators are expensive, and we greatly appreciate your support for your student's study of mathematics in this way. If you have any questions, please contact me, Trish Morris.