ST. BRENDAN HIGH SCHOOL

Calculus Honors
Summer Math Packet

Congratulations! You made it to Calculus!

This summer math packet is a review of concepts learned in Algebra and Pre-Calculus classes that are needed when you begin your Calculus course in August. Completion of the packet will assure that all students begin the school year on the same page and with equal opportunity to learn and build upon mathematical concepts that should have been learned in previous courses.

Instructions for completing the packet:

- Please print the packet or use loose leaf paper to complete the packet by hand showing all work. Work must be neat and legible.
- Please use your Pre-Calculus notes or the websites provided to help you if you need reminders on how to complete some practice problems.
- Take notes as you complete your work. You will be given a quiz on this material the first week of school.
- Work on the packet with your friends. Help each other. Every student is responsible for knowing the material in this packet when you return in August. We will review as a team and everyone will be expected to participate.
- Bring your packet to our first class together. It will be collected for a grade. Only packets done with paper and pencil will be accepted.

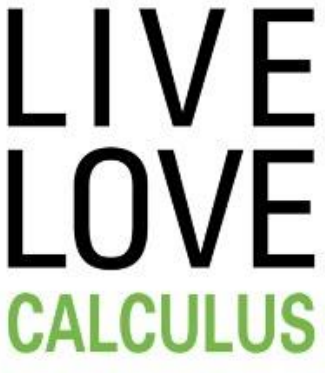
A FRIENDLY SABRE SUGGESTION –
begin your summer with a SCHEDULE for completing your packet.

Helpful Websites:

http://www.mathtv.com/
http://www.purplemath.com/modules/index.htm
https://www.khanacademy.org

Helpful for graphing functions:
https://www.education.ti.com/en/resources/family-of-functions

CALCULUS HONORS SUMMER PACKET



DUE: First day of school.
This assignment is to be done at your leisure during the summer. It is designed to help you become comfortable with your graphing calculator as well as give you practice in necessary math skills which are basic knowledge for Calculus. It is important that you gain these skills over the summer so that we can spend more time talking about calculus and less time on how to use the calculator and do basic algebra.

Part 1: Graph. Analyze each.

1. Parent Function: $y = x^2$
   a. $y = x^2 - 5$
   b. $y = x^2 + 3$
   c. $y = (x - 10)^2$
   d. $y = (x + 8)^2$
   e. $y = 4x^2$
   f. $y = 0.25x^2$
   g. $y = -x^2$
   h. $y = -(x + 3)^2 + 6$
   i. $y = (x + 4)^2 - 8$
   j. $y = -2(x + 1)^2 + 4$
   k. $y = \frac{1}{3}(x - 6)^2 - 6$
   l. $y = -3(x + 2)^2 - 2$
2. Parent Function: $y = \sin(x)$
   a. $y = \sin(2x)$
   b. $y = 2\sin(x)$
   c. $y = \sin(x) - 2$
   d. $y = \sin(2x) + 2$
3. Parent Function: $y = \cos(x)$
   a. $y = \cos(3x)$
   b. $y = \cos\left(\frac{x}{2}\right)$
   c. $y = 2\cos(x) + 2$
   d. $y = -2\cos(x) - 1$
4. Parent Function: $y = x^3$
   a. $y = x^3 + 2$
   b. $y = -x^3$
   c. $y = x^3 - 5$
   d. $y = -x^3 + 3$
   e. $y = (x - 4)^3$
   f. $y = (x - 1)^3 - 4$
   g. $y = -2(x + 2)^3 + 1$
   h. $y = x^3 + x$
5. Parent Function: $y = \sqrt{x}$
   a. $y = \sqrt{x} - 2$
   b. $y = \sqrt{-x}$

c. $y=\sqrt{x}+5$

d. $y=\sqrt{6-x}$

e. $y=-\sqrt{x}$

f. $y=-\sqrt{-x}$

g. $y=\sqrt{x+2}$

h. $y=\sqrt{2x-6}$

i. $y=-2\sqrt{x}$

j. $y=-\sqrt{4-x}$

6. Parent Function: $y=\ln x$

a. $y=\ln(x+3)$

b. $y=\ln(x)+3$

c. $y=\ln(x-2)$

d. $y=\ln(-x)$

e. $y=-\ln x$

f. $y=\ln|x|$

g. $y=\ln(2x)-4$

h. $y=-3\ln(x)+1$

7. Parent Function: $y=e^x$

a. $y=e^{2x}$

b. $y=e^{x-2}$

c. $y=e^{2-x}$

d. $y=e^{2x}+3$

e. $y=-e^x$

f. $y=e^{-x}$

g. $y=2-e^x$

h. $y=e^{0.5x}$

8. Parent Function: $y=2^x$

a. $y=3^x$

b. $y=0.5^x$

c. $y=3^{-x}$

d. $y=4^{x-3}$

e. $y=2^{x-3}+2$

f. $y=2^{-x}+3$

9. Parent Function: $y=\frac{1}{x}$

a. $y=\frac{1}{(x-2)}$

b. $y=-\frac{1}{x}$

c. $y=\frac{1}{(x+4)}$

d. $y=\frac{2}{(5-x)}$

10. Parent Function: $y=|x|$

a. $y=|x|+2$

b. $y=|x-3|$

c. $y=-2|x|$

d. $y=|x-2|+3$

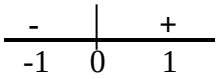
Part 2: Determine the domain and zeros (nearest 0.001) of each of the following functions. Use a sign chart to determine the intervals where the function is positive and negative. Use your calculator and table for help.

For example $f(x) = x$

Domain: {all reals}

Zeros: x= 0

Sign chart**:**



11. Given: $f(x) = x^4 - 3x^3 + 2x^2 - 7x - 11$

12. Given: $f(x) = 0.7x^2 + 3.2x + 1.5$

13. Given: $f(x) = x^4 - 8x^2 + 5$

14. Given: $f(x) = x^3 + 3x^2 - 10x - 1$

15. Given: $f(x) = 100x^3 - 203x^2 + 103x - 1$

16. Given: $f(x) = \frac{(x+1)}{(x+2)}$

Part 3: Linear equation: Write the following equations in point-slope $y - y_1 = m(x - x_1)$ form.

17. The line containing point (4,-7) and having slope of 2.

18. The line containing the point (-13,5) and parallel to $4x + 2y = -7$

19. The line containing the point (0,-2) and perpendicular to $x - 4y = -7$

20. The line containing the point (2,9) and having a slope of 0.

21. The perpendicular bisector of the segment between (-5,3) and (12,3)

Part 4: Graph the following inequalities.

22. $y \le -x^2 + x + 6$
23. $y > x^3 - 4x$, for $\{x | -6 \le x \le 6\}$
24. $y < -(x-2)^2 - 4$, for $\{x | -4 \le x \le 8\}$

Part 5: For questions 25-27: Copy and complete each (use your calculator)

a. Sketch the graph of $f(x)$.
b. Sketch the graph of $|f(x)|$.
c. Sketch the graph of $f(|x|)$.
d. Sketch the graph of $f(2x)$.
e. Sketch the graph of $2 \cdot f(x)$.

25. $f(x) = x^2 - 5x - 3$

26. $f(x) = 2\sin(3x)$

27. $f(x) = -x^3 - 2x^2 + 3x - 4$

28. Given: $f(x) = 3x + 2$
$g(x) = -4x - 2$
Find the point of intersection.

29. Given: $f(x) = x^2 - 5x + 2$
$g(x) = 3 - 2x$
Find the coordinates of any point(s) of intersection.

30. Given: $f(x) = x^4 - 7x^3 + 6x^2 + 8x + 9$
a. Find the lowest point on the graph.
b. Find the highest and lowest values of *f*(*x*) over the interval $-10 \le x \le 10$.
c. Find the highest and lowest values of *f*(*x*) over the interval $-2 \le x \le 6$.

Part 6: Factor each of the expressions

31. $x^2 - 8x + 12$

32. $3x^2 + 8x + 5$

33. $x^2 - 25$

34. $4x^2 - 81$

Part 7: Composition of Functions: Given $f(x) = 4x - 1$ and $g(x) = x + 6$, find the following compositions:

35. $g(f(x)$

36. $f(g(x)$

37. $f(f(x)$

38. $g(f(g(x)))$

Part 8: Simplify Expressions

39. ln (1)

40. e^0

41. $e^{1+\ln(x)}$

42. $e^{l3lnx)}$

43. $\frac{\sqrt{x}}{x}$

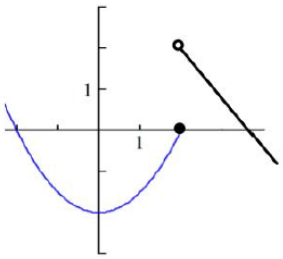
44. e^{ln3}

Part 9: Piecewise function:

45. Sketch $y = |x|$ as a piecewise function

46. Sketch a graph of the piecewise function $f(x) = \begin{cases} -x^2, -2 \le x < 1 \\ -2, \quad x = 1 \\ 3x + 5, 1 < x \le 3 \end{cases}$

47. The function $f(x)$ is graphed below. Find the following.



a) $f(2)$ b) $f(0)$ c) $f(x) = 0$

Summer MC Review

Pre Calc – domain – range / graphing / solving / trig / factoring / odd-even

[1969 AP Calculus AB #1] Which of the following defines a function *f* for which *f(-x) = -f(x)?*

A) $f(x) = x^2$ B) $f(x) = \sin x$ C) $f(x) = \cos x$

D) $f(x) = \log x$ E) $f(x) = e^x$

[1969 AP Calculus AB #2] $\ln(x-2) < 0$ if and only if

A) $x < 3$ B) $0 < x < 3$ C) $2 < x < 3$

D) $x > 2$ E) $x > 3$

[1985 AP Calculus AB #15] The domain of the function defined by $f(x) = \ln(x^2 - 4)$ is the set of all real numbers *x* such that

A) $|x| < 2$ B) $|x| \leq 2$ C) $|x| > 2$

D) $|x| \geq 2$ E) *x* is a real number

[1985 AP Calculus AB #21] The domain of the function *f* given by $f(x) = \dfrac{1}{1-x^2}$ is $\{x : |x| > 1\}$, what is the range of *f*?

A) $\{x : -\infty < x < -1\}$ B) $\{x : -\infty < x < 0\}$ C) $\{x : -\infty < x < 1\}$

D) $\{x : -1 < x < \infty\}$ E) $\{x : 0 < x < \infty\}$

[1985 AP Calculus AB #26] The graph of $y^2 = x^2 + 9$ is symmetric to which of the following?

I. The *x*-axis
II. The *y*-axis
III. The origin

A) I only B) II only C) III only

D) I and II only E) I, II and III

[1988 AP Calculus AB #2] What is the domain of the function *f* given by $f(x) = \dfrac{\sqrt{x^2-4}}{x-3}$

A) $\{x : x \neq 3\}$ B) $\{x : |x| \leq 2\}$ C) $\{x : |x| \geq 2\}$

D) $\{x : |x| \geq 2 and x \neq 3\}$ E) $\{x : x \geq 2 and x \neq 3\}$

[1988 AP Calculus AB #32] Which of the following does NOT have a period of π?

A) $f(x) = \sin\left(\frac{1}{2}x\right)$ B) $f(x) = |\sin x|$ C) $f(x) = \sin^2 x$

D) $f(x) = \tan x$ E) $f(x) = \tan^2 x$

[1988 AP Calculus AB #42] The graph of which of the following equations has $y = 1$ as an asymptote?

A) $y = \ln x$ B) $y = \sin x$ C) $y = \frac{x}{x+1}$

D) $y = \frac{x^2}{x-1}$ E) $y = e^{-x}$

[1988 AP Calculus AB #42] The graph of which of the following equations has $y = 1$ as an asymptote?

A) $y = \ln x$ B) $y = \sin x$ C) $y = \frac{x}{x+1}$

D) $y = \frac{x^2}{x-1}$ E) $y = e^{-x}$

[1988 AP Calculus AB #44] Let *f* and *g* be odd functions. If *p*, *r*, and *s* are nonzero functions defined as follows, which must be odd?

I. $p(x) = f(g(x))$
II. $r(x) = f(x) + g(x)$
III. $s(x) = f(x)g(x)$

A) I only B) II only C) I and II only
D) II and III only E) I, II and III

[1993 AP Calculus AB #13] The fundamental period of $2\cos(3x)$ is

A) $\frac{2\pi}{3}$ B) 2π C) 6π

D) 2 E) 3

[1993 AP Calculus AB #35] If the graph of $y = \frac{ax+b}{x+c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a + c =$

A) -5 B) -1 C) 0

D) 1 E) 5

[1969 AP Calculus AB #1] Which of the following defines a function *f* for which *f(-x) = -f(x)?*

A) $f(x) = x^2$ B) $f(x) = \sin x$ C) $f(x) = \cos x$

D) $f(x) = \log x$ E) $f(x) = e^x$

[1969 AP Calculus AB #2] $\ln(x-2) < 0$ if and only if

A) $x < 3$ B) $0 < x < 3$ C) $2 < x < 3$

D) $x > 2$ E) $x > 3$

Name _____

0.1: Monomial Factors

Factor as indicated

1. $3x^4 + 4x^3 - x^2 = x^2(\_\_\_\_)$

2. $2\sqrt{x} + 6x^{\frac{3}{2}} = 2\sqrt{x}(\_\_\_\_)$

3. $e^{-x} - xe^{-x} + 2x^2e^{-x} = e^{-x}(\_\_\_\_)$

4. $x^{-1} - 2 + x = x^{-1}(\_\_\_\_)$

5. $\frac{x}{2} - 6x^2 = \frac{x}{2}(\_\_\_\_)$

6. $\sin^2 x + \tan x = \sin x(\_\_\_\_)$

7. $\frac{1}{2x^3 + 4x} = \frac{1}{2x}(\_\_\_\_)$

0.2: Binomial Factors

Factor as indicated

1. $(x-1)^2(x)-3(x-1)=(x-1)(\_\_\_\_\_)$

2. $3(x^2+4)(x^2+1)+6(x^2+4)^2=(x^2+4)(\_\_\_\_\_)$

3. $\sqrt{x^2+1}-\dfrac{x^2}{\sqrt{x^2+1}}=\dfrac{1}{\sqrt{x^2+1}}(\_\_\_\_\_)$

4. $(x-3)^3(x+2)-2(x-3)^2(x+2)^2=(x-3)^2(x+2)(\_\_\_\_\_)$

5. $(2x+1)^{\frac{3}{2}}(x^{\frac{1}{2}})+(2x+1)^{\frac{5}{2}}(x^{-\frac{1}{2}})=(2x+1)^{\frac{3}{2}}(x^{-\frac{1}{2}})(\_\_\_\_\_)$

0.3: Factoring Quadratic Expressions

Factor as indicated.

1. $x^2 - 3x + 2 = (\quad)(\quad)$

2. $x^2 - 121 = (\quad)(\quad)$

3. $x^2 + 5x - 6 = (\quad)(\quad)$

4. $x^2 + 5x + 6 = (\quad)(\quad)$

5. $2x^2 + 5x - 3 = (\quad)(\quad)$

6. $e^{2x} + 2 + e^{-2x} = (\quad)(\quad)$

7. $x^4 - 7x^2 + 12 = (\quad)(\quad)(\quad)$

8. $\sin x - \sin^3 x = (\quad)(\quad)(\quad)$

0.4: Simplify Complex Fractions

Reduce each expression to lowest terms:

1. $\dfrac{3x+9}{6x}$

2. $\dfrac{x^{5}}{x^{\frac{1}{2}}}$

3. $\dfrac{(x+1)^{3}(x-2)+3(x+1)^{2}}{(x+1)^{4}}$

4. $\dfrac{x^{\frac{1}{2}}-x^{\frac{1}{3}}}{x^{\frac{1}{6}}}$

5. $\dfrac{\sqrt{x-1}+(x-1)^{\frac{3}{2}}}{\sqrt{x-1}}$

6. $\dfrac{1-(\sin x+\cos x)^{2}}{2\sin x}$

0.5: Quadratic Formula

For each equation, solve for the indicated expression.

1. $x^2 - 4x - 1 = 0$, *for* x

2. $2x^2 + x - 3 = 0$, *for* x

3. $\cos^2 x + 3\cos x + 2 = 0$, *for* $\cos x$, *then for* x

4. $x^2 - xy - (1 + y^2) = 0$, *for* x

5. $x^4 - 4x^2 + 2 = 0$, *for* x^2

0.6: Synthetic Division

Use synthetic division to factor as indicated.

1. $x^3 - 4x^2 + 2x + 1 = (x - 1)(\quad)$

2. $2x^3 + 5x + 7 = (x + 1)(\quad)$

3. $x^4 - 3x^3 + x^2 + x + 2 = (x - 2)(\quad)$

4. $4x^4 + 3x^2 - 1 = (2x - 1)(\quad)$

0.7: Special Products

Factor completely (into linear or irreducible quadratic factors):

1. $x^3 - 27$

2. $x^3 - 3x^2 + 3x - 1$

3. $x^3 + 6x^2 + 12x + 8$

4. $x^4 - 25$ (Note: into three factors)

*5. $x^4 - 8x^3 + 24x^2 - 32x + 16$ (Pascal's Triangle)

0.8: Factoring by Grouping

Factor completely (into linear or irreducible quadratic factors):

1. $x^3 + 4x^2 - 2x - 8$

2. $x^3 + 2x^2 + 3x + 6$

*3. $5\cos^2 x - 5\sin^2 x + \sin x + \cos x$

*4. $\cos^2 x + 4\cos x + 4 - \tan^2 x$

0.9: Simplifying

Rewrite each of the following in simplest form:

1. $\dfrac{(x-1)(x+3)-(x+1)^2}{x+1}$

2. $\dfrac{\sqrt{x^2+1}-\dfrac{1}{\sqrt{x^2+1}}}{x^2+1}$

3. $\dfrac{x^2-5x+6}{x^2-4x+4}$

4. $\dfrac{1}{x+1}-\dfrac{1}{x-1}-\dfrac{2}{x^2-1}$

5. $\dfrac{x(-2x)}{2\sqrt{1-x^2}}+\sqrt{1-x^2}+\dfrac{1}{\sqrt{1-x^2}}$

0.10: Rationalizing

Remove the sum or difference from the denominator by multiplying the numerator and denominator by the conjugate of the denominator.

1. $\dfrac{1}{1-\cos x}$

2. $\dfrac{x}{1-\sqrt{x^2+1}}$

3. $\dfrac{2}{x+\sqrt{x^2+1}}$

0.11: Algebraic Errors to Avoid

Error	*Correct form*	*Comments*
$a - (x - b) \neq a - x - b$	$a - (x - b) = a - x + b$	Change all signs when distribution negative through parentheses.
$(a + b)^2 \neq a^2 + b^2$	$(a + b)^2 = a^2 + 2ab + b^2$	Don't forget middle term when squaring binomials.
$\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right) \neq \frac{1}{2}ab$	$\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right) = \frac{1}{4}(ab)$	1/2 occurs twice as a factor.
$\frac{a}{x + b} \neq \frac{a}{x} + \frac{a}{b}$	Leave as $\frac{a}{x + b}$	Don't add denominators when adding fractions.
$\frac{1}{a} + \frac{1}{b} \neq \frac{1}{a + b}$	$\frac{1}{a} + \frac{1}{b} = \frac{a + b}{ab}$	Use definition for adding fractions.
$\frac{\frac{x}{a}}{b} \neq \frac{bx}{a}$	$\frac{\frac{x}{a}}{b} = \left(\frac{x}{a}\right)\left(\frac{1}{b}\right) = \frac{x}{ab}$	Multiply by reciprocal of the denominator.
$\frac{1}{3x} \neq \frac{1}{3}x$	$\frac{1}{3x} = \frac{1}{3} \cdot \frac{1}{x}$	Use definition for multiplying fractions.
$1/x + 2 \neq \frac{1}{x + 2}$	$1/x + 2 = \frac{1}{x} + 2$	Be careful when using a slash to denote division.
$(x^2)^3 \neq x^5$	$(x^2)^3 = x^{2 \cdot 3} = x^6$	Multiply exponents when an exponential form is raised to a power.
$2x^3 \neq (2x)^3$	$2x^3 = 2(x^3)$	Exponents have priority over coefficients.
$\frac{1}{x^2 + x^3} \neq x^{-2} + x^{-3}$	Leave as $\frac{1}{x^2 + x^3}$	Don't shift term-by-term from denominator to numerator.
$\sqrt{5x} \neq 5\sqrt{x}$	$\sqrt{5x} = \sqrt{5}\sqrt{x}$	Radicals apply to every factor inside radical.
$\sqrt{x^2 + a^2} \neq x + a$	Leave as $\sqrt{x^2 + a^2}$	Don't apply radicals term-by-term.
$\frac{a + bx}{a} \neq 1 + bx$	$\frac{a + bx}{a} = 1 + \frac{b}{a}x$	Cancel common factor, *not* common terms.
$\frac{a + ax}{a} \neq a + x$	$\frac{a + ax}{a} = 1 + x$	Factor *before* canceling.

Extra Topic 1: Solving Inequalities

Solve the following inequalities and graph the solution on the real number line.

1. $|9-2x|<1$

2. $|x-a|<b \quad , b>0$

3. $x^4 - x \le 0$

4. $2x^2+1<9x-3$

Extra Topic 2: Function Transformation

If $f(x)=x^2-1$, describe in words what the following would do to the graph of $f(x)$.

1. $f(x)-4$

2. $f(x-4)$

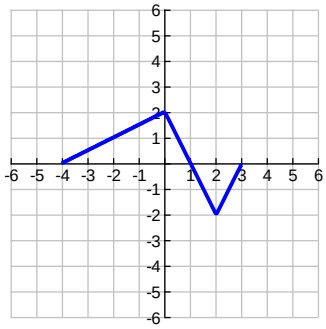
3. $-f(x+2)$

4. $5f(x)+3$

5. $f(2x)$

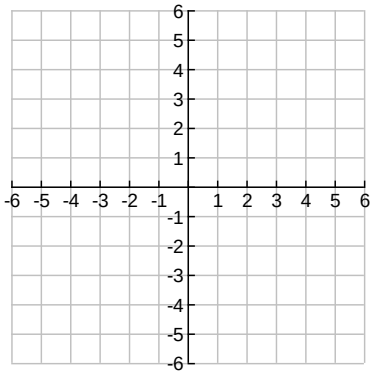
6. $|f(x)|$

Here is a graph of $y = f(x)$.

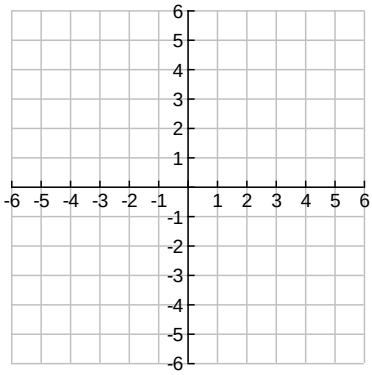


Sketch the following graphs.

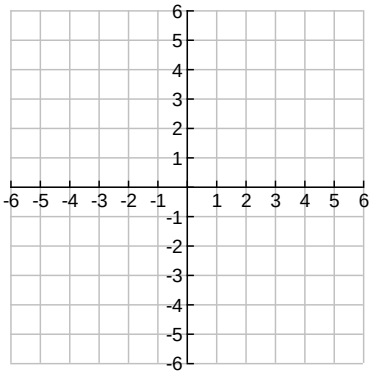
7. $y = 2f(x)$



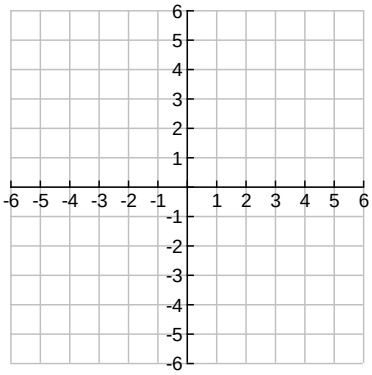
8. $y = -f(x)$



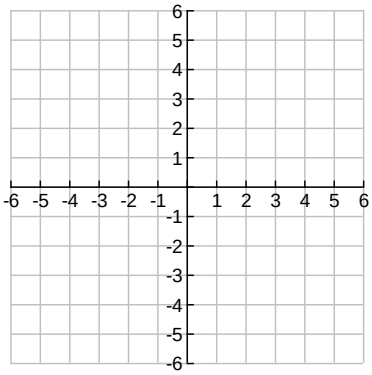
9. $y = f(x-1)$



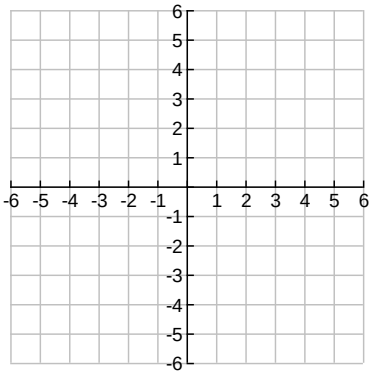
10. $y = f(x+2)$



11. $y = |f(x)|$



12. $y = f|x|$



Extra Topic 3: Trigonometry Review

Solve the following problems.

If point P is on the terminal side of θ, find all 6 trig functions of θ. Draw a picture.

1. $P(-2, 4)$

2. $P\left(\sqrt{5}, -2\right)$

3. If $\cos\theta = \frac{-5}{13}$, θ is quadrant II, find all 6 trig functions of θ.

4. If $\cot\theta = 3$, θ in quadrant III, find all 6 trig functions of θ.

For problems 3-11, solve each equation on the interval $[0, 2\pi)$.

3. $\sin x = \frac{1}{2}$

4. $\cos^2 x = \cos x$

5. $2\cos x + \sqrt{3} = 0$

6. $4\sin^2 x = 1$

7. $2\sin^2 x + \sin x = 1$

8. $\cos^2 x + 2\cos x = 3$

9. $2\sin x\cos x + \sin x = 0$

10. $8\cos^2 x - 2\cos x = 1$

11. $\sin^2 x - \cos^2 x = 0$