Name_____

Calculus Summer Review Packet

I. Introduction:

- This is the Calculus Summer Review Packet. It has multiple parts, so please read this introduction thoroughly. By completing this work carefully and accurately, you will be academically prepared to begin Calculus in September/January.

- If you have already completed Pre-Calculus, complete this work over the summer while the material is still familiar. If you are not scheduled for Calculus until Second Semester, be sure to periodically look over the material throughout the first semester.

- If you are taking Pre-Calculus during the first semester next year, **you are still responsible for completing this packet!** You are prepared to complete much of this even after only Algebra 2, so get those parts done over the summer. Then I suggest you periodically and systematically work on completing this throughout the first semester. **Do not wait until you have finished Pre-Calculus** to begin this packet because you will be too overwhelmed in those couple days between semesters!

II. Suggestions:

- Show any necessary work.
- If there is any confusion, there will be time for brief questions during the first several days of class.
- Feel free to get together and work on this, but **everyone must hand in his/her own assignment with the appropriate work.**
- You will be using a graphing calculator in this course, so be sure yours is in good working condition.

III. **Timeline** for the first week of class (brief general questions will be addressed in class):

- Day 1: Review: Real Numbers and Real Line (I.I), Cartesian Plane (I.II)
- Day 2: Review: Graphs of equations (I.III), Lines in the plane (I.IV)
- Day 3: Graphing Quiz (without a calculator!), Review: functions (I.V), trig. functions (I.VI)
- Day 4: Review logarithmic, exponential, and other transcendental functions (I.VII)
- Day 5: Summer Packet Test and Packet due.

IV. Graphs: be sure you are able to graph the following equations <u>without a calculator</u>:

$y=x$	$y=[x]$	$y=\frac{1}{x}$	$y=\sqrt{x}$	$y=\sin x$	$y=\csc x$
$y=\lvert x\rvert$	$y=x^3$	$y=c$	$y=e^x$	$y=\cos x$	$y=\sec x$
$y=x^2$	$y=\sqrt[3]{x}$	$x=c$	$y=\ln x$	$y=\tan x$	$y=\cot x$

V. Algebra/Pre-Calculus Outline:

Besides completing the actual written work, you need to be familiar and comfortable with the information in the following outline. **Before Unit 1 begins**, it is expected that you are familiar and comfortable with the terminology, notation, procedures, and graphs listed.

A. Functions

1. Formal definition and notation
2. Terminology
   a. domain and range
   b. odd and even
   c. increasing and decreasing
3. Inverse functions
   a. finding inverses
   b. testing if an inverse is also a function
   c. testing if two given functions are inverses
4. Composite functions
5. Intervals
   a. open (neighborhood)
   b. closed
   c. half-open half-closed
   d. notation
6. Compound/Piecewise Functions – evaluating

B. Linear Algebra

1. Graphing
2. Slopes
   a. formal definition
   b. parallel and perpendicular lines
3. Three forms and names
   a. point-slope
   b. slope-y-intercept
   c. standard/general **(new Ax+By+C=0)**
4. Finding linear equations including parallel and perpendicular to a given line

C. Absolute Values

1. Graphing
2. Solving

D. Trigonometry

1. Unit Circle
   a. definition
   b. equation
2. Radian measure
3. Graphs of all 6 functions $[-2\pi;2\pi]$
4. Identities:
   a. Pythagorean
   b. Reciprocal
   c. Cofunction

E. Quadratic equations

1. Graphing
2. Solving
   a. factoring
   b. completing the square
   c. quadratic formula

F. Miscellaneous

1. Distance formula
2. Midpoint formula

G. Exponents and logarithms

1. General notation
2. Logarithm Rules
3. Change of base formula
4. "e" "ln"
   a. definition of "e"
   b. relationship between "e" and "ln"
5. Other bases
6. General evaluating and solving

Print, Solve and Hand in this Problem set: show all necessary work.

I.I. Real Numbers and the Real Line

In Exercises 1-10, complete the table by filling in the appropriate interval notation, set notation, and graph on the real line:

	Inequality Notation	Interval Notation	Graph
1.	$x < 3$ or $x \geq 5$		
2.		$(-4,0) \cup (1,\infty)$	
3.			
4.	$-5 \leq x < 4$		
5.		$(-\infty,2) \cup (2,\infty)$	
6.			
7.	$x < -3$ or $-2 \leq x \leq 3$		
8.		$(-\infty,-4] \cup (-2,\infty)$	
9.			
10.	$x \leq -2$ or $-1 < x < 4$		

Solve inequality and graph the solution on the real line:

11. $2x+7<3$

12. $\frac{x}{2}-\frac{x}{3}>5$

13. $|x+2|<5$

14. $|3x+1|\geq 4$

15. In the manufacture and sale of a certain product, the revenue for selling x units is R=115.95x and the cost of producing x units is C=95x+750. In order for a profit to be realized, R must be greater than C. For what values of x will this product return a profit?

16. A utility company has a fleet of vans. The annual operating cost of each van is estimated to be C=0.32m+2300, where C is measured in dollars and m is measured in miles. If the company wants the annual operating cost of each van to be less than $10,000, then m must be less than what value?

17. A business had annual retail sales of $110,000 in 1993 and $224,000 in 1996. Assuming that the annual increase in sales followed a linear pattern, what were the retail sales in 1995?

I.II. The Cartesian Plane

In exercises 18-19, a) plot the points, b) find the distance between the points, and c) find the midpoint of the line segment joining the points:

18. (2,1), (4,5)

19. $(\frac{2}{3},-\frac{1}{3}),(\frac{5}{6},1)$

In exercises 20, find x so that the distance between the points is 5:

20. (2,-1), (x,2)

In exercises 21, find y so that the distance between the points is 8:

21. (5,1), (5,y)

I.III. Graphs of Equations

In exercises 22-27, match the given equation with its graph:

22. y=x-2

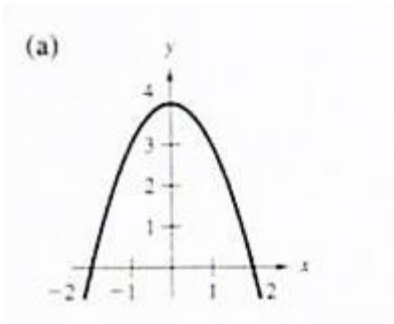
23. $y=-\frac{1}{2}x+2$

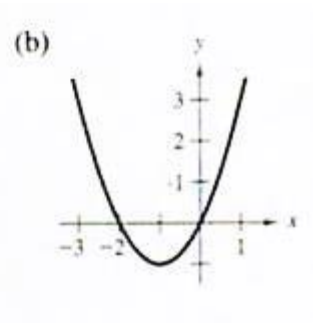
24. $y=x^2+2x$

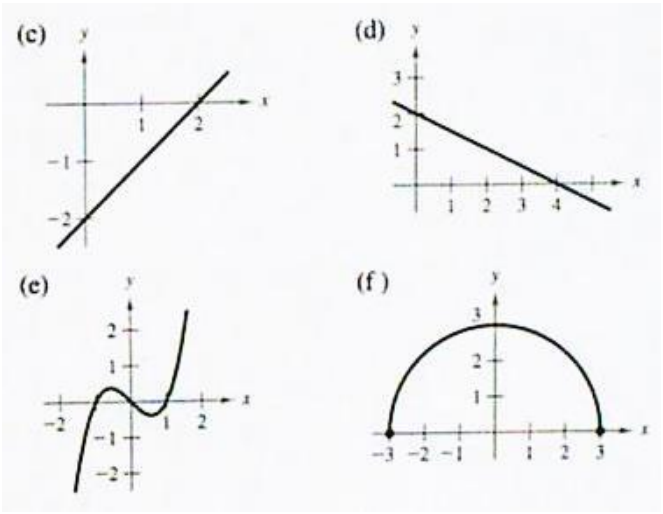
25. $y=\sqrt{9-x^2}$

26. $y=4-x^2$

27. $y=x^3-x$







In exercises 28-30, find intercepts:

28. $y=(x-1)(x-3)$

29. $y=x^3-4x$

30. $xy=4$

In exercises 31-32, sketch the graph of each equation, and identify the intercepts:

31. $y=2x-3$

32. $y=x^2+3$

In exercises 33-34, find the points of intersection of the graphs of the equations:

33. $-x^2+y=-14,\ 2x+y=10$

34. $x^2+y=5,\ -x-y=1$

In exercises 35, find the sales necessary to break even (R=C) for the given cost C of x units and the given revenue R obtained by selling x units:

35. $C=8650x+250{,}000 \quad R=9950x$

I.IV. Lines in the Plane

In exercises 36-37, plot the given pair of points and find the slope of the line passing through them:

36. (-2,1), (4,-3)

37. $(\frac{7}{8},\frac{3}{4}),(\frac{5}{4},-\frac{1}{4})$

In exercises 38-39. use the given point on the line and the slope of the line to find three additional points that the line passes through:

38. (-3,4), m is undefined

39. (-2,-2), m=2

In exercises 40-41, find the slope and y-intercept (if possible) of the line specified by the given equation:

40. 6x-5y=15

41. $y = -1$

In exercises 42-44, find an equation for the line that passes through the given points, and sketch the graph of the line:

42. (2,1), (0,-3)

43. (1,-2),(3,-2)

44. $\left(\frac{7}{8},\frac{3}{4}\right),\left(\frac{5}{4},-\frac{1}{4}\right)$

In exercises 45-47, find an equation of the line that passes through the given point and has the indicated slope:

45. (0,3) m=$\frac{3}{4}$

46. (-1,2) m is undefined

47. (0,4) m=0

In exercises 48-49, write an equation of the line through the given point (a) parallel to the given line and (b) perpendicular to the given line:

48. (2,1) 4x-2y=3

49. $\left(\frac{7}{8},\frac{3}{4}\right)$ 5x+3y=0

I.V. Functions

50. Given f(x)=2x-3, find the following:

(a) f(0) (b) f(-3) (c) f(b) (d) f(x-1)

51. Given f(x)=3x-1, find $\frac{f(x)-f(1)}{x-1}$

In exercises 52-57, find the domain and range of the given function, and sketch its graph:

52. $f(x)=4-x^2$

53. $f(x)=\frac{4}{x}$

54. $f(x)=\sqrt{x-1}$

55. $f(x)=\frac{1}{2}x^3+2$

56. f(x)=|x-2|

57. $f(x)=\frac{|x|}{x}$

58. Use the graph of f(x)= $\sqrt{x}$ to sketch the graph of each of the following:

(a) y= $\sqrt{x}+2$ (b) y= $-\sqrt{x}$ (c) y= $\sqrt{x-2}$

(d) y= $\sqrt{x+3}$ (e) y= $\sqrt{x}-4$ (f) y= $2\sqrt{x}$

59. Given f(x)= $\frac{1}{x}$ and g(x)=x^2-1, find the following:

(a) f(g(x)) (b) g(f(x)) (c) f(g($\frac{1}{\sqrt{2}}$))

(d) g(f($\frac{1}{\sqrt{2}}$)) (e) g(f(2)) (f) f(g(2))

In exercises 60, find the composite functions f(g(x)) (which is the same as f◦g) and g(f(x)) (which is the same as g◦f) . What is the domain of each function? Are the two composite functions equal?

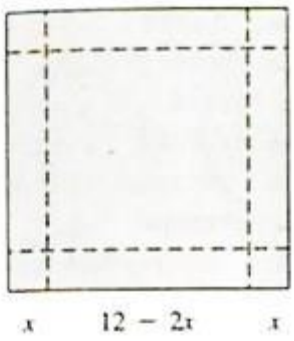
60. f(x)=x^3 , $g(x)=\sqrt[3]{x}$

In exercises 61-62, find the real zeros of the function:

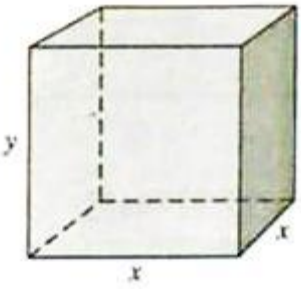
61. $f(x)=x^2-9$

62. $f(x)=x^3-x$

63. An open box is to be made from a square piece of material 12 inches on a side, by cutting equal squares from each corner and turning up the sides. Express the Volume V as a function of x.



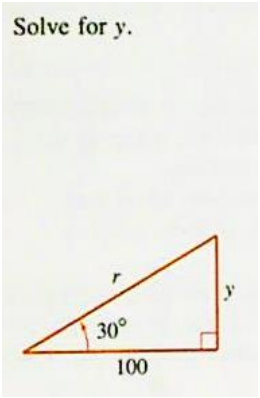
64. A closed box with a square base of side x has a surface area of 100 square feet. Express the volume V of the box as a function of x.



I.VI. Trigonometric Functions

In exercises 65-66, solve for x, y, or r as indicated:

#65.



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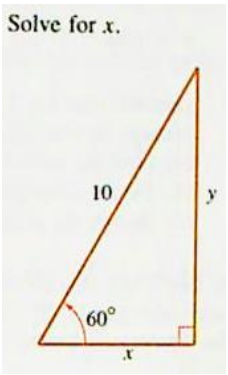

I.VII. Logarithmic, Exponential, and other Transcendental Functions

In exercises 67, use the properties of logarithms to write each expression as a sum, difference, or multiple of logarithms:

a. $\ln\frac{2}{3}$ b. $\ln\frac{xy}{z}$ c. $\ln(\frac{x^2-1}{x^3})^3$

d. $\ln\sqrt{a-1}$ e. $\ln\sqrt{2^3}$ f. $\ln\frac{1}{5}$ g. $\ln 3e^2$

In exercises 68, write each expression as a logarithm of a single quantity:

a. ln(x-2)-ln(x+2) b. 3lnx+2lny-4lnz c. $\frac{1}{3}\left[2\ln(x+3)+\ln x-\ln(x^2-1)\right]$

In exercises 69, solve for x:

(a) $e^{\ln x}=4$ (b) $\ln e^{2x}=3$

(c) $e^{\ln 2x}=12$ (d) $\ln e^{-x}=0$

(e) $\ln x=2$ (f) $e^{x}=4$

(g) $\ln x^2=10$ (h) $e^{-4x}=5$

(i) $\log_{10}1000=x$

(j) $\log_{10}.1=x$

(k) $\log_4 \frac{1}{64} = x$

(l) $\log_5 25=x$

(m) $\log_3 x=-1$

(n) $\log_2 x=-4$

(o) $\log_b 27=3$

(p) $\log_b 125=3$

(q) $x^2-x=\log_5 25$

(r) $3x+5=\log_2 64$

(s) $\log_3 x+\log_3(x-2)=1$

(t) $\log_{10}(x+3)-\log_{10}x=1$