

JUSTIFY ALL ANSWERS!!!!

DUE DATE : Monday, August 31st, 2022

This packet contains a review of most of the prerequisite skills required to be successful in AP Calculus. All problems should be completed in pencil and, unless otherwise noted, without the use of any calculator. If you do not understand a problem, or do not remember how to do a problem, you will need to use other resources to review the material. Justification must be shown for each answer, as it is your responsibility to effectively communicate your solution to me. All multiple-choice answers need to be written in the space provided next to the question number and graphs completed when directed. It is my expectation that all solutions and answers will be neat and accurate by 8/31/2022, your third day of school in August, when it will be collected and graded for accuracy. This will be your first quiz grade of first quarter.

You may choose to complete all problems in the spaces provided (which I prefer you do). OR use your own paper to complete all problems. Do NOT complete part of your problems on the packet and part of your problems on your own paper. If you use your own paper, you will need to write only on the front side of each sheet (do NOT write on the back side.) Copy each question verbatim, provide the solution and box your final answer. Skip a space and begin the next problem. Each problem should start and end on the same page -- please do not start a problem on the bottom of a page knowing you do not have ample room to complete its solution.

____ 1. Simplify: $(19x^{-6}y^{11})(-6xy^5)$

- a. $-114x^{-5}y^{16}$ b. $\frac{13y^{16}}{x^5}$ c. $\frac{-114y^6}{x^6}$ d. $-114x^{-7}y^{-24}$

____ 2. Simplify: $\frac{4}{16+\sqrt{11}}$

- a. $\frac{64+4\sqrt{11}}{-245}$ b. $\frac{64+4\sqrt{11}}{245}$ c. $\frac{64-\sqrt{11}}{245}$ d. $\frac{64-4\sqrt{11}}{245}$

____ 3. Simplify: $y^{\frac{2}{11}}y^{\frac{4}{11}}$

- a. $y^{\frac{2}{11}}$ b. $y^{\frac{6}{11}}$ c. $y^{\frac{12}{11}}$ d. $y^{\frac{8}{11}}$

____ 4. Simplify: $\frac{3}{4x^2-25} + \frac{2}{2x+5}$

- a. $\frac{4x+7}{(2x+5)(2x-5)}$ b. $\frac{4x-10}{(2x-5)(2x+5)}$ c. $\frac{4x-7}{(2x+5)(2x-5)}$ d. $\frac{5}{(4x^2+2x-20)}$

____ 5. Simplify: $\frac{8}{y+2} - \frac{3y}{y^2-4}$

- a. $\frac{5y-16}{(y+2)(y-2)}$ b. $\frac{8-3y}{(y+2)(y^2-4)}$ c. $\frac{5y-16}{(y+2)(y^2-4)}$ d. $\frac{5y+16}{(y+2)(y-2)}$

_____ 6. Factor Completely: $30x^3 - 50x^2 + 27x - 45$

- a. $10x^2(3x - 5) - 9(3x - 5)$ c. $10x^2(3x - 5) - 27x + 45$
b. $(10x^2 + 9)(3x - 5)$ d. $(30x^3 - 50x^2) + (27x - 45)$

_____ 7. Factor Completely: $4x^2 - 13x + 9$

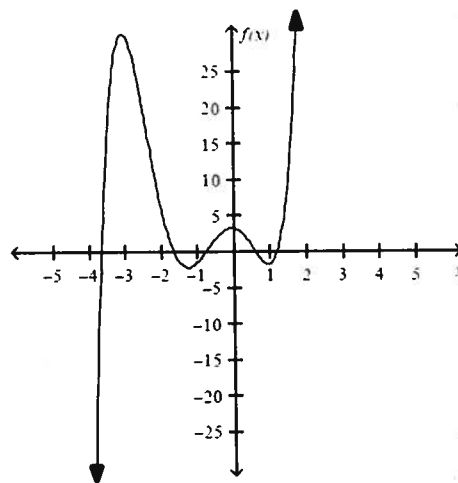
- a. $4x^2 - 4x - 9x + 9$ c. $4x(x - 1) - 9(x - 1)$
b. $4x^2 - 3x - 10x + 9$ d. $(4x - 9)(x - 1)$

_____ 8. Simplify: $\frac{30x^2+53x+22}{70x^2+17x-66}$ Assume that no denominator is equal to zero.

- a. $\frac{3x-2}{7x+6}$ b. $\frac{3x-2}{7x-6}$ c. $\frac{3x+2}{7x-6}$ d. $\frac{3x+2}{7x+6}$

_____ 9. Which of the following are true about the following graph?

- a. It is an odd-degree polynomial function.
The function has five real zeros.
 $\lim_{x \rightarrow -\infty} f(x) = +\infty$ and $\lim_{x \rightarrow +\infty} f(x) = +\infty$
- b. It is an odd-degree polynomial function.
The function has five real zeros.
 $\lim_{x \rightarrow -\infty} f(x) = -\infty$ and $\lim_{x \rightarrow +\infty} f(x) = +\infty$
- c. It is an odd-degree polynomial function.
The function has four real zeros.
 $\lim_{x \rightarrow -\infty} f(x) = -\infty$ and $\lim_{x \rightarrow +\infty} f(x) = +\infty$
- d. It is an even-degree polynomial function.
The function has five real zeros.
 $\lim_{x \rightarrow -\infty} f(x) = -\infty$ and $\lim_{x \rightarrow +\infty} f(x) = +\infty$



_____ 10. For the given values, determine consecutive values of x between which each real zero is located.

$$f(x) = -2x^4 - 4x^3 - 2x^2 + 3x + 8$$



- a. There is a zero between $x = 1$ and $x = 2$.
b. There are zeros between $x = 2$ and $x = 3$, between $x = 1$ and $x = 0$, and between $x = -2$ and $x = -3$.
c. There are zeros between $x = 1$ and $x = 2$ and between $x = -1$ and $x = -2$.
d. There is a zero between $x = -1$ and $x = -2$.

11. Estimate the x-coordinates at which the relative maxima and relative minima occur for the function

$$f(x) = x^3 - 4x^2 - 11x + 30$$



- The relative maximum is at $x = -3$ and the relative minimum is at $x = 5$.
- The relative maximum is at $x = 36$ and the relative minimum is at $x = -14.815$.
- The relative maximum is at $x = 30$ and the relative minimum is at $x = -11$.
- The relative maximum is at $x = -1$ and the relative minimum is at $x = 3.667$.

12. Find all the zeros of the function $f(x) = x^3 - 15x^2 + 73x - 111$ (No calculator.)

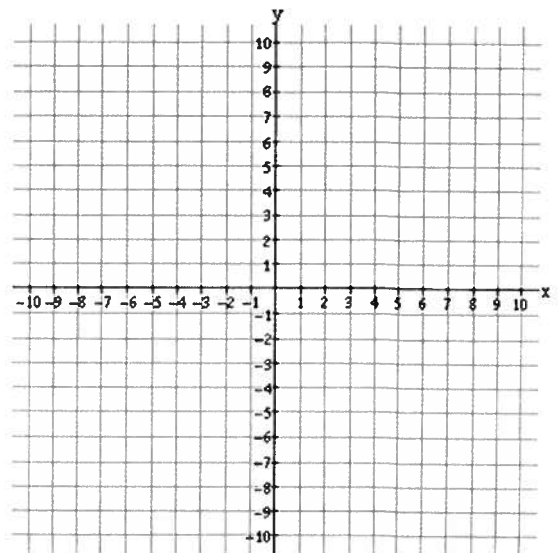
- $3, 6 + i, 6 - i$
- $6 + i, 6 - i$
- $3, 6 - i$
- $-3, 6 + i, 6 - i$

13. Calculate the inverse of the function given by $f(x) = \frac{7x-3}{16}$

- $f^{-1}(x) = \frac{16x-3}{7}$
- $f^{-1}(x) = \frac{16x+3}{7}$
- $f^{-1}(x) = \frac{7x+16}{3}$
- $f^{-1}(x) = \frac{7x-16}{3}$

14. Graph the function given by $f(x) = -\sqrt{4x+12} + 5$, labeling four (x, y) coordinate points on $f(x)$ and determine the domain and range of $f(x)$.

- domain: $x \geq 4$
range: $y \geq 5$
- domain: $x \leq -12$
range: $y \geq 5$
- domain: $x \geq -3$
range: $y \leq 5$
- domain: \mathbb{R}
range: \mathbb{R}



_____ 15. In $\triangle ABC$, $m\angle ABC = 90^\circ$, $AC = 25$ and $BC = 20$. Determine the $\sin(\angle A)$ and the $\cot(\angle A)$.

a. $\sin(\angle A) = \frac{3}{5}$ and $\cot(\angle A) = \frac{3}{4}$

b. $\sin(\angle A) = \frac{4}{5}$ and $\cot(\angle A) = \frac{3}{4}$

c. $\sin(\angle A) = \frac{4}{5}$ and $\cot(\angle A) = \frac{4}{3}$

d. $\sin(\angle A) = \frac{3}{5}$ and $\cot(\angle A) = \frac{4}{3}$

_____ 16. Simplify: $7 \sin(\theta) \sec(\theta)$.

a. $7 \cot(\theta)$

b. $7 \cos(\theta)$

c. $7 \tan(\theta)$

d. 7

_____ 17. Simplify: $\frac{\cos^2(\theta) - 1}{12 \sin^2(\theta)}$.

a. $-\frac{\sin(\theta)}{12}$

b. $-\frac{1}{12}$

c. $\frac{1}{12}$

d. $\frac{\cos(\theta)}{12}$

_____ 18. Give the domain of the function given by $(x) = \frac{x^2 - 9x + 20}{x^2 - 4x + 3}$.

a. $\mathbb{R}/x \neq 3, x \neq 1$

b. $\mathbb{R}/x \neq -5, x \neq -4$

c. $\mathbb{R}/x \neq -3, x \neq -1$

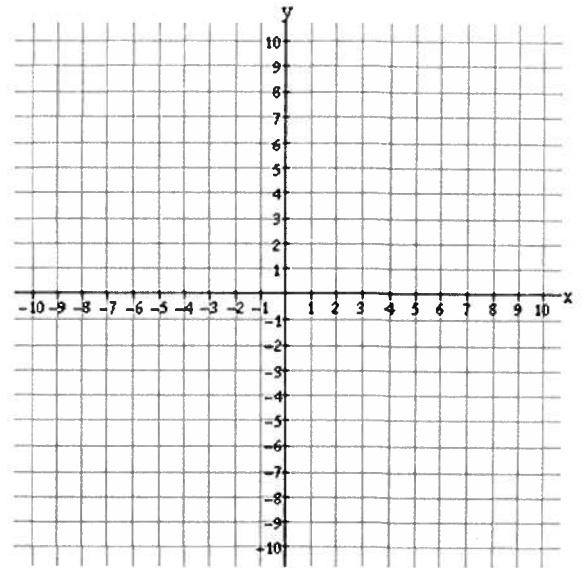
d. $\mathbb{R}/x \neq -5, x \neq 4$

_____ 19. Find the domain of the function given by $f(x) = \sqrt{-2x - 4}$.

- a. $x \geq 0$
- b. $x \leq -2$
- c. $x \geq -2$
- d. $x \leq 0$

_____ 20. Graph the function given by $f(x) = -x^2 - 4$ labeling four (x, y) coordinate points on $f(x)$ and determine the domain and range of $f(x)$.

- a. domain: $(-\infty, +\infty)$
range: $(-4, +\infty)$
- b. domain: $(-\infty, +\infty)$
range: $(-\infty, +\infty)$
- c. domain: $(-\infty, +\infty)$
range: $(-\infty, -4]$
- d. domain: $(-\infty, +\infty)$
range: $[-4, +\infty)$



_____ 21. Find the vertical asymptotes(s), if any, for $f(x) = \frac{3x-7}{x^2-5x+6}$.

- a. $x = 7, x = 2$
- b. $x = 2, x = 3, x = 7$
- c. $x = 2, x = 3$
- d. No vertical asymptotes

_____ 22. Determine if the graph of the rational function has a slant asymptote. If it does, find the equation of the slant asymptote.

$$f(x) = \frac{2x^3 + x^2 - 6x + 2}{x^2 - x - 2}$$

- a. $y = 2x$
- b. $y = 2x - 3$
- c. $y = 2x + 3$
- d. No slant asymptotes

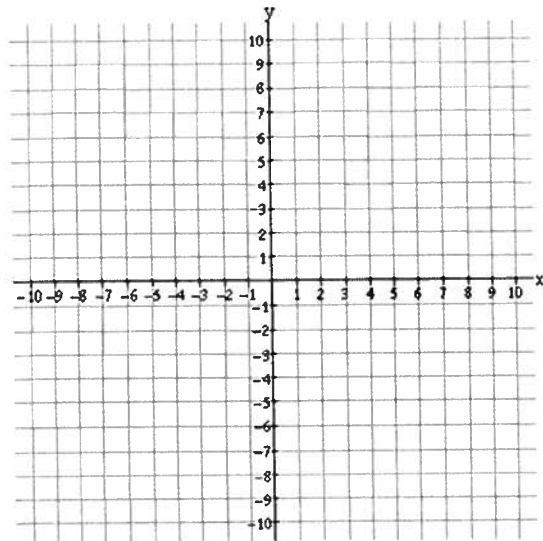
_____ 23. Determine the horizontal asymptote, if any of the graph of $f(x) = \frac{2x^2+8}{3x^2+4x-1}$.

- a. $y = \frac{2}{3}$ b. $y = -8$ c. $y = 0$ d. No horizontal asymptote

_____ 24. Sketch the graph of the following rational function labeling four (x, y) coordinate points on $f(x)$ and any asymptotes.

$$f(x) = \frac{2x^2-5x-3}{x^2-4}$$

- a. asymptotes: $x = -3, x = 2, y = 3$
 b. asymptotes: $x = -2, x = 2, y = 2$
 c. asymptotes: $x = -3, x = 3, y = 2$
 d. asymptotes: $x = -1, x = 3, y = 3$



_____ 25. Find $f(x+h), f(x+h) - f(x)$, and $\frac{f(x+h)-f(x)}{(x+h)-(x)}$ for the function $f(x) = \frac{2}{x+1}$.

- a. $\frac{2}{xh+1}, \frac{-2xh}{(x-h-1)(x-1)}, \frac{-2}{(x+h+1)(x+1)}$
 b. $\frac{2}{x+h+1}, \frac{-2xh}{(x-h-1)(x-1)}, \frac{2}{(x-h-1)(x+2)}$
 c. $\frac{2}{xh+1}, \frac{-2h}{(x+h+1)(x-1)}, \frac{2}{(x-h-1)(x+2)}$
 d. $\frac{2}{x+h+1}, \frac{-2h}{(x+h+1)(x-1)}, \frac{-2}{(x+h+1)(x+1)}$

_____ 26. Evaluate the expression: $\ln(e^2)$

- a. e^2 b. 2^e c. $\ln(2^e)$ d. 2

_____ 27. Evaluate the expression: $e^{\ln(14)}$.

a. $\ln(14^e)$

b. 14

c. $\ln e^{14}$

d. e^{14}

_____ 28. Solve the given equation algebraically: $5e^{-0.04x} + 67 = 97$.



a. -0.557

b. -44.794

c. -150.000

d. -19.454

_____ 29. Solve the given equation algebraically: $\frac{600}{1+e^{-x}} = 575$.



a. 3.135

b. -0.715

c. 3.062

d. -0.672

_____ 30. Find the value of x: $3 \ln(4x) = 13$.



a. 19.049

b. 1.083

c. 17.333

d. 0.367

_____ 31. What is $3 \ln(5) - \ln(2)$ written as a single logarithm?

a. $\ln(7.5)$

b. $\ln(27)$

c. $\ln\left(\frac{5}{2}\right)^2$

d. $\ln(62.5)$

___ 32. What is the solution of $e^{x+1} = 13$?

- a. $x = \ln(13) + 1$
- b. $x = \ln(13) - 1$
- c. $x = \ln(13)$
- d. $x = \ln(12)$

___ 33. Find the coordinates of the point of intersection of a -240° angle and the unit circle.

- a. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
- b. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
- c. $\left(\frac{-\sqrt{3}}{2}, -\frac{1}{2}\right)$
- d. $\left(-2, -\frac{2\sqrt{3}}{3}\right)$

___ 34. Approximate the value of $\arctan(-0.667)$.



- a. -33.703 radians
- b. $-.588$ radians
- c. $-.787$ radians
- d. $.787$ radians

___ 35. Graph $y = \sin(\theta)$, where $0 \leq \theta \leq 2\pi$.
Use the graph to evaluate $\sin\left(\frac{3\pi}{2}\right)$.

- a. $-\frac{\sqrt{2}}{2}$
- b. -1
- c. 0
- d. 1



___ 36. Use the graph of $y = \sin(\theta)$, where $0 \leq \theta \leq 2\pi$ to evaluate $\sin^{-1}(1)$.

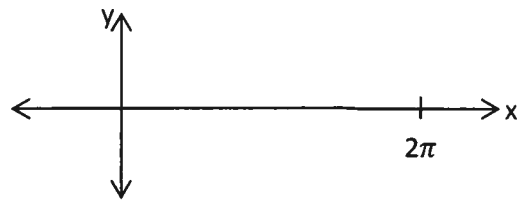
- a. $\frac{\pi}{2}$
- b. $\frac{\pi}{3}$
- c. $\frac{\pi}{4}$
- d. Not possible

37. Use the graph of $y = \sin(\theta)$, where $0 < \theta < 2\pi$ to evaluate $\text{rctan}(0)$.

- a. $\frac{\pi}{2}$ b. $\frac{\pi}{4}$ c. 0 d. π

38. Graph $y = \cos(\theta)$, where $0 \leq \theta \leq 2\pi$.
Use the graph to evaluate $\cos\left(\frac{3\pi}{2}\right)$.

- a. $-\frac{\sqrt{2}}{2}$ b. -1 c. 0 d. 1



39. Simplify: $\tan\left(\arcsin\left(\frac{1}{2}\right)\right)$.

- a. $\frac{\sqrt{3}}{3}$ b. $\sqrt{3}$ c. $\frac{\sqrt{3}}{2}$ d. $\frac{\sqrt{2}}{2}$

40. Factor the expression and use the fundamental identities to simplify $\cos^2(x)\sec^2(x) - \cos^2(x)$.

- a. $\cos^2(x)\cot^2(x)$
b. $\cos^2(x)$
c. 1
d. $\sin^2(x)$

41. Determine the expression that completes the identity: $\frac{1-\cos(u)}{\sin(u)} + \frac{\sin(u)}{1-\cos(u)} =$

- a. $2\csc(u)$
b. 0
c. $2\sin(u)$
d. $2 + \cos(u)$

_____ 42. Find all the x -values that are solution of the equation: $5\cot^2(x) - 15 = 0$, where $0 \leq \theta \leq 2\pi$.

a. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

b. $\frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6}$

c. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

d. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

_____ 43. Find the x -values that are solutions of the equation: $8\cos(x) - 4\sqrt{3} = 0$, where $0 \leq x \leq 2\pi$.

a. $\frac{\pi}{6}, \frac{11\pi}{6}$

b. $\frac{5\pi}{6}, \frac{7\pi}{6}$

c. $\frac{7\pi}{6}, \frac{11\pi}{6}$

d. $\frac{\pi}{6}, \frac{7\pi}{6}$

_____ 44. Find all the solutions of the equation in the interval $[0, 2\pi)$ where $4\cos(3x) - 2\sqrt{3} = 0$.

a. $\frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}$

b. $\frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}$

c. $\frac{\pi}{18}, \frac{11\pi}{18}, \frac{2\pi}{3}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}$

d. $\frac{\pi}{18}, \frac{13\pi}{18}$

_____ 45. Find all solutions of: $\sin(2x) - \frac{\sqrt{2}}{2} = 0$.

a. $\frac{\pi}{8} + n\pi, \frac{3\pi}{8} + n\pi$

b. $\frac{\pi}{8} + 2n\pi, \frac{3\pi}{8} + 2n\pi$

c. a. $\pm \frac{\pi}{3} + n\pi$

d. a. $\pm \frac{\pi}{3} + 2n\pi$

_____ 46. Classify the conic represented by the equation $x^2 + 11xy - 19y^2 + 2x + 8y + 22 = 0$.

- a. parabola
- b. hyperbola
- c. ellipse or circle
- d. none of these

_____ 47. Determine which equation that represents a parabola:

- a. $5y^2 + 6x - 2y = -11$
- b. $2x^2 + 6xy + 2y^2 + 5y - 2 = 0$.
- c. $5x^2 + 6x + 10y^2 - 2y = -2$.
- d. $-2y^2 + 6y + 2x^2 + 2x + 5 = 0$.

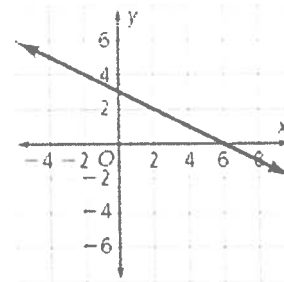
_____ 48. Divide $(8x^4 - 20x^3 - 14x^2 + 8x + 1)$ by $(x + 1)$ using synthetic division.

- a. $8x^3 + 28x^2 - 14x + \frac{11}{x+1}$
- b. $8x^3 + 36x^2 + 18x + 10 + \frac{11}{x+1}$
- c. $8x^3 - 28x^2 - 14x - 6 + \frac{7}{x+1}$
- d. $8x^3 + 28x^2 - 14x + 8$

_____ 49. Which equation represents a line through $(-1, 1)$ with a slope of $\frac{2}{3}$?

- a. $y + 1 = \frac{2}{3}(x + 1)$
- b. $y + 1 = \frac{2}{3}(x - 1)$
- c. $y - 1 = \frac{2}{3}(x - 1)$
- d. $y - 1 = \frac{2}{3}(x + 1)$

_____ 50. Which of the following equations is shown in the following graph?



a. $y + 2 = -\frac{1}{2}(x + 2)$

b. $y + 3 = -\frac{1}{2}(x + 6)$

c. $y - 3 = -\frac{1}{2}(x - 6)$

d. $y - 2 = -\frac{1}{2}(x - 2)$

_____ 51. Which of the following systems of equations has the solution $(4, -1)$?

a. $-2x + 4y = 6$ and $-3x + 6y = -2$

b. $3x - y = 0$ and $4x + 3y = 26$

c. $3x - 2y = 14$ and $2x + 2y = 6$

d. $4x + 9y = 1$ and $4x + 6y = -2$

_____ 52. What function has a graph with a removable discontinuity at $(5, \frac{1}{9})$?

a. $y = \frac{(x-5)}{(x+4)(x-5)}$

b. $y = \frac{1}{(x-5)}$

c. $y = \frac{(4x-1)(x-5)}{(5x+1)(x-5)}$

d. $y = \frac{x+1}{5x-4}$

_____ 53. Which expression equals that of $\frac{\frac{2}{m}+6}{\frac{1}{n}}$?

a. $\frac{12n}{m}$

b. $\frac{2n+6mn}{m}$

c. $\frac{6m+2}{mn}$

d. $\frac{m}{2n+6mn}$

_____ 54. Write $(12)^{\frac{1}{3}} \cdot (45)^{\frac{1}{3}} \cdot (50)^{\frac{1}{3}}$ in simplest form.

- a. $\sqrt{27,000}$
- b. 30
- c. $(107)^{\frac{1}{3}}$
- d. 27,000

_____ 55. What is $(-32x^{10}y^{35})^{-\frac{1}{5}}$?

- a. $2x^2y^7$
- b. $-\frac{2}{x^2y^7}$
- c. $-\frac{1}{2x^2y^7}$
- d. $\frac{2}{x^2y^7}$

_____ 56. Give the meaning of $\sum_{i=5}^8 (3i + 4)$

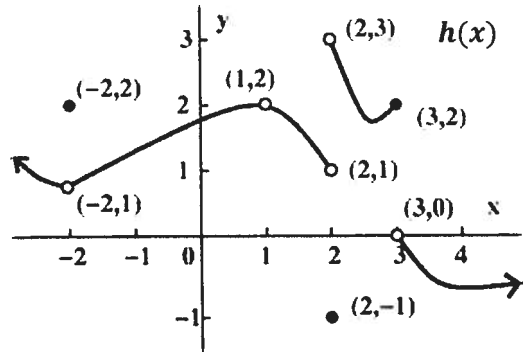
- a. 7, 10, 13, 16, 19, 22, 25, 28
- b. $19 + 22 + 25 + 28 = 94$
- c. $(3(5) + 4) + (3(8) + 4) = 47$
- d. $3 \frac{(8)(8+1)}{2} = 108$

_____ 57. Determine the number of terms in the finite arithmetic sequence: -3, 1, 5, 9, 13, ... 825.

- a. 208
- b. 275
- c. 825
- d. 828

58. Determine $\lim_{x \rightarrow -2} h(x)$

- a. does not exist
- b. -2
- c. 1
- d. 2



59. Determine $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 5x + 6}$

- a. does not exist
- b. -1
- c. 5
- d. $+\infty$

60. A certain type of radioactive element has a half-life of 15 years.

Suppose that the initially there were 22 grams. After how many years would there be 8.5 grams?



- a. 1.467 years
- b. 3.864 years
- c. 11.591 years
- d. 20.580 years

★ BONUS ★

Given $f(x) = \frac{x-2}{x+1}$, determine $f'(x)$ then evaluate $f'(-2)$ and explain the significance of $f'(-2)$.