Middle School Summer Review Packet for West Bloomfield Middle School Grade 8

(students who have completed Math 8 or Compacted Math 2)

Ten Topics

(Exercises begin on page 2, Explanations begin on page 21, and the Answer Key begins on page 37.)

____/10 1. **Proportional Relationships and Solving Story Problems**
(score) ***(Book 3 Chapter 1)***

____/10 2. **Simplifying and Evaluating Algebraic Expressions Equations**
(score) ***(Book 3 Chapter 2)***

____/10 3. **Solving Equations and the Distributive Property**
(score) ***(Book 3 Chapter 3)***

____/10 4. **Slope-Intercept Form ($y = mx + b$) and Graphs, Tables, Rules**
(score) ***(Book 3 Chapter 3 and 4)***

____/10 5. **Solving Equations with Fractional Coefficients and Systems of Equations** ***(Book 3 Chapter 5)***
(score)

____/10 6. **Rigid Transformations, Dilations, Similar Figures** ***(Book 3 Chapter 6)***
(score)

____/10 7. **Circle Graphs, Scatterplots, Best Line of Fit, Writing Equations from the Line of Best Fit** ***(Book 3 Chapter 7)***
(score)

____/20 8. **Compound Interest/Simple Interest, Scientific Notation, Laws of Exponents, Recognizing Functions** ***(Book 3 Chapter 8)***
(score)

____/10 9. **Parallel Line Angle Relationships, Unknown Angles in Triangles Pythagorean Theorem** ***(Book 3 Chapter 9)***
(score)

/10 10. **Volume and Surface Area of Cylinders, Pyramids, Cones, and Spheres** ***(Book 3 Chapter 10)***
____
(score)

______ **/160 TOTAL**

1. Proportional Relationships and Story Problems

Set up a proportion (two fractions equal to each other), then solve. No calculators!

Example: Henry can make 7 origami (folded-paper) cranes in 10 minutes. He read a story about a girl who made 1505 cranes, so he was curious about how long it would take him to make that many without stopping.

$$\frac{10\text{ minutes}}{7\text{ cranes}} = \frac{?\text{ minutes}}{1505\text{ cranes}}$$

$(1505 \cdot 10) \div 7 = 2150$ minutes, or 35 hours 50 minutes

1. The Abbott Middle School WEB class decided to have a taco sale. The school has 800 students, but the WEB students weren't sure how many to make. They took a survey to see how many students would buy a taco, and 25 out of 40 students surveyed said they would definitely buy a taco. How many tacos should the WEB students expect to sell?

2. If Sally can read 75 pages in 4 hours, how long will it take her to read 450 pages?

3. In line at the movies are 162 people in front of you. If you count 9 tickets sold in 70 seconds, how many minutes will it take before you buy your ticket?

4. When Sarah does her math homework, she finishes 10 problems every 12 minutes. How long will it take her to complete 35 problems?

5. While constructing his house, the little piggy found that 4 bundles of sticks contained 208 sticks. If each bundle has the same number of sticks, how many sticks in each bundle?

6. In the past year, Anna has spent $7440 on her rent. What is the unit cost (dollars per 1 month)?

Define a variable and then write and solve an equation for each problem below.

Example: Todd is 10 years older than Jordan.
The sum of their ages is 64.
How old are Todd and Jordan?

Let x = Jordan's age

So x + 10 = Todd's age

$$x + x + 10 = 64$$
$$2x + 10 = 64$$
$$\frac{2x}{2} = \frac{54}{2}$$
$$x = 27 \text{ (Jordan) and Todd is } 37$$

7. Sam and Susan are shooting marbles. Sam has five more marbles than Susan, and they have a total of 73 marbles. How many marbles does each of them have?

8. A cable 84 meters long is cut into two pieces so that one piece is 18 meters longer than the other. Find the length of each piece of cable.

9. West High School's population is 250 students fewer than twice the population of East High School. The two schools have a total of 2858 students. How many students attend East High School?

10. Mr. Smith needs to separate $385 into three parts to pay some debts. The second part must be five times as large as the first part. The third part must be $35 more than the first part. How much money must be in each part?

2. Simplifying and Evaluating Algebraic Expressions

1. $2x^2 + 1 + xy + x^2 + 2xy + 5$

2. $4m + 2mn + m^2 + m + 3m^2$

3. $8x^2 + 3x - 13x^2 + 10x^2 - 25x - x$

4. $3(5x - 2) - 2(-2x + 6)$

5. $20 + 5xy + 4y^2 + 10 + 10 + y^2 + xy$

Evaluate the expressions below for the given values.

6. $6m + 2n^2$ when $m = 7$ and $n = 3$

7. $(6x)^2 - \frac{x}{5}$ when $x = 10$

8. $6x^2 - \frac{x}{5}$ when $x = 10$

9. $(k - 3)(k + 2)$ when $k = 1$

10. $\frac{5x}{3} - 2$ when $x = -18$

3. Solving Equations and the Distributive Property

Solve each equation. Show all work.

1. $3 - 2x = 2x - 5$

2. $1 + 2x - 4 = -3 + x$

3. $x + (-4) + 3x = 2x - 1 + 3$

4. $2x - 7 = -x - 1$

5. $3(4 + x) = x + 6$

6. $6 - x - 4 = 4(x - 2)$

7. $-(-x - 3) = -4(x + 3)$

8. $-(x - 6) = -2(2x - 6)$

9. $5 + 2 - (x + 1) = 3x - 6$

10. $4x - (-3x + 2) = 7x - 3$

4. Slope-Intercept Form ($y = mx + b$) and Graphs, Tables, Rules

1. Look at the figures below. How many tiles would be in the 100th figure? Find as many ways as you can to justify your conclusion.

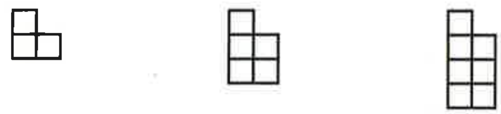


Figure 1 Figure 2 Figure 3

2. Study the tile pattern below.

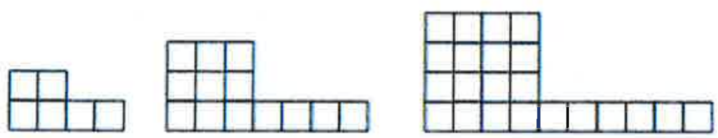


Figure 1 Figure 2 Figure 3

a. Draw Figure 3 and Figure 4. Explain how the pattern grows.

b. Write an equation (rule) for the number of tiles in the pattern.

3. Study the figures in the pattern below.

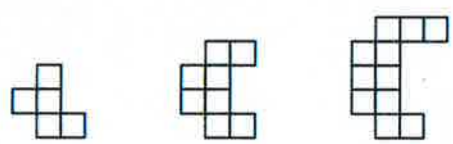


Figure 1 Figure 2 Figure 3

a. Draw Figures 4 and 5 on graph paper.

b. How many tiles will Figure 13 have?

c. How tall will Figure 106 be?

4. The figures below show progressively larger arrangements of squares.

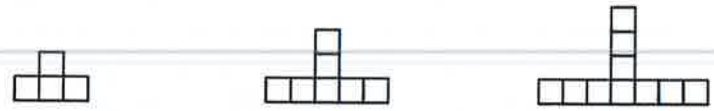


Figure 1 Figure 2 Figure 3

a. Copy and complete the table below.

x (figure number)	0	1	2	3	4	5	6
y (number of squares)							

b. Write an algebraic rule (equation) to show the relationship between y (the number of squares) and x (the figure number).

c. How many squares would be in Figure 100?

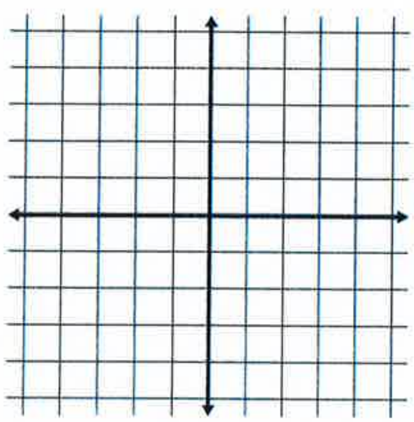
5. The points (2, 3) and (6, 11) lie on a line.

a. What is the growth rate of this line?

b. What is the y-intercept?

c. What is the equation of the line?

6. Make a complete graph of $y = \frac{1}{2}x - 3$.



7. Each box below contains information about cell phone use at a different high school. Based on the information you are given about each school, decide which one has had the greatest growth in cell phone use.

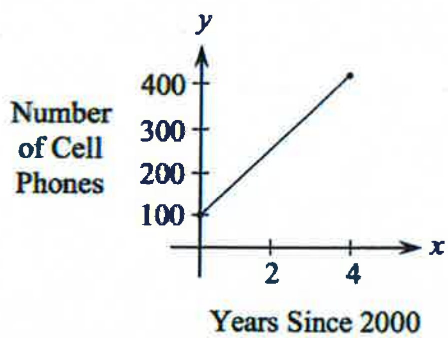
Kennedy High School (1750 students):

At this high school, only 350 students had a cell phone five years ago. Now, five years later (in 2004), 1167 students have cell phones.

American High School (1600 students):

Year	# of Phones
2000	200
2001	250
2002	280
2003	320
2004	400

Newark High School (1800 students):



Irvington High School (1450 students):

y = number of cell phones
x = years since 1999
$y = 45x + 600$ (as of 2004)

8. Write a "y =" equation for each of the following lines:

a. A line with a slope of –2 passing through the point (–11, 17).

b. A line with a slope of 0.25 and an <u>x-intercept</u> of –4.

9. Find the equation of the line that passes through the points (–4, 3) and (6, –2).

10. Use the patterns in the table to write a "y =" equation for the relationship.

input (x)	-3	-2	-1	0	1	2	3	4	5
output (y)	-11	-8	-5	-2	1	4	7	10	13

5. Solving Equations with Fractional Coefficients and Systems of Equations

Solve each equation.

1. $\frac{3}{4}x = 60$

2. $-\frac{8}{3}x = 6$

3. $\frac{x}{2} + \frac{x}{5} = 7$

4. $\frac{m}{3} - \frac{2m}{5} = \frac{1}{5}$

5. $\frac{(2x-3)}{6} = \frac{2x}{3} + \frac{1}{2}$

Solve each equation for the specified variable.

6. Solve for y: $5x + 3y = 15$

7. Solve for x: $y = \frac{1}{4}x + 1$

Find the point of intersection (x,y) for each system of linear equations. Use the "equal values" method.

8. $y = x - 6$
$y = 12 - x$

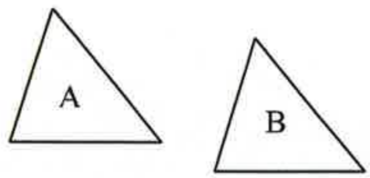
9. $y = 3x - 5$
$y = 2x + 14$

10. ***Write a system of linear equations to represent the problem, then solve.***
Larry and his sister, Betty, are saving money to buy their own laptop computers. Larry has \$215 and can save \$35 each week. Betty has \$380 and can save \$20 each week. When will Larry and Betty have the same amount of money?

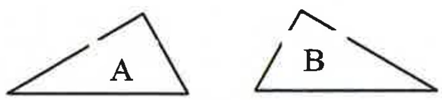
6. Rigid Transformations, Dilations, Similar Figures

Describe the transformation (translation, rotation, reflection) that moves triangle A to the location of triangle B.

1.



2.



Consider the point (3, -4).

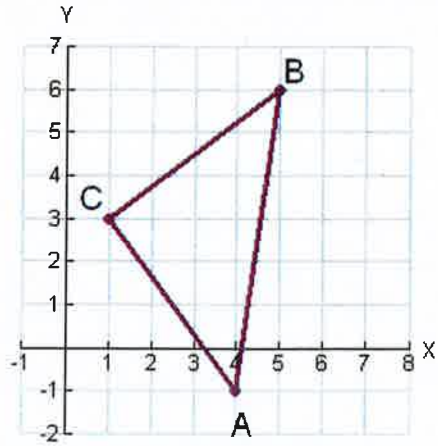
3. What are the coordinates of the point after a reflection across the y-axis?

4. What are the coordinates of the point after a reflection across the x-axis?

5. What are the coordinates of the point after a rotation 90° clockwise about the origin?

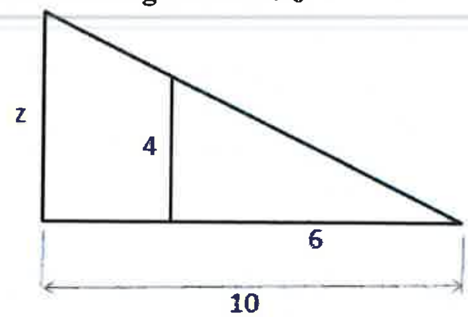
6. What are the coordinates of the triangle below after a dilation of 2?

A(,) B(,) C(,)

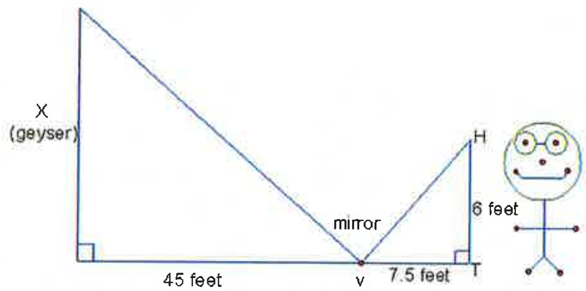


The following figures are similar. Set up a proportion and solve to find the missing side.

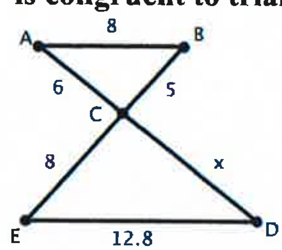
7. Find the length of side z.



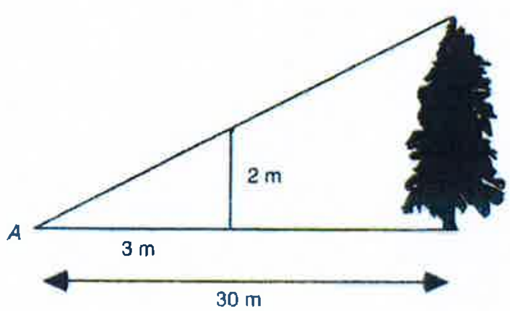
8. How tall is the geyser?



9. Triangle ABC is congruent to triangle DEC. Find the length of side x.

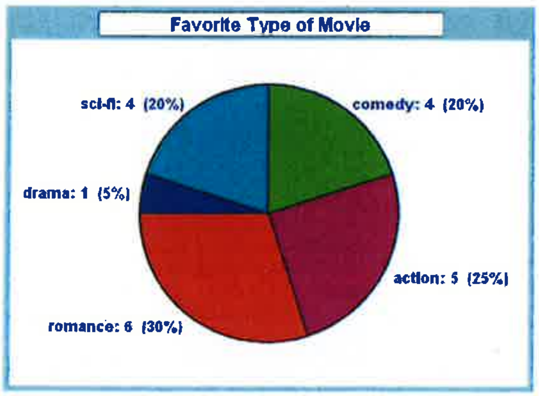


10. How tall is the tree?



7. Circle Graphs, Scatterplots, Best Line of Fit, Writing Equations from the Line of Best Fit

Answer the following questions about the circle graph below.

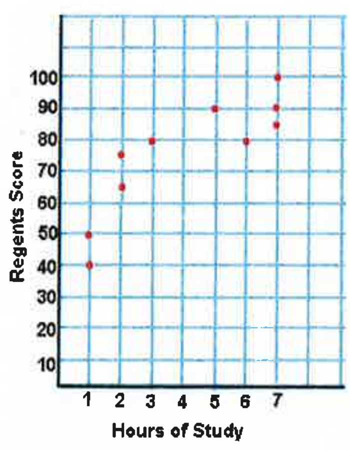


1. How many students are represented by the survey?

2. How many degrees should the central angle for the "action" sector receive?

3. How many degrees should the central angle for the "drama" sector receive?

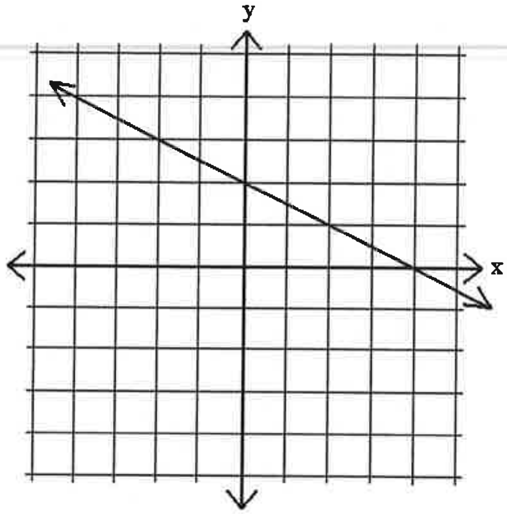
For questions 4 and 5, use the scatterplot below.



4. Which type of correlation does the scatterplot show: positive, negative, or no correlation?

5. Draw a best line of fit. What is the "y =" equation for your line?

For questions 6, 7, and 8, use the graph of the line below.

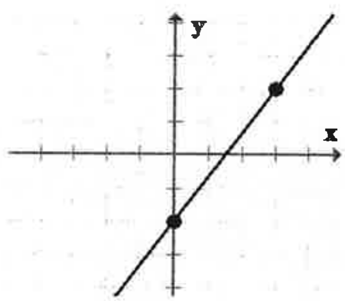


6. What is the slope of the line?

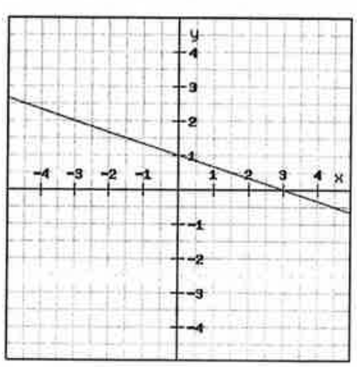
7. What is the y-intercept?

8. What is the equation of the line? (y =)

9. What is the equation of the line pictured below?



10. What is the equation of the line pictured below?



8. Compound Interest/Simple Interest, Scientific Notation, Laws of Exponents, Recognizing Functions

1. Using the formula for simple interest, ($I = prt$), calculate how much interest you would earn if you deposited \$200 for 5 years at an annual simple interest rate of 4%.

2. Using the formula for compound interest, $A = P(1 + r)^n$, calculate how much money you would have if Paris Hilton gave you \$2,000,000 and you invested it at 6% for 20 years.

3. Write the following number in scientific notation: .00000781

4. Write the following number in standard form: 2,340,000,000,000

Simplify each expression.

5. 5^{-3} **6.** $x^3 \cdot x^4$ **7.** $\frac{5^{16}}{5^{14}}$ **8.** $(x^4)^3$ **9.** $(4x^2y^3)^4$

10. $4(a^2b^{-2})^3$ **11.** $\frac{x^5y^4z^2}{x^4y^3z^2}$ **12.** $4(x^3)^2x^2$ **13.** $(4x^3)^2x^2$

Determine if the relationship is a function.

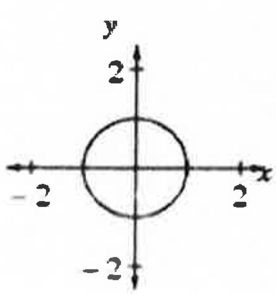
14.

x	7	–2	0	4	9	–3	6
y	6	–3	4	2	10	–3	0

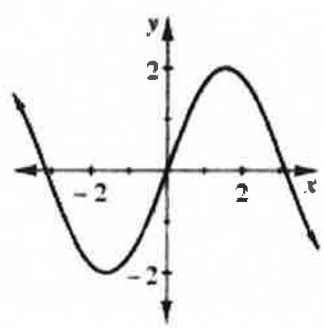
15.

x	3	–1	2	0	1	2	9
y	4	–5	9	7	4	–8	2

16.



17.

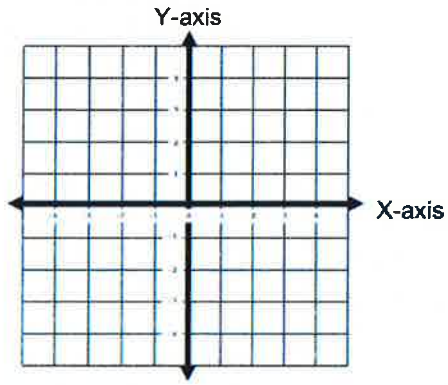


18. $y = \sqrt{x}$

19. $y = 2x^2$

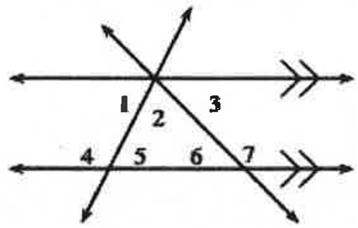
20. Graph the following points on the grid below and connect them in a smooth curve. Is it a function?

x	0	1	1	2	2
y	0	1	–1	4	–4

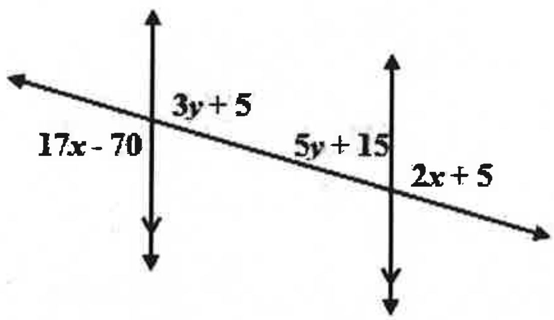


9. Parallel Line Angle Relationships, Unknown Angles in Triangles Pythagorean Theorem

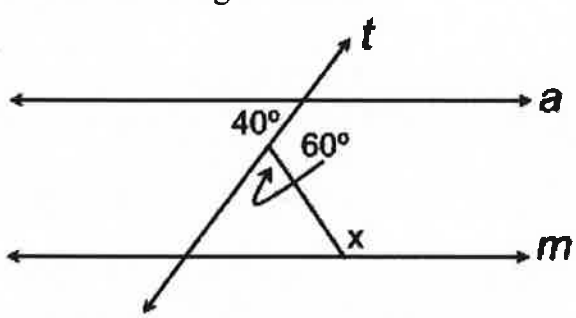
1. If the m∠ 5 = 53° and the m∠ 7 = 125°, find the measure of each numbered angle.



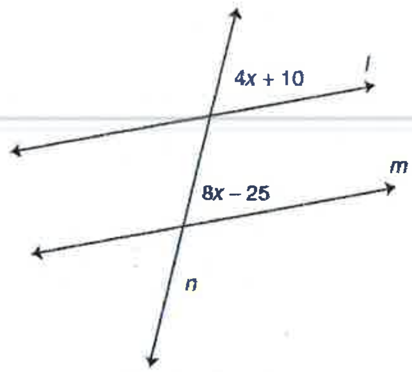
2. Solve for x and y in the diagram below.



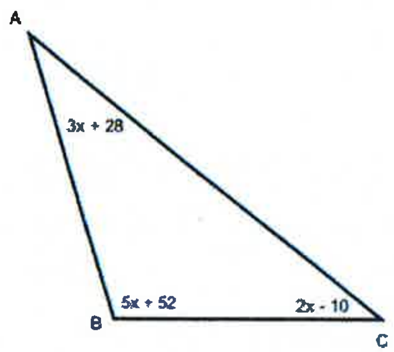
3. Solve for x in the diagram below.



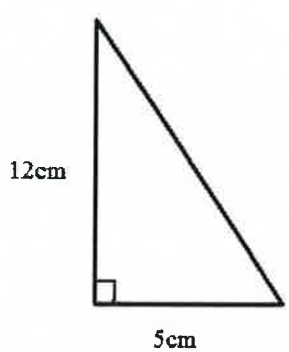
4. Solve for x in the diagram below.



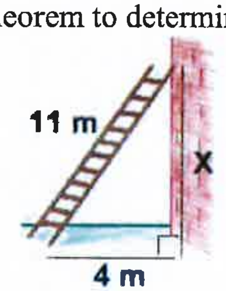
5. Solve for x in the diagram below.



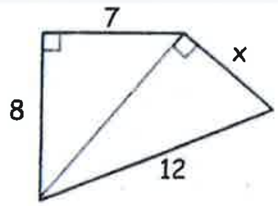
6. Use the Pythagorean Theorem to find the value of x.



7. Use the Pythagorean Theorem to determine how far up the wall the ladder reaches.

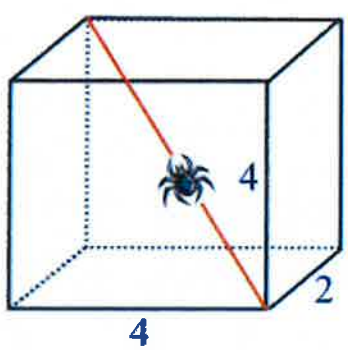


8. Use the Pythagorean Theorem to find the value of *x*.



9. To get from point A to point B you must avoid walking through a pond. To avoid the pond, you must walk 34 meters south and 41 meters east. To the nearest meter, how many meters would be saved if it were possible to walk through the pond? Draw a picture to represent the situation, then solve.

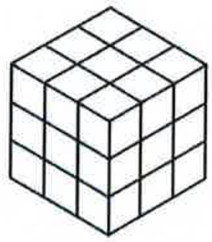
10. Find the length of the interior diagonal, shown in red.



10. Volume and Surface Area of Cylinders,

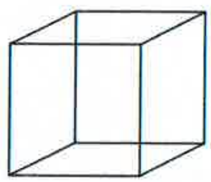
Pyramids, Cones, and Spheres

1. Find the surface area of the cube below:

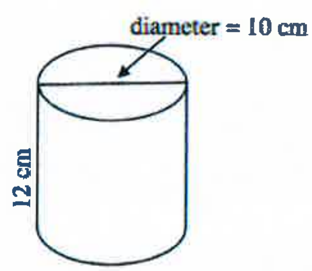


2. Find the volume of the cube in problem #1.

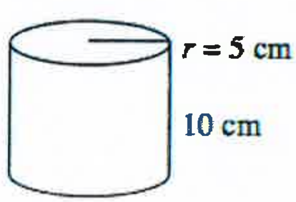
3. If the volume of the cube below is 4913 mm^3, how long is each side?



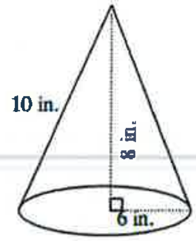
4. Find the volume of the cylinder below:



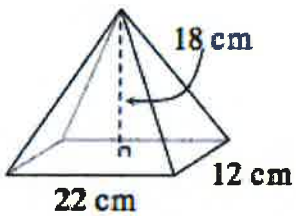
5. Find the surface area of the cylinder below:



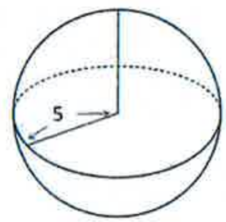
6. Find the volume of the cone at right:



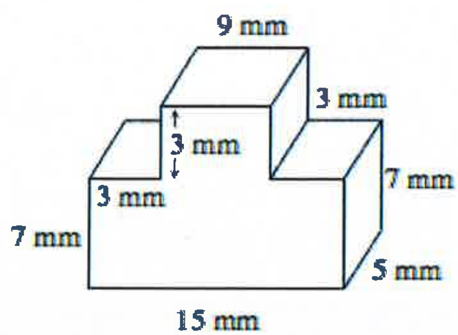
7. Find the volume of the pyramid below:



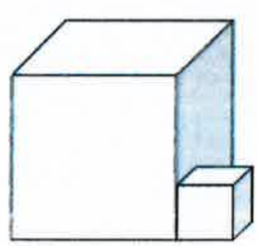
8. Find the volume of the sphere at right. The radius is 5 m.



9. Find the volume of the prism below. All angles are right angles.



10. The office building where Ryan works is made up of two cube-shaped pieces. At lunchtime, Ryan and his other office workers walk around the edge of the building for exercise. If the volume of the larger part is 64,000 m^3 and the volume of the smaller part is 4096 m^3, what is the distance around the edge of the building?



Explanations

1. Proportional Relationships and Story Problems

Solving Proportions

If a relationship is known to be proportional, ratios from the situation are equal. An equation stating that two ratios are equal is called a **proportion**. Some examples of proportions are:

$$\frac{6\text{ mi}}{2\text{ hr}} = \frac{9\text{ mi}}{3\text{ hr}} \qquad \frac{5}{7} = \frac{50}{70}$$

Setting up a proportion is one strategy for solving for an unknown part of one ratio. For example, if the ratios $\frac{9}{2}$ and $\frac{x}{16}$ are equal, setting up the proportion $\frac{x}{16} = \frac{9}{2}$ allows you to solve for x.

Cross-Multiplication: This method of solving the proportion is a shortcut for using a Fraction Buster (multiplying each side of the equation by the denominators).

Fraction Buster

$$\frac{x}{16} = \frac{9}{2}$$
$$2 \cdot 16 \cdot \frac{x}{16} = \frac{9}{2} \cdot 2 \cdot 16$$
$$2 \cdot x = 9 \cdot 16$$
$$2x = 144$$
$$x = 72$$

Cross-Multiplication

$$\frac{x}{16} = \frac{9}{2}$$
$$\frac{x}{16} \times \frac{9}{2}$$
$$2 \cdot x = 9 \cdot 16$$
$$2x = 144$$
$$x = 72$$

Solving Story Problems by Writing Equations using the 5–D Process

The 5-D Process is an organized method to help write equations and solve problems. The D's stand for Describe/Draw, Define, Do, Decide, and Declare. An example of this work is shown below.

Example Problem: The base of a rectangle is 13 centimeters longer than the height. If the perimeter is 58 centimeters, find the base and the height of the rectangle.

Describe/Draw: The shape is a rectangle, and we are looking at the perimeter.

	Define		Do	Decide
	Height (trial)	Base (height + 13)	Perimeter 2(base) + 2(height)	58?
Trial 1:	10	10 + 13 = 23	2(23) + 2(10) = 66	66 is too high

Use any trial value.

Use the relationships stated in the problem to determine the values of the other quantities (such as base and perimeter).

Now use the trial to create an equation by defining and adding a variable line.

Let x represent the height in cm	x	$x + 13$	$2(x) + 2(x + 13)$	$2(x) + 2(x + 13) = 58$

Now use your algebra skills to solve the equation.

$$2(x) + 2(x+13) = 58$$
$$2x + 2x + 26 = 58$$
$$4x + 26 = 58$$
$$4x = 32$$
$$x = 8$$

Declare: The base is 21 centimeters, and the height is 8 centimeters.

2. Simplifying and Evaluating Algebraic Expressions

Simplifying Algebraic Expressions by Combining Like Terms

Combining tiles that have the same area to write a simpler expression is called **combining like terms**. See the example shown below.

$x^2 + 2x + 2$

When you are not working with actual tiles, it can help to picture the tiles in your mind. You can use these images to combine the terms that are the same. Here are two examples:

Example 1: $2x^2 + xy + y^2 + x + 3 + x^2 + 3xy + 2 \quad \Rightarrow \quad 3x^2 + 4xy + y^2 + x + 5$

Example 2: $3x^2 - 2x + 7 - 5x^2 + 3x - 2 \quad \Rightarrow \quad -2x^2 + x + 5$

A **term** is an algebraic expression that is a single number, a single variable, or the product of numbers and variables. The simplified algebraic expression in Example 2 above contains three terms. The first term is $-2x^2$, the second term is x, and the third term is 5.

Evaluating Expressions and the Order of Operations

To **evaluate** an algebraic expression for particular values of the variables, replace the variables in the expression with their known numerical values and simplify. Replacing variables with their known values is called **substitution**. An example is provided below.

Evaluate $4x - 3y + 7$ for $x = 2$ and $y = 1$.

Replace x and y with their known values of 2 and 1, respectively, and simplify.

$4(2) - 3(1) + 7$
$= 8 - 3 + 7$
$= 12$

When evaluating a complex expression, you must remember to use the **Order of Operations** that mathematicians have agreed upon. As illustrated in the example below, the order of operations is:

Original expression: $(10-3\cdot 2)\cdot 2^2 - \frac{13-3^2}{2} + 6$

Circle expressions that are groupedwithin parentheses or by a fraction bar: $(10-3\cdot 2)\cdot 2^2 - \frac{13-3^2}{2} + 6$

Simplify *within* circled terms using the order of operations:

- Evaluate exponents.
- Multiply and divide from left to right
- Combine terms by adding and subtracting from left to right.

$$(10-3\cdot 2)\cdot 2^2 - \frac{13-3\cdot 3}{2} + 6$$

$$(10-6)\cdot 2^2 - \frac{13-9}{2} + 6$$

$$4\cdot 2^2 - \frac{4}{2} + 6$$

Circle the remaining terms: $4\cdot 2\cdot 2 - \frac{4}{2} + 6$

Simplify *within* circled terms using the Order of Operations as described above.

$$16 - 2 + 6$$

$$20$$

3. Solving Equations and the Distributive Property

Solving a Linear Equation

When solving an equation like the one shown below, several important strategies are involved.

Simplify. Combine like terms and "make zeros" on each side of the equation whenever possible.

Keep equations balanced. The equal sign in an equation indicates that the expressions on the left and right are balanced. Anything done to the equation must keep that balance.

Get x alone. Isolate the variable on one side of the equation and the constants on the other.

$$
\begin{aligned}
3x - 2 + 4 &= x - 6 \\
3x + 2 &= x - 6 && \text{combine like terms} \\
-x \quad &= -x && \text{subtract } x \text{ on both sides} \\
\hline
2x + 2 &= -6 \\
-2 &= -2 \\
\hline
\frac{2x}{2} &= \frac{-8}{2} && \text{subtract 2 on both sides} \\
x &= -4 && \text{divide both sides by 2}
\end{aligned}
$$

Undo operations. Use the fact that addition is the opposite of subtraction and that multiplication is the opposite of division to solve for x. For example, in the equation $2x = -8$, since the 2 and x are multiplied, dividing both sides by 2 will get x alone.

The Distributive Property

The **Distributive Property** states that for any three terms a, b, and c:

$$a(b + c) = ab + ac$$

That is, when a multiplies a group of terms, such as $(b + c)$, it multiplies *each* term of the group. For example, when multiplying $2(x + 4)$, the 2 multiplies both the x and the 4. This can be represented with algebra tiles, as shown below.

$$2(x+4) = 2 \cdot x + 2 \cdot 4 = 2x + 8$$

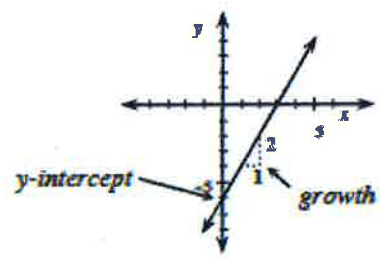
The 2 multiplies each term.

4. Slope-Intercept Form ($y = mx + b$) and Graphs, Tables, Rules

Linear Equations

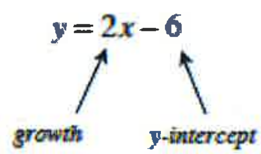
A **linear equation** is an equation that forms a line when it is graphed. This type of equation may be written in several different forms. Although these forms look different, they are equivalent; that is, they all graph the same line.

Standard form: An equation in $ax + by = c$ form, such as $-6x + 3y = 18$.

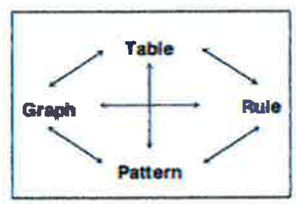


***y* = *mx* + *b* form:** An equation in $y = mx + b$ form, such as $y = 2x - 6$.

You can quickly find the **growth** and ***y*-intercept** of a line in $y = mx + b$ form. For the equation $y = 2x - 6$, the growth is 2, while the y-intercept is (0, −6



Representations of Patterns Web (graphs, tables, rules)



Consider the **tile patterns** below. The number of tiles in each figure can also be represented in an $x \rightarrow y$ **table**, on a **graph**, or with a **rule** (equation).

Remember that in this course, tile patterns will be considered to be elements of continuous relationships and thus will be graphed with a continuous line or curve.

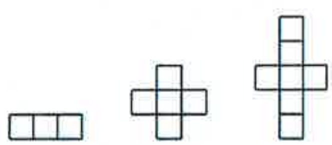
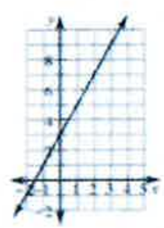


Figure 0 Figure 1 Figure 2

Tile Pattern



Graph

$y = 2x + 3$

Rule (Equation)

Figure Number (x)	0	1	2
Number of Tiles (y)	3	5	7

$x \rightarrow y$ Table

5. Solving Equations with Fractional Coefficients and Systems of Equations

Solving Equations with Fractions (also known as Fraction Busters)

Example: Solve $\frac{x}{3} + \frac{x}{5} = 2$ for x.

This equation would be much easier to solve if it had no fractions. Therefore, the first goal is to find an equivalent equation that has no fractions.

$\frac{x}{3} + \frac{x}{5} = 2$

To eliminate the denominators, multiply both sides of the equation by the common denominator. In this example, the lowest common denominator is 15, so multiplying both sides of the equation by 15 eliminates the fractions. Another approach is to multiply both sides of the equation by one denominator and then by the other.

The lowest common denominator of $\frac{x}{3}$ *and* $\frac{x}{5}$ *is* 15.

$15 \cdot (\frac{x}{3} + \frac{x}{5}) = 15 \cdot 2$

$15 \cdot \frac{x}{3} + 15 \cdot \frac{x}{5} = 15 \cdot 2$

Either way, the result is an equivalent equation without fractions:

$5x + 3x = 30$

$8x = 30$

Solving Systems of Equations Using the Equal Values Method

The **Equal Values Method** is a non-graphing method to find the point of intersection or solution to a system of equations.

Start with two equations in $y = mx + b$ form, such as $y = -2x + 5$ and $y = x - 1$. Take the two expressions that equal y and set them equal to each other. Then solve this new equation to find x. See the example at right.

$$-2x + 5 = x - 1$$

$$-3x = -6$$

$$x = 2$$

Once you know the x-coordinate of the point of intersection, substitute your solution for x into *either* original equation to find y. In this example, the first equation is used.

$$y = -2x + 5$$

$$y = -2(2) + 5$$

$$y = 1$$

A good way to check your solution is to substitute your solution for x into*both* equations to verify that you get equal y-values.

$$y = x - 1$$

$$y = (2) - 1$$

$$y = 1$$

Write the solution as an ordered pair to represent the point on the graph where the equations intersect.

(2 ,1)

6. Rigid Transformations, Dilations, Similar Figures

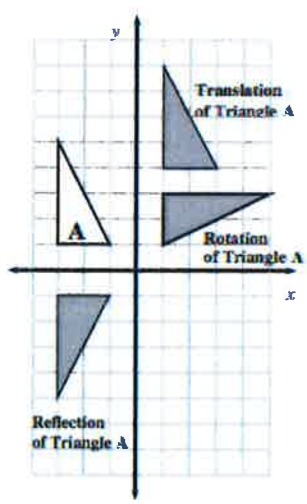
Rigid Transformations

Rigid transformations are ways to move an object while not changing its shape or size. Specifically, they are translations (slides), reflections (flips), and rotations (turns). Each movement is described at right.

A **translation** slides an object horizontally (side-to-side), vertically (up or down), or both. To translate an object, you must describe which direction you will move it, and how far it will slide. In the example at right, Triangle A is translated 4 units to the right and 3 units up.

A **reflection** flips an object across a line (called a **line of reflection**). To reflect an object, you must choose a line of reflection. In the example at right, Triangle A is reflected across the x-axis.

A **rotation** turns an object about a point. To rotate an object, you must choose a point, direction, and angle of rotation. In the example at right, Triangle A is rotated 90° clockwise (↻) about the origin (0, 0).



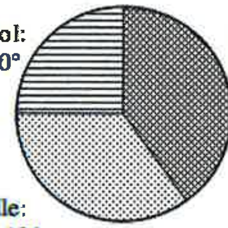
Dilating a Figure

To dilate a figure, multiply each of the original coordinates by the dilation factor.

Similar Figures

Two figures that have the same shape but not necessarily the same size are similar. In similar figures the measures of the corresponding angles are equal and the ratios of the corresponding sides are proportional. This ratio is called the scale factor.

7. Circle Graphs, Scatterplots, Line of Best Fit, Writing Equations from the Line of Best Fit



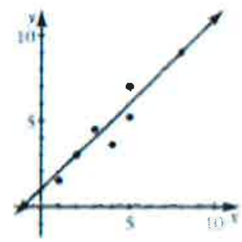
Circle Graphs

A **circle graph** (sometimes called a **pie chart**) is similar to a bar graph in that it deals with categorical data (such as make of car or grade in school) and not continuous data (such as age or height).

Each category of data is put into its own sector of the circle. The measure of the central angle bounding the sector is proportional to the percent of elements of that type of the whole. For example, if Central Schools has 40% of its students in elementary school, 35% in middle school, and 25% in high school, then its circle graph would have a central angle of 144° (0.4 times 360°) for the sector showing the elementary school, 126° for the sector showing the elementary school, and 90° for the sector showing the high school.

Line of Best Fit

A **line of best fit** is a straight line drawn through the center of a group of data points plotted on a scatterplot. It represents a set of data for two variables. It does not need to intersect each data point. Rather, it needs to approximate the data. A line of best fit looks and "behaves" like the data, as shown in the example at right.



Slope of a Line

The **slope** of a line is the ratio of the change in y to the change in x between any two points on the line. To find slope, you compute the *ratio* that indicates how y-values are changing with respect to x-values. Essentially, slope is the unit rate of change, because it measures how much y increases or decreases as x changes by one unit. If the slope is positive (+), the$y$-values are increasing. If it is negative (–), the y-values are decreasing. The graph of a line goes up for positive slopes and down for negatives slopes as the line moves across the graph from left to right.

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}}$$

Some textbooks write this ratio as $\frac{rise}{run}$..

8. Compound Interest/Simple Interest, Scientific Notation, Laws of Exponents, Recognizing Functions

Simple Interest

Simple interest is interest paid only on the original amount of the principal at each specified interval (such as annually, or monthly). The formula to calculate simple interest is:

$I = Prt$ where P = Principal
I = Interest
r = Rate
t = Time

Example: Theresa invested $1425.00 in a savings account at her local bank. The bank pays a simple interest rate of 3.5% annually. How much money will Theresa have after 4 years?

$$I = Prt \quad \Rightarrow \quad I = 1425(0.035)(4) = \$199.50$$

$$\Rightarrow \quad P + I = \$1425 + \$199.50 = \$1624.50$$

Theresa will have $1624.50 after 4 years.

Compound Interest

Compound interest is interest paid on both the original principal (amount of money at the start) and the interest earned previously.

The formula for compound interest is: $A = P(1 + r)^n$

where A = total amount including previous interest earned,
P = principal,
r = interest rate for each compounding period, and
n = number of time periods

Example: Theresa has a student loan that charges a 1.5% monthly compound interest rate. If she currently owes $1425.00 and does not make a payment for a year, how much will she owe at the end of the year (12 months)?

$$A = P(1+r)^n \quad \Rightarrow \quad A = 1425(1+0.015)^{12}$$

$$\Rightarrow \quad 1425(1.015)^{12} = 1425 \cdot 1.1956 = \$1703.73$$

Theresa will owe $1703.73 after 12 months (1 year).

Scientific Notation

Scientific notation is a way of writing very large and very small numbers compactly. A number is said to be in scientific notation when it is written as a product of two factors as described below.

- The first factor is less than 10 and greater than or equal to 1.
- The second factor has a base of 10 and an integer exponent.
- The factors are separated by a multiplication sign.

Scientific Notation	Standard Form
5.32×10^{6}	5,320,000
3.07×10^{-4}	0.000307
2.61×10^{-15}	0.00000000000000261

Laws of Exponents

Expressions that include exponents can be expanded into factored form and then rewritten in simplified form.

Expression	Factored Form	Simplified Form
$(5x)^3(2y)(x^2)y$	$5 \cdot x \cdot 5 \cdot x \cdot 5 \cdot x \cdot 2 \cdot y \cdot x \cdot x \cdot y$	$250x^5y^2$

The **Laws of Exponents** summarize several rules for simplifying expressions that have exponents. The rules below are true if $x \neq 0$ and $y \neq 0$

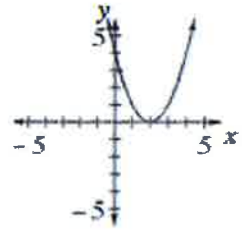
$$x^a \cdot x^b = x^{(a+b)} \qquad (x^a)^b = x^{ab} \qquad \frac{x^a}{x^b} = x^{(a-b)}$$

$$x^0 = 1 \qquad (x^a y^b)^c = x^{ac} y^{bc} \qquad x^{-a} = \frac{1}{x^a}$$

Functions

A relationship between inputs and outputs is a **function** if there is no more than one output for each input. A function is often written in a form where y is set equal to some expression involving x. In this " $y =$ " form, x is the input and y is the output. Below is an example of a function.

$$y = (x - 2)^2$$

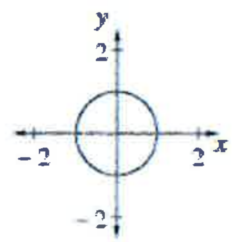


x	−2	−1	0	1	2	3	4	5
y	16	9	4	1	0	1	4	9

In the example above the value of y depends on x. Therefore, y is called the **dependent variable** and x is called the **independent variable**.

The equation $x^2 + y^2 = 1$ is not a function because there are two y-values (outputs) for some x-values, as shown below. .

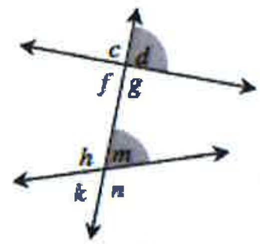
$$x^2 + y^2 = 1$$



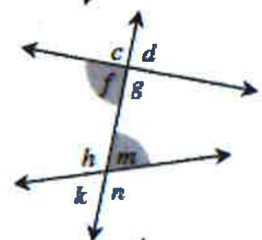
x	−1	0	0	1
y	0	−1	1	0

9. Parallel Line Angle Relationships, Unknown Angles in Triangles Pythagorean Theorem

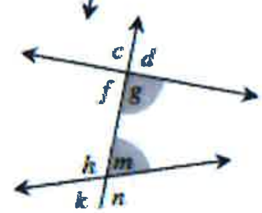
Parallel Lines and Angle Pairs



Corresponding angles lie in the same position but at different points of intersection of the transversal. For example, in the diagram at right, $\angle m$ and $\angle d$ form a pair of corresponding angles, since both of them are to the right of the transversal and above the intersecting line. Corresponding angles are congruent when the lines intersected by the transversal are parallel.

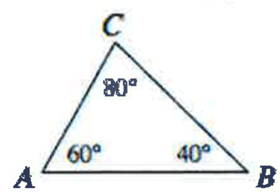


$\angle f$ and $\angle m$ are **alternate interior angles** because one is to the left of the transversal, one is to the right, and both are between (inside) the pair of lines. Alternate interior angles are congruent when the lines intersected by the transversal are parallel.



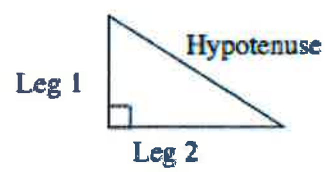
$\angle g$ and $\angle m$ are **same side interior angles** because both are on the same side of the transversal and both are between the pair of lines. Same side interior angles are supplementary when the lines intersected by the transversal are parallel.

Angle Sum Theorem for Triangles



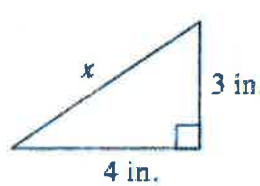
The measures of the angles in a triangle add up to 180°. For example, in ΔABC at right, $m\angle A + m\angle B + m\angle C = 180°$.

Right Triangles and the Pythagorean Theorem



A right triangle is a triangle in which the two shorter sides form a right (90°) angle. The shorter sides are called **legs**. The third and longest side, called the **hypotenuse**, is opposite the right angle.
The **Pythagorean Theorem** states that for any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. $(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$

Example:



$3^2 + 4^2 = x^2$
$9 + 16 = x^2$
$25 = x^2$
$5 = x$

10. Volume and Surface Area of Cylinders, Pyramids, Cones, and Spheres

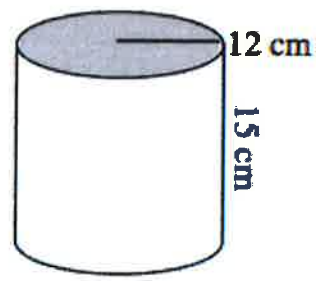
Volume of a Cylinder

The volume of a cylinder can be calculated in exactly the same way as the volume of a prism. First divide the cylinder into layers that are each one unit high. Then, to calculate the total volume, multiply the volume of one layer by the number of layers it takes to fill the shape. The volume of a cylinder can also be calculated by multiplying the area of the base (B) by the height (h).

The volume of a cylinder can also be calculated by multiplying the area of the base (B) by the height (h).

Volume = (area of base)(height)

$V = Bh = (r^2\pi)(h)$



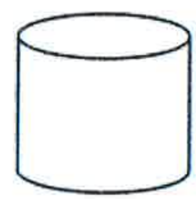
For example, for the cylinder at right,

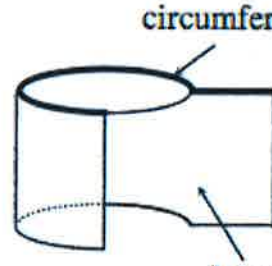
Area of the base: $B = (12)^2\pi = 144\pi$

Volume: $V = 144\pi(15) = 2160\pi \approx 6785.84 \text{ cm}^3$

Surface Area of a Cylinder

A **cylinder** has two congruent, circular bases. The **lateral surface** of the cylinder, when opened flat, forms a rectangle with a height equal to the height of the cylinder and a width equal to the circumference of the cylinder's base.





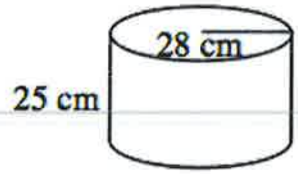
The surface area of a cylinder is the sum of the two base areas and the lateral surface area. The formula for the surface area is:

$S.A. = 2r^2\pi + \pi\, dh$

$= 2r^2\pi + 2\pi\, rh$

where r = radius, d = diameter, and h = height of the cylinder.

For example, to find the surface area of the cylinder at right:



Area of the two circular bases:

$2(28\text{ cm})^2\,\pi = 1568\pi\text{ cm}^2$

Area of the lateral face:

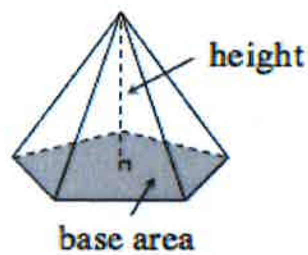
$\pi(56)(25) = 1400\text{ cm}^2$

Total Surface Area = $1568\pi\text{ cm}^2 + 1400\pi\text{ cm}^2 = 2968\text{ cm}^2$

$\approx 9324.25\text{ cm}^2$

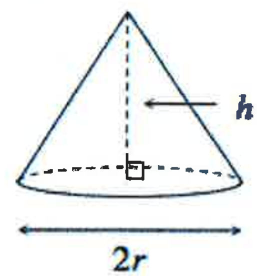
Volume of a Pyramid and a Cone

In general, the **volume of a pyramid** is one-third of the volume of the prism with the same base area and height. Thus:



$$V = \frac{1}{3}(\text{base area})(\text{height})$$

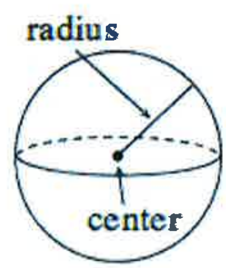
The **volume of a cone** is one-third of the volume of the cylinder with the same radius and height. Therefore, the volume of a cone can be found using the formula shown below, where r is the radius of the base and h is the height of the cone.



$$V = \frac{1}{3}(\text{base area})(\text{height}) = \frac{1}{3}\pi r^2 h$$

Volume of a Sphere

A sphere is formed by a set of points that are equidistant from a fixed point, its center. It is three-dimensional.



The volume of a sphere is twice the volume of a cone with the same radius and height. Since the volume of a cone with radius r and height $2r$ is $V = \frac{1}{3}\pi r^2(2r) = \frac{2}{3}\pi r^3$, the volume of a sphere with radius r is:

$$V = \frac{4}{3}\pi r$$

Answer Key

1. Proportional Relationships and Story Problems

1. 500 students
2. 24 hours
3. 21 minutes
4. 42 minutes
5. 52 sticks
6. $620 a month
7. Susan has 34 marbles, and Sam has 39 marbles.
8. 33 meters and 51 meters
9. East High School: 1036 students
   West High School: 1822 students
10. First part = $50; Second part = $250;
    Third part = $85

2. Simplifying and Evaluating Algebraic Expressions

1. $3x^2 + 3xy + 6$

2. $4m^2 + 2mn + 5m$

3. $5x^2 - 23x$

4. $19x - 18$

5. $5y^2 + 6xy + 40$

6. 60

7. 3598

8. 598

9. -6

10. -32

3. Solving Equations and the Distributive Property

1. $x = 2$

2. $x = 0$

3. $x = 3$

4. $x = 2$

5. $x = -3$

6. $x = 2$

7. $x = -3$

8. $x = 2$

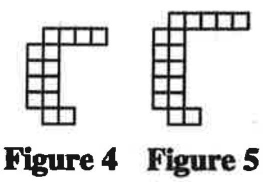
9. $x = 3$

10. No solution

4. Slope-Intercept Form ($y = mx + b$) and Graphs, Tables, Rules

1. The 100th figure will have 201 tiles.

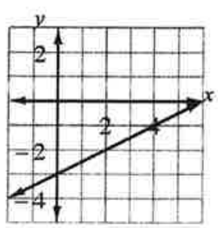
2. a) The "square" portion should be (figure # + 1)^2, and each arm should be 2 times the figure #.
   b) $y = (x + 1)^2 + 2x$, or $y = x^2 + 4x + 1$

3. a) 

   b) 41 tiles
   c) since the height of each figure is 2 more than the figure number, the height of Figure 106 will be 108

4. a) 4, 7, 10, 13, 16, 19
   b) $y = 3x + 1$
   c) 301 squares

5. a) 2 b) (0, –1) c) $y = 2x - 1$

6. 

7. The following are the growth rates for each school:

Kennedy: about 163 cell phones per year; American: an average of 50 cell phones per year; Newark: about 50 cell phones per year; Irvington: 45 cell phones per year. Kennedy has the greatest growth in cell phone use.

8. a) $y = -2x - 5$ b) $y = \frac{1}{4}x + 1$

9. $y = -\frac{1}{2}x + 1$

10. $y = 3x - 2$

5. Solving Equations with Fractional Coefficients and Systems of Equations

1. $x = 80$

2. $x = -\frac{9}{4}$

3. $x = 10$

4. $m = -3$

5. $x = -3$

6. $y = -\frac{5}{3}x + 5$

7. $x = 4y - 4$

8. (9, 3)

9. (19, 52)

10. Let x = weeks, and y = total money saved.
 Larry: $y = 35x + 215$ Betty: $y = 20x + 380$
 The solution is (11, 600). They will both have $600 in 11 weeks.

6. Rigid Transformations, Dilations, Similar Figures

1. translation

2. reflection

3. (-3, -4)

4. (3, 4)

5. (-4, -3)

6. A (8, -2) B (10,12) C (2, 6)

7. $z = 6\frac{2}{3}$

8. x = 36 feet

9. x = 9.6

10. 20 meters

7. Circle Graphs, Scatterplots, Line of Best Fit, Writing Equations from the Line of Best Fit

1. 20 students

2. 90°

3. 18°

4. positive correlation

5. Answers will vary. Approximately, $y = 10x + 40$ or $y = 20/3\ x + 50$, or thereabout.

6. $-\frac{1}{2}$

7. 2

8. $y = -\frac{1}{2}x + 2$

9. $y = \frac{4}{3}x - 2$

10. $y = -\frac{1}{3}x + 1$

8. Compound Interest/Simple Interest, Scientific Notation, Laws of Exponents, Recognizing Functions

1. \$40

2. \$6,414,270.94

3. 7.81×10^{-6}

4. 2.34×10^{12}

5. 1/125

6. x^7

7. 5^2

8. x^{12}

9. $256x^8y^{12}$

10. $4a^6b^{-6}$ or $\frac{4a^6}{b^6}$

11. xy

12. $4x^8$

13. $16x^8$

14. yes

15. no

16. no

17. yes

18. no

19. yes

20. no

9. Parallel Line Angle Relationships, Unknown Angles in Triangles Pythagorean Theorem

1. $m\angle 1 = 53°$ $m\angle 2 = 72°$ $m\angle 3 = 55°$ $m\angle 4 = 127°$ $m\angle 6 = 55°$
2. $x = 5$; $y = 20$
3. $x = 100°$
4. $x = 8\ 3/4$
5. $x = 11$
6. 13 cm
7. 10.24 m
8. 5.56 units
9. 53 m
10. $x = 6$ units

10. Volume and Surface Area of Cylinders, Pyramids, Cones, and Spheres

1. 54 units

2. 27 $units^3$

3. 17 mm

4. 942 cm^3

5. 471 cm^2

6. 301.44 in^3

7. 1584 in^3

8. ≈ 523 1/3 m^3

9. 660 mm^3

10. 192 m