AB CALCULUS
SUMMER WORK
2022 - 2023

Because of the cumulative nature of math, you have learned that you need to have mastered concepts before you can learn new ones. The problems you will be completing will help you to review material from Algebra I through Pre-Calculus. I have chosen these problems because it is necessary that you not only understand how to do them but also handle them with a certain amount of ease, confidence, and speed.

Please follow these directions:

1. **Show all work neatly, thoroughly and in pencil on a separate sheet of paper. Careful documentation of your work is *extremely important*!**
2. **Follow directions in each problem CAREFULLY!!!**
3. **Round any decimals to THREE DECIMAL PLACES.**
4. **Unless told otherwise, you may use your graphing calculator.**

I anticipate that this assignment should take you anywhere from four to eight hours. I would suggest that you don't attempt to start it until August. It is due on the first full day of classes. I will be available to answer individual questions beginning in mid-August. Once we know how school is going to begin, you will hear from me about specific dates. You will have a test on this material and Chapter 2 (Limits) around Labor Day. You may work together on this assignment. However, you will be taking the assessment without the benefit of anyone else's expertise, so you should help each other but don't just copy someone's work. In the long run, that won't pay off. Here are some words of advice from past students:

You definitely shouldn't try to just "wing-it." Practice a bit each day and go in for help whenever you need it- you will be glad!

It is very possible to be successful in AB Calculus if you do your homework and stay focused in class. Don't spend more than an hour on homework, but try to make sure you understand it. Reading the text is a helpful review of the day's notes and it gives you a lot of good examples. Try to stay focused in class and understand what is going on or you get behind and be lost. Overall this class has been challenging, but it was definitely my best class of the year. REMEMBER YOUR TRIG!

I think that the main thing that will help you out next year is to try and do the homework every night. If you do a small amount of homework each night it will save you the trouble of innumerable hours of studying before a particular test. You also need to do your best to pay close attention and participate in class. If you do pay close attention you will find homework and tests will be much easier.

If you're going to take Calculus you should expect to not have everything to click instantly. Some of the subject matter is a bit abstract and it can take a little while for it to sink in. There's not much to do in terms of plugging into formulas and getting an answer as in Algebra or Trigonometry. One nice thing about Calculus is that there is often a real world application that you can see and, most of the time, hold in your hands. The mathematics is truly beautiful when it all works out and you'll be proud of yourself when you finally grasp the concept behind the material as well as being able to use the actual equations and theories well for class.

I would periodically review my previous chapter test and work. I actually found this is very important, since this course kind of builds on itself. And problems in early chapters tend to "snowball" to later chapters.

Focus really hard on beginning chapters because a lot of what you learn later branches off these units.

Be ready to work hard, and if you've been able to slack off in math in the past, reverse those bad habits quickly or you may have already begun to dig yourself into a hole. Admit when you need help and get it as soon as a problem arises.

This class is certainly a challenge, but one that is definitely feasible if you work hard, and work with consistency. Getting a good grade on a test does not mean that you can stop paying attention for a couple of days – your grade will suffer. Keep track of all notes, quizzes, and tests – don't ever throw anything away!

It is the accumulation of all previous math plus more. You cannot forget the math you have learned because it all builds on in this class. However, it is not unbearable but rather manageable if you stick with it and work hard. If I could do it again, I would try to focus 110% every day and not take occasional days off.

I. Algebra II Review, Inequalities

1) Simplify each of the following, eliminating negative exponents and complex fractions:

a) $\dfrac{\dfrac{2}{x+1}-\dfrac{x}{x^2-x-2}}{\dfrac{3}{x+1}-\dfrac{2}{x-2}}$

b) $\dfrac{a^{-2}-b^{-2}}{(a+b)^{-1}}$

c) $|3x+4|<1$

d) $|5-3x|\geq 10$

e) $x^3>x$

f) $\dfrac{2x-1}{x+1}>0$

g) $\dfrac{2x-1}{x+1}<1$

2) Solve the following equations for the indicated variable:

a) $2x-2yd=y+xd$, for d

b) $\dfrac{2x}{4\pi}+\dfrac{1-x}{2}=0$, for x

II. Exponential and Logarithmic Functions

1) Solve for x without a calculator.

a) $\log_2 x=3$

b) $\log_3 x^2=2\log_3 4-4\log_3 5$

c) The answer to this equation lies between what two integers: $2\log_2 9-\log_2 3=x$

2) Simplify:

a) $\log(10)^{1/2}$

b) $\log\left(\dfrac{1}{10^x}\right)$

c) $2\log\sqrt{x}+3\log x^{1/3}$

d) $\ln 5+\ln(x^2-1)-\ln(x-1)$

III. Calculator Basics

1) Determine which of the following gives a complete graph for the indicated equation:

a) $y=-x^3+8x^2-x+5$

(i) [-10,10] x [-10,10]
(ii) [0,10] x [-10,80]
(iii) [-5,10] x [-10,80]
(iv) [-50,50] x [-100,100]

b) $f(x)=\dfrac{3x^2+x-5}{x^2+1}$ ***

(i) [-10,10] x [-10,10]
(ii) [-2,20] x [-20,20]
(iii) [0,20] x [-5,5]
(iv) [-5,20] x [-5,5]

***Not even these windows will catch a subtlety that only calculus can find for us! Stay **tuned!!!**

2) Estimate the maximum and minimum values (This means ***give the y-values*** of each of the following functions on their given intervals.

a) $f(x) = 2^x + x^2$; interval: [-4,1]

(i) Justify, in words, why you selected the minimum value. (Do not say "because my calculator said so"! Talk about the behavior of the curve.).

(ii) ***Where*** does the maximum value occur?

b) Go back to #1b) on the bottom of the previous page; interval: [15,17]

Please don't go back and change your answer to 1b). That was a good choice for a pre-cal student and will remain a good choice for a student who has taken calculus as well. However, calculus provides us with new skills to help us determine changes that aren't always so obvious, particularly to the human eye. Please give a "y" window where the graph of $f(x)$ does not look like a horizontal line.

c) $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$; interval: (-2,2)

(i) Explain why this function has no maximum on this interval.

(ii) Could you give an interval where this function would achieve a maximum value? Using your interval, what is the maximum value and where does it occur?

(NOTE: There is no one right answer to this question.)

3) Graph $y = \dfrac{x^2 - 9}{x - 3}$ on your TI-83.

a) Explain why this graph is a linear function instead of a rational function having a vertical asymptote at $x = 3$.

b) Zoom in on the graph around x = 3. Give the viewing window when you first see the empty pixel at x = 3 or you notice the graph getting "jagged" around there. (The newness of your calculator will determine which of these pictures you get.)

c) If you had to fill in a y-value when x = 3, what would it be?

4) Use the cubic equation from 1a) $y = -x^3 + 8x^2 - x + 5$.

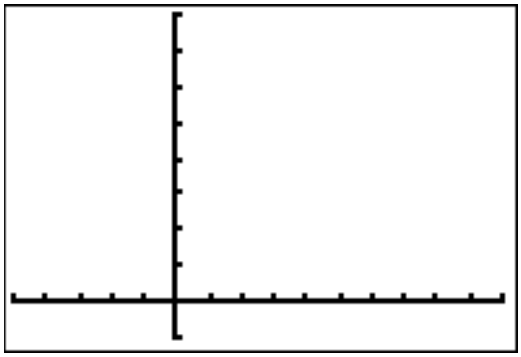
a) Find $y(-1)$ and $y(6)$.

b) Find the equation of the line passing through these two points.

c) Using your graphing calculator, graph the cubic and this line using a straight edge. Show this graph in the window on the next page .

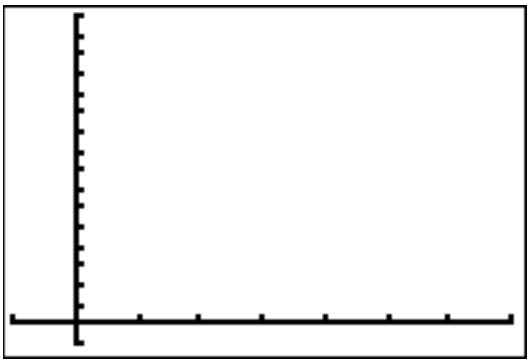
d) Using a ruler, draw two other lines (again using a straight edge) that are parallel to the line you found in part b) and are tangent to the graph of the cubic equation. (Don't worry – these lines will intersect the curve at some other point.) Give the x-value of the points of tangency of each line to the curve, correct to the nearest integer.

For #4c:



[-5,10] x [-10,80]

For #5b:



[,] x [,]

5) An open box is to be made from cutting squares of side s from each corner of a piece of cardboard 25" by 30".

a) Write an expression for the volume, V, of the box in terms of s.

b) Draw a graph of $V(s)$ on the graph to the upper right. You may use your graphing calculator. Fill in the window below the graph. Identify the domain and range of this graph.

c) What domain and range make sense in this problem situation? Highlight this on your original graph.

d) Find the value of s that will give the maximum volume. What is the maximum volume?

e) What value(s) of s will give a volume of 1225 cubic units?

IV. Cartesian Plane Basics

1) For what value of k is $5x + ky = 3$ parallel to $2x - 3y = 5$? For what value of k are the two lines perpendicular?

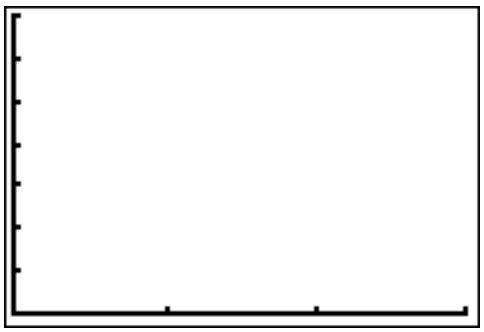
2) Find the line that passes through (-1,3) and the point of intersection of the lines $x + 3y = 1$ and $2x - y = -5$. Leave your answer in point slope form.

3) You need to rent a jet for one day. Knowing that Swissair rents a Lear jet with a pilot for $2000 a day and $1.75 per mile, while Air France rents a Lear jet with a pilot for $1500 a day and $2.00 a mile, find the following:

a) For each company, write a formula giving cost as a function of distance traveled.

b) Sketch graphs of both functions below, labeling intercepts and point of intersection.

c) If cost were the only issue, when would you rent from Air France?

For 3b: 

V. TOOLKIT Functions

You should be very familiar with the following functions and be able to envision them in your head without the advantage of a graphing calculator:

1. constant function: $f(x) = k$, k is a constant
2. linear function: $f(x) = mx + b$
3. identity function: $f(x) = x$
4. absolute value function: $f(x) = |x|$
5. piece-wise (or partitioned domain) function: $f(x) = \begin{cases} \sin x, \text{ when } x \le -1 \\ x - 3, \text{ when } x > -1 \end{cases}$
6. greatest integer function: $f(x) = [x]$
7. quadratic function: $f(x) = ax^2 + bx + c$
8. polynomial function: $f(x) = q_0 x^n + q_1 x^{n-1} + \ldots + q_1 x + p$
9. radical function: $f(x) = \sqrt{x}$
10. rational function: $f(x) = \dfrac{ax - b}{cx - d}$
11. exponential function: $f(x) = b^x$ where $b > 0, \neq 1$
12. logarithmic function: $f(x) = \log_b x$

1) Identify the domain and range of the following functions. ***Try*** to draw the graphs ***without*** the benefit of your calculator.

a) $k(t) = \begin{cases} |t-3| & \text{if } -3 \le t < 1 \\ 2 - t^2 & \text{if } 1 \le t < 4 \\ \frac{1}{2}t + 1 & \text{if } t \ge 4 \end{cases}$

b) $g(x) = 2\left[\dfrac{x}{2}\right] - 1$

c) $h(x) = 1 - 3\sqrt{2 + x}$

d) $f(x) = \dfrac{x^2 + 1}{x}$ (Use a calculator for now!)

2) Describe how each graph can be obtained from the graph of $f(x)=\sqrt{x}$, $g(x)=\frac{1}{x}$, h(x)=|x|, k(x)=x^3, l(x)=log$x$, or $m(x)=2^x$. (**HINT**: Think about order of operations!)

a) $y=2(x+3)^3-5$ b) $y=|x-1|+2$ c) $y=\frac{1}{2(3-x)}+1$

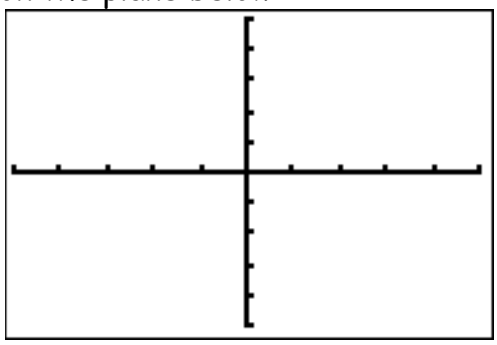
d) $y=2\sqrt{x-1}-3$ e) $y=-\frac{1}{2}\log(x+4)+2$ f) $y=2^{(2-x)}-4$

3) Given $f(x)=-(x+3)^2$ when $x\geq -3$.

a) Find the inverse, $f^{-1}(x)$, of $f(x)$.

b) Identify the domain and range of $f(x)$ and $f^{-1}(x)$.

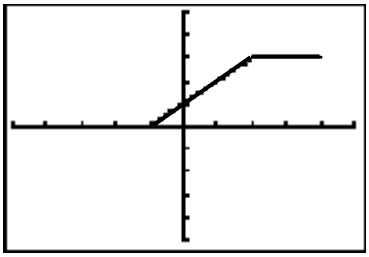
c) Graph both functions on the plane below:



[-5,5] x [-5,5]

d) Analytically prove $f(f^{-1}(x))=x$.

4) Given the following graph of $f(x)$:



[-5,5] x [-5,5]

Copy the given function onto your own paper and sketch a graph of the following functions using what you know about function transformations:

a) $f(x+2)$ b) $f(x)-1$ c) $3+f(x-1)$ d) $-\frac{1}{2}f(x)+3$

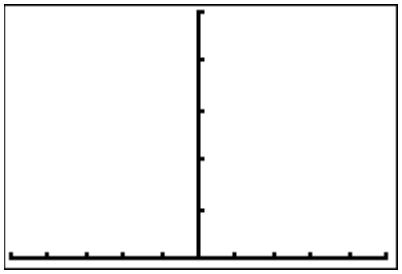
5) Identify the following functions as odd, even or neither:

a) $y = -x^3 + 2x - 11$ b) $y = 5x^2 - x^4$ c) $y = \dfrac{-1}{x^2 + 4}$ d) $y = -2x^3 + 4x$

VI. <u>Parametric Equations</u>: *You may use your graphing calculator.*

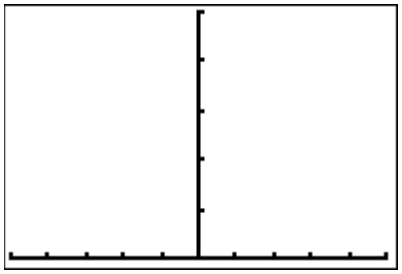
1) Parametric equations give us the ability to graph curves that are ***<u>not</u>*** functions. Graph the following three sets of curves, determine their corresponding Cartesian equations, and tell how you can determine which direction (up/down or left/right) the parabola (if, indeed, you do have the graph of a parabola) will open based on their parametric equations. Identify the initial and terminal points:

a) $\begin{matrix} x(t) = 2t - 1 \\ y(t) = t + 1 \end{matrix}$ on $\left[0,2\right]$



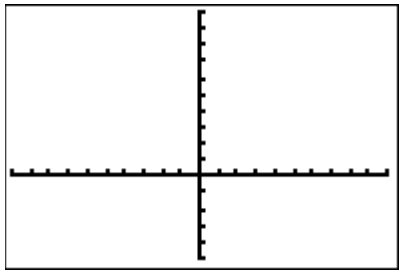
[-5,5] x [0,5]

b) $\begin{matrix} x(t) = \sqrt{t - 1} \\ y(t) = t - 1 \end{matrix}$ on [1,5]



[-5,5] x [0,5]

c) $\begin{matrix} x(t) = -t^2 + 3 \\ y(t) = 2t - 1 \end{matrix}$ on [-2,3]

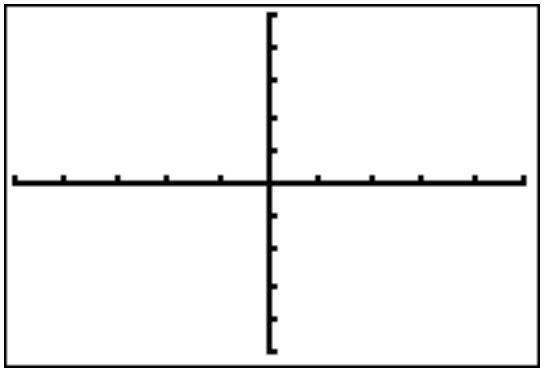


[-10,10] x [-5,10]

2) Graph $x = y^2 - 6y + 11$

a) in function mode using two functions (HINT: complete the square)

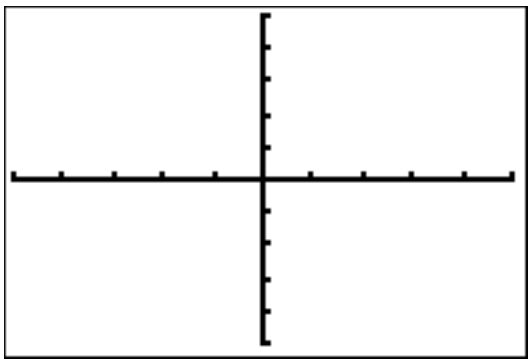
b) in parametric mode (You need to come up with the parametric equations.)



[-5,5] x [-5,5]

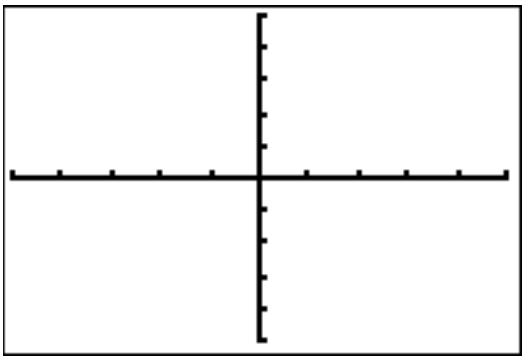
3) Graph and identify the kind of conic the graph represents:

a) $\begin{matrix} x(t)=3\cos t \\ y(t)=3\sin t \end{matrix}$ on $[0,2\pi]$



[-5,5] x [-5,5]

b) $\begin{matrix} x(t)=2\cos t \\ y(t)=5\sin t \end{matrix}$ on $[0,2\pi]$



[-5,5] x [-5,5]

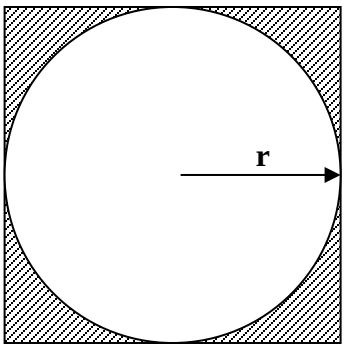
VII. Introduction to Calculus

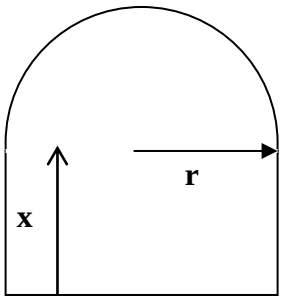
1) Simplify $\dfrac{f(x+h)-f(x)}{h}$ where

a) $f(x)=2x+3$ b) $f(x)=\dfrac{1}{x+1}$ c) $f(x)=x^2$

2) a) Find the ratio of the shaded area inside to the area of the square in the picture in (a) below.

b) Find a formula for the perimeter of a window of the shape in the picture in (b) below.





c) Two cars start moving from the same point. One travels south at 100 km/hr, while the other heads west at 50 km/hr. How far apart are they two hours later?

d) A kite is 100 m above the ground. If there are 200 m of string out, what is the angle between the string and the horizontal, assuming that the string is taut.

NO CALCULATOR on the ENTIRE SECTION

VIII. Trig Review

Your life will be a lot easier next year if you can remember your trig values for special angles, the basic shape of the six trig functions, their domain and range, and these identities:

$$\sin^2\theta+\cos^2\theta=1$$
$$\tan^2\theta+1=\sec^2\theta$$
$$1+\cot^2\theta=\csc^2\theta$$
$$\sin 2\theta=2\sin\theta\cos\theta$$
$$\sin^2\theta=\frac{1-\cos 2\theta}{2}$$
$$\cos^2\theta=\frac{1+\cos 2\theta}{2}$$

We always use radians and never work in degrees.

1) Evaluate paying attention to signs and number of rotations:

a) $\sin\pi$ b) $\cos\left(\frac{3\pi}{2}\right)$ c) $\tan\left(\frac{5\pi}{6}\right)$ d) $\sin\left(\frac{4\pi}{3}\right)$

e) $\cos\left(\frac{7\pi}{4}\right)$ f) $\sec\left(\frac{2\pi}{3}\right)$ g) $\cot\left(\frac{5\pi}{4}\right)$ h) $\csc\left(\frac{11\pi}{6}\right)$

i) $\sin\left(-\frac{27\pi}{6}\right)$ j) $\sin\left(-\frac{17\pi}{3}\right)$ k) $\sin\left(\frac{108\pi}{8}\right)$ l) $\cos\left(-\frac{11\pi}{4}\right)$

2) Change the following to radians paying attention to signs and number of rotations:

a) 510° b) 120° c) 135° d) -210° e) -315°

3) Identify which of the six trig functions are

a) odd
b) even
c) neither

4) Which equations have the same graph?

a) $y=\sin x$ b) $y=\sin(-x)$ c) $y=\cos(x)$
d) $y=-\sin(x)$ e) $y=-\cos(x)$

5) Find $\sin\theta$ or $\cos\theta$, whichever is not given, and state the quadrant θ lies in.

a) $\cos\theta=\frac{3}{5},\ \sin\theta>0$ b) $\sin\theta=\frac{1}{\sqrt{2}},\ \cos\theta<0$

c) $\sin\theta=-\frac{12}{13},\ \cos\theta>0$ d) $\cos\theta=\frac{\sqrt{5}}{5},\ \sin\theta>0$

6) Simplify:

a) $\cos\frac{2\pi}{3}\cdot\cos\frac{\pi}{6}+\sin\frac{2\pi}{3}\sin\frac{\pi}{6}$

b) $\cos\frac{2\pi}{3}\cdot\sin\frac{\pi}{6}+\cos\frac{\pi}{6}\cdot\sin\frac{2\pi}{3}$

7) Evaluate:

a) $\sin\left(\arcsin\frac{1}{2}+\arccos\frac{\sqrt{3}}{2}\right)$

b) $\tan\left(\arctan\sqrt{3}+\frac{\pi}{3}\right)$

c) $\tan\left(\arcsin\frac{1}{2}\right)$

d) $\sec\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$

8) Arrange these in order from least to greatest:

a) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$, $\arccos 0$, $\arccos\left(\frac{1}{2}\right)$

b) $\arctan\left(-\sqrt{3}\right)$, $\arctan 0$, $\arctan\left(\frac{1}{2}\right)$

IX. Functions

Be sure you can quickly and easily graph each of the following functions as well as fill in the following chart with pertinent information about each of them.

function	*domain*	*range*	*odd/even*	*periodic*	*one-to-one*
$y=x$					
$y=x^2$					
$y=x^3$					
$y=\lvert x\rvert$					
$y=\lfloor x\rfloor$					
$y=\frac{1}{x}$					
$y=\frac{1}{x^2}$					
$y=\sqrt{x}$					
$y=b^x$					
$y=\log_b x$					
$y=\sin x$					
$y=\cos x$					
$y=\tan x$					
$y=\sec x$					

2022 – 2023
AB Summer Work - Odd Answers

THIS NEEDS UPDATING!!!! SOME SECTIONS CHANGED/REMOVED

I.

1a) $\frac{x-4}{x-8}$ 1b) $\frac{3x(3x+2)}{(2x+1)^{3/2}}$ 1c) $\frac{(a+b)^2(b-a)}{a^2b^2}$ 1d) $\left(-\frac{5}{3},-1\right)$

1e) $\left(-\infty,-\frac{5}{3}\right]\cup[5,\infty)$ 1f) $(-1,0)\cup(1,\infty)$ 1g) $(-\infty,-1)\cup\left(\frac{1}{2},\infty\right)$ 1h) (-1,2)

II.

1a) {8} 1b) $\left\{\pm\frac{4}{25}\right\}$ 1c) between 4 and 5

III.

1a) iii. 1b) i. 3a) The numerator factors and cancels with the denominator.

3b) (2,4) 3c) $y(3)=6$

5a) $V(s)=s(30-2s)(25-2s)$

5b) See calculator; Domain: reals; Range: reals

5c) Domain: (0,12.5); Range: (0,1525)

5d) side $\cong 4.527''$; V(4.527) $\cong 1512.0403$ cu in

5e) V = 1225 at $s \cong 2.411''$ & $7.035''$

IV.

1) parallel: $-\frac{15}{2}$; perpendicular: $\frac{10}{3}$

3a) Swissair: C = 1.75d + 2000; Air France: C = 2d + 1500

3b) intersection point: (2000 mi, $5500)

4c) If your trip was less than 2000 miles

V.

1a) domain: $\Re \geq -3$; range: $(-14,1]\cup(2,\infty)$

1b) domain: reals; range: odd integers

1c) domain: $\Re \geq -2$; range: $\Re \leq 1$

1d) domain $\Re \neq 0$; range: $|\Re| \geq 2$

3a) $f^{-1}(x)=-3+\sqrt{-x}$

3b) $\text{domain}_f: \Re \geq -3$

$\text{range}_f: \Re \leq 0$

$\text{domain}_{f^{-1}}: \Re \leq 0$

$\text{range}_{f^{-1}}: \Re \geq -3$

3c) see calculator

3d) analysis

5a) neither 5b) even 5c) even 5d) odd

VI.

1a) $y=\frac{1}{2}x+\frac{3}{2}$; initial: (-1,1); terminal: (3,3)

1b) $y=x^2$; initial: (0,0); terminal: (2,4)

1c) $(y+1)^2 = 12 - 4x$; initial: (-1,-5); terminal: (-6,5)

If you do have the graph of a parabola, how do you tell if it opens □ or ↑↓?

3) See calculator – make sure you're in *radians!*

3a) is a circle with center at (0,0) and radius of 3 3b) is an ellipse with center at (0,0) and equation $\frac{x^2}{4} + \frac{y^2}{25} = 1$

VII.

1a) 2 1b) $-\frac{1}{(x+h+1)(x+1)}$ 1c) $2x+h$

VIII.

1a) 0 1b) 0 1c) $-\frac{\sqrt{3}}{3}$ 1d) $-\frac{\sqrt{3}}{2}$ 1e) $\frac{\sqrt{2}}{2}$ 1f) -2 1g) 1

1h) -2 1i) -1 1j) $\frac{\sqrt{3}}{2}$ 1k) -1 1l) $-\frac{\sqrt{2}}{2}$

3) odd: sin, csc, tan, cot; even: cos, sec; neither: none

5a) yes

5b) large: 1; small: -1

5c) large: 1; small: 0

5d) domain: real; range: [0,1]

7a) $\frac{4}{5}$; quad I

7b) $-\frac{1}{\sqrt{2}}$; quad II

7c) $\frac{5}{13}$; quad IV

7d) $\frac{2\sqrt{5}}{5}$; quad I

9a) $\frac{2\sqrt{2}}{3}$ 9b) $\frac{\sqrt{15}}{4}$ 9c) $\frac{\sqrt{3}}{3}$ 9d) $\frac{5}{3}$

11a) $\frac{\sqrt{3}}{2}$ 11b) $\frac{1}{2}$ 11c) $\frac{23}{25}$ 11d) $-\sqrt{3}$

I pledge that the work on this paper is my own and I abided by the directions given at the beginning of the assignment

I only used a calculator where it was indicated to do so was permissible.

I also pledge that I will <u>NOT</u> discuss the contents of this assignment with anyone else other than an AB Calculus classmate until it is graded and returned to me.