



AP CALCULUS AB
SUMMER PACKET
2022-2023

This summer math packet is a review of some of the concepts you learned in Pre-calculus that are needed when you begin your Calculus class in August. It will assure that all students will be on the same page as to what they are expected to know.

Instructions for completing the packet:

- ✓ Please print the packet or use loose leaf paper to complete the packet by hand showing all work when necessary. Work must be neat and legible.
- ✓ Please use your Pre-Calculus notes of the websites provided to help you if you need reminders on how to complete some practice problems.
- ✓ Take notes as you complete your work. You will be given a quiz on this material the first week of school.
- ✓ Work on the packet with your friends. Help each other. Every student is responsible for knowing the material in this packet.
- ✓ Bring your packet the first day of school. It will be collected for a grade. Only packets done with paper and pencil will be accepted.

Helpful Websites:

<http://www.mathtv.com/>

<http://www.purplemath.com/modules/index.htm>

<http://www.khanacademy.org>

AP CALCULUS SUMMER WORKSHEET

DUE: First day of school.

This assignment is to be done at your leisure during the summer. It is designed to help you become comfortable with your graphing calculator as well as give you practice in necessary math skills which are basic knowledge for Calculus. You will need to read your calculator manual to understand how your calculator works. It is important that you gain these skills over the summer so that we can spend more time talking about calculus and less time on how to use the calculator and do basic algebra

For questions 1 through 10 indicate the type of transformation applied to the parent function. Use the following abbreviations:

VSU : vertical shift up ; *VSD* : vertical shift down ; *HSR* : horizontal shift right ; *HSL* : horizontal shift left ; *VC* : vertical compression ; *VS* : vertical stretch ; *HC* : horizontal compression ; *HS* : horizontal stretch ; *RX* : reflection over x-axis ; *RY* : reflection over y-axis

1. Parent Function: $y = x^2$

a. $y = x^2 - 5$

c. $y = (x - 10)^2$

e. $y = 4x^2$

g. $y = -x^2$

i. $y = (x + 4)^2 - 8$

k. $y = \frac{1}{3}(x - 6)^2 - 6$

b. $y = x^2 + 3$

d. $y = (x + 8)^2$

f. $y = 0.25x^2$

h. $y = -(x + 3)^2 + 6$

j. $y = -2(x + 1)^2 + 4$

l. $y = -3(x + 2)^2 - 2$

2. Parent Function: $y = \sin(x)$

a. $y = \sin(2x)$

c. $y = \sin(x) - 2$

b. $y = 2\sin(x)$

d. $y = \sin(2x) + 2$

3. Parent Function: $y = \cos(x)$

a. $y = \cos(3x)$

c. $y = 2\cos(x) + 2$

b. $y = \cos\left(\frac{x}{2}\right)$

d. $y = -2\cos(x) - 1$

4. Parent Function: $y = x^3$

a. $y = x^3 + 2$

c. $y = x^3 - 5$

e. $y = (x - 4)^3$

g. $y = -2(x + 2)^3 + 1$

b. $y = -x^3$

d. $y = -x^3 + 3$

f. $y = (x - 1)^3 - 4$

h. $y = x^3 + x$

5. Parent Function: $y = \sqrt{x}$

a. $y = \sqrt{x} - 2$

c. $y = \sqrt{x} + 5$

e. $y = -\sqrt{x}$

g. $y = \sqrt{x + 2}$

i. $y = -2\sqrt{x}$

b. $y = \sqrt{-x}$

d. $y = \sqrt{6 - x}$

f. $y = -\sqrt{-x}$

h. $y = \sqrt{2x - 6}$

j. $y = -\sqrt{4 - x}$

6. Parent Function: $y = \ln x$

a. $y = \ln(x + 3)$

c. $y = \ln(x - 2)$

e. $y = -\ln x$

g. $y = \ln(2x) - 4$

b. $y = \ln(x) + 3$

d. $y = \ln(-x)$

f. $y = \ln|x|$

h. $y = -3\ln(x) + 1$

7. Parent Function: $y = e^x$

a. $y = e^{2x}$

c. $y = e^{2-x}$

e. $y = -e^x$

g. $y = 2 - e^x$

b. $y = e^{x-2}$

d. $y = e^{2x} + 3$

f. $y = e^{-x}$

h. $y = e^{0.5x}$

8. Parent Function: $y = 2^x$

a. $y = 3^x$

c. $y = 3^{-x}$

e. $y = 2^{x-3} + 2$

b. $y = 0.5^x$

d. $y = 4^{x-3}$

f. $y = 2^{-x} + 3$

9. Parent Function: $y = \frac{1}{x}$

a. $y = \frac{1}{(x-2)}$

c. $y = \frac{1}{(x+4)}$

b. $y = -\frac{1}{x}$

d. $y = \frac{2}{(5-x)}$

10. Parent Function: $y = |x|$

a. $y = |x| + 2$

c. $y = -2|x|$

b. $y = |x-3|$

d. $y = |x-2| + 3$

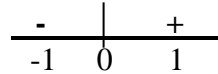
For questions 11 through 16, determine the domain and zeros (nearest 0.001) of each of the following functions. Use a sign chart to determine the intervals where the function is positive and negative. Use your calculator and table for help.

For example $f(x) = x$

Domain: {all reals}

Zeros: $x = 0$

Sign chart:



11. Given: $f(x) = x^4 - 3x^3 + 2x^2 - 7x - 11$

12. Given: $f(x) = 0.7x^2 + 3.2x + 1.5$

13. Given: $f(x) = x^4 - 8x^2 + 5$

14. Given: $f(x) = x^3 + 3x^2 - 10x - 1$

15. Given: $f(x) = 100x^3 - 203x^2 + 103x - 1$

16. Given: $f(x) = \frac{(x+1)}{(x+2)}$

For questions 17 to 21 use the given information to write the following equations in point-slope $y - y_1 = m(x - x_1)$ form.

17. The line containing point (4,-7) and having slope of 2.

18. The line containing the point (-13,5) and parallel to $4x + 2y = -7$

19. The line containing the point $(0,-2)$ and perpendicular to $x - 4y = -7$

20. The line containing the point $(2,9)$ and having a slope of 0.

21. The perpendicular bisector of the segment between $(-5,3)$ and $(12,3)$

Graph the following inequalities.

22. $y \leq -x^2 + x + 6$

23. $y > x^3 - 4x$, for $\{x | -6 \leq x \leq 6\}$

24. $y < -(x-2)^2 - 4$, for $\{x | -4 \leq x \leq 8\}$

For questions 25-27 sketch the following graphs for the given function. You might use a calculator.

25. $f(x) = x^2 - 5x - 3$

- a. Sketch the graph of $f(x)$.
- b. Sketch the graph of $|f(x)|$.
- c. Sketch the graph of $f(|x|)$.
- d. Sketch the graph of $f(2x)$.
- e. Sketch the graph of $2 \cdot f(x)$.

26. $f(x) = 2 \sin(x)$

- a. Sketch the graph of $f(x)$.
- b. Sketch the graph of $|f(x)|$.
- c. Sketch the graph of $f(|x|)$.
- d. Sketch the graph of $f(2x)$.
- e. Sketch the graph of $2 \cdot f(x)$.

27. $f(x) = -x^3 - 2x^2 + 3x - 4$

- a. Sketch the graph of $f(x)$.
- b. Sketch the graph of $|f(x)|$.
- c. Sketch the graph of $f(|x|)$.
- d. Sketch the graph of $f(2x)$.
- e. Sketch the graph of $2 \cdot f(x)$.

28. Given: $f(x) = x^2 - 5x + 2$
 $g(x) = 3 - 2x$

Find the coordinates of the point(s) of intersection.

Factor each of the expressions

29. $x^2 - 8x + 12$

30. $3x^2 + 8x + 5$

31. $x^2 - 25$

32. $4x^2 - 81$

Given $f(x) = 4x - 1$ and $g(x) = x + 6$, find the following compositions:

35. $g(f(x))$

36. $f(g(x))$

37. $f(f(x))$

38. $g(f(g(x)))$

Simplify Expressions

39. $\ln(1)$

40. e^0

41. $e^{1+\ln(x)}$

42. $e^{3\ln x}$

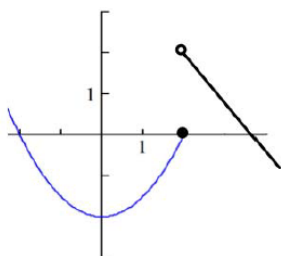
43. $\frac{\sqrt{x}}{x}$

44. $e^{\ln 3}$

45. Sketch $y = |x|$ as a piecewise function

46. Sketch a graph of the piecewise function $f(x) = \begin{cases} -x^2, & -2 \leq x < 1 \\ -2, & x = 1 \\ 3x + 5, & 1 < x \leq 3 \end{cases}$

47. The function $f(x)$ is graphed below. Find the following.



a) $f(2)$

b) $f(0)$

c) $f(x) = 0$

48. Which of the following defines a function f for which $f(-x) = -f(x)$?

A) $f(x) = x^2$

B) $f(x) = \sin x$

C) $f(x) = \cos x$

D) $f(x) = \log x$

E) $f(x) = e^x$

49. $\ln(x - 2) < 0$ if and only if

A) $x < 3$

B) $0 < x < 3$

C) $2 < x < 3$

D) $x > 2$

E) $x > 3$

50. The domain of the function defined by $f(x) = \ln(x^2 - 4)$ is the set of all real numbers x such that

A) $|x| < 2$

B) $|x| \leq 2$

C) $|x| > 2$

D) $|x| \geq 2$

E) x is a real number

51. The domain of the function f given by $f(x) = \frac{1}{1-x^2}$ is $\{x : |x| > 1\}$, what is the range of f ?

A) $\{x : -\infty < x < -1\}$

B) $\{x : -\infty < x < 0\}$

C) $\{x : -\infty < x < 1\}$

D) $\{x : -1 < x < \infty\}$

E) $\{x : 0 < x < \infty\}$

52. The graph of $y^2 = x^2 + 9$ is symmetric to which of the following?

I. The x -axis

II. The y -axis

III. The origin

A) I only

B) II only

C) III only

D) I and II only

E) I, II and III

53. What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2 - 4}}{x - 3}$

- A) $\{x : x \neq 3\}$ B) $\{x : |x| \leq 2\}$ C) $\{x : |x| \geq 2\}$
 D) $\{x : |x| \geq 2 \text{ and } x \neq 3\}$ E) $\{x : x \geq 2 \text{ and } x \neq 3\}$

54. Which of the following does NOT have a period of π ?

- A) $f(x) = \sin\left(\frac{1}{2}x\right)$ B) $f(x) = |\sin x|$ C) $f(x) = \sin^2 x$
 D) $f(x) = \tan x$ E) $f(x) = \tan^2 x$

55. The graph of which of the following equations has $y = 1$ as an asymptote?

- A) $y = \ln x$ B) $y = \sin x$ C) $y = \frac{x}{x+1}$
 D) $y = \frac{x^2}{x-1}$ E) $y = e^{-x}$

55. The graph of which of the following equations has $y = 1$ as an asymptote?

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56. Let f and g be odd functions. If p , r , and s are nonzero functions defined as follows, which must be odd?

- I. $p(x) = f(g(x))$
 II. $r(x) = f(x) + g(x)$
 III. $s(x) = f(x)g(x)$

- A) I only B) II only C) I and II only
 D) II and III only E) I, II and III

57. The fundamental period of $2 \cos(3x)$ is

- A) $\frac{2\pi}{3}$ B) 2π C) 6π
 D) 2 E) 3

58. If the graph of $y = \frac{ax+b}{x+c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a + c =$

A) -5
D) 1

B) -1
E) 5

C) 0

Factor as indicated

59. $3x^4 + 4x^3 - x^2 = x^2(\underline{\hspace{2cm}})$

60. $2\sqrt{x} + 6x^{\frac{3}{2}} = 2\sqrt{x}(\underline{\hspace{2cm}})$

61. $e^{-x} - xe^{-x} + 2x^2e^{-x} = e^{-x}(\underline{\hspace{2cm}})$

62. $x^{-1} - 2 + x = x^{-1}(\underline{\hspace{2cm}})$

63. $\frac{x}{2} - 6x^2 = \frac{x}{2}(\underline{\hspace{2cm}})$

64. $\sin^2 x + \tan x = \sin x(\underline{\hspace{2cm}})$

$$65. \frac{1}{2x^3 + 4x} = \frac{1}{2x} (\underline{\hspace{2cm}})$$

Factor as indicated

$$66. (x-1)^2(x) - 3(x-1) = (x-1)(\underline{\hspace{2cm}})$$

$$67. 3(x^2 + 4)(x^2 + 1) + 6(x^2 + 4)^2 = (x^2 + 4)(\underline{\hspace{2cm}})$$

$$68. \sqrt{x^2 + 1} - \frac{x^2}{\sqrt{x^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}} (\underline{\hspace{2cm}})$$

$$69. (x-3)^3(x+2) - 2(x-3)^2(x+2)^2 = (x-3)^2(x+2)(\underline{\hspace{2cm}})$$

$$70. (2x+1)^{\frac{3}{2}}(x^{\frac{1}{2}}) + (2x+1)^{\frac{5}{2}}(x^{-\frac{1}{2}}) = (2x+1)^{\frac{3}{2}}(x^{-\frac{1}{2}})(\underline{\hspace{2cm}})$$

Factor as indicated.

$$71. x^2 - 3x + 2 = (\quad)(\quad)$$

$$72. x^2 - 121 = (\quad)(\quad)$$

$$73. x^2 + 5x - 6 = (\quad)(\quad)$$

$$74. x^2 + 5x + 6 = (\quad)(\quad)$$

$$75. 2x^2 + 5x - 3 = (\quad)(\quad)$$

$$76. e^{2x} + 2 + e^{-2x} = (\quad)(\quad)$$

$$77. x^4 - 7x^2 + 12 = (\quad)(\quad)(\quad)$$

$$78. \sin x - \sin^3 x = (\quad)(\quad)(\quad)$$

Reduce each expression to lowest terms:

$$79. \frac{3x+9}{6x}$$

$$80. \frac{x^5}{x^{\frac{1}{2}}}$$

$$81. \frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$$

$$82. \frac{x^{\frac{1}{2}} - x^{\frac{1}{3}}}{x^{\frac{1}{6}}}$$

$$83. \frac{\sqrt{x-1} + (x-1)^{\frac{3}{2}}}{\sqrt{x-1}}$$

$$84. \frac{1 - (\sin x + \cos x)^2}{2 \sin x}$$

For each equation, solve for the indicated expression.

85. $x^2 - 4x - 1 = 0$, for x

86. $2x^2 + x - 3 = 0$, for x

87. $\cos^2 x + 3\cos x + 2 = 0$, for $\cos x$, then for x

88. $x^2 - xy - (1 + y^2) = 0$, for x

89. $x^4 - 4x^2 + 2 = 0$, for x^2

Use synthetic division to factor as indicated.

90. $x^3 - 4x^2 + 2x + 1 = (x - 1)(\quad)$

91. $2x^3 + 5x + 7 = (x + 1)(\quad)$

92. $x^4 - 3x^3 + x^2 + x + 2 = (x - 2)(\quad)$

93. $4x^4 + 3x^2 - 1 = (2x - 1)(\quad)$

Factor completely (into linear or irreducible quadratic factors):

94. $x^3 - 27$

95. $x^3 - 3x^2 + 3x - 1$

96. $x^3 + 6x^2 + 12x + 8$
*

97. $x^4 - 25$ (Note: into three factors)

98. $x^4 - 8x^3 + 24x^2 - 32x + 16$ (Pascal's Triangle)

Factor completely (into linear or irreducible quadratic factors):

99. $x^3 + 4x^2 - 2x - 8$

100. $x^3 + 2x^2 + 3x + 6$

101. $5\cos^2 x - 5\sin^2 x + \sin x + \cos x$

102. $\cos^2 x + 4\cos x + 4 - \tan^2 x$

Rewrite each of the following in simplest form:

$$103. \frac{(x-1)(x+3) - (x+1)^2}{x+1}$$

$$104. \frac{\sqrt{x^2+1} - \frac{1}{\sqrt{x^2+1}}}{x^2+1}$$

$$105. \frac{x^2 - 5x + 6}{x^2 - 4x + 4}$$

$$106. \frac{1}{x+1} - \frac{1}{x-1} - \frac{2}{x^2-1}$$

$$107. \frac{x(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}}$$

Remove the sum or difference from the denominator by multiplying the numerator and denominator by the conjugate of the denominator.

$$108. \frac{1}{1 - \cos x}$$

$$109. \frac{x}{1 - \sqrt{x^2 + 1}}$$

$$110. \frac{2}{x + \sqrt{x^2 + 1}}$$

Solve each equation on the interval $[0, 2\pi)$.

111. $\sin x = \frac{1}{2}$

112. $\cos^2 x = \cos x$

113. $2\cos x + \sqrt{3} = 0$

114. $4\sin^2 x = 1$

$$115. 2\sin^2 x + \sin x = 1$$

$$116. \cos^2 x + 2\cos x = 3$$

$$117. 2\sin x \cos x + \sin x = 0$$

$$118. 8\cos^2 x - 2\cos x = 1$$

$$119. \sin^2 x - \cos^2 x = 0$$