Review of Princeton Mathematics K-12 Program of Studies
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Current high school math offerings:

- 2 tracks of Algebra (Regular, Plus)
- 3 tracks of Geometry (Regular, Accelerated, and Plus)
- 4 tracks of Algebra II (Elements, Regular, Accelerated, and Plus)
- Applications and Modeling of Math
- 3 tracks of PreCalculus (Regular, Accelerated, and Plus)
- Regular Calculus, AB Calc, BC Calc
- Multivariable Calculus and Linear Algebra
- Intro to Stat, Data Analysis and Probability
- AP Statistics
- Discrete Mathematics

Current K-8 school offerings

- Kindergarten
- Grade 1
- Grade 2
- Grade 3
- Grade 4
- Grade 5
- Grade 6 Pre Algebra Accelerated
- Grade 7 Algebra Part 1
- Grade 8 Algebra Part 2
- Algebra I mixed grades

The district has the traditional set of mathematics course offerings in high school that are seen at the majority of high schools in America. In middle school, the district has mixed the middle school NJSLS in grade level courses that are all mistitled as algebra. My comments with respect to examining the curriculum for compliance to NJSLS and QSAC are below. It should be noted that even though the district displays multiple tracks of many courses, the curriculum guides do not show the differentiation of content between the tracks and does not provide guides for the ELL courses.

HIGH SCHOOL

Algebra I - Note the course description mentions students will also experience an introduction to irrational numbers, data analysis and trigonometry. I do not see any experiences with trigonometry in the units below (note that such is not necessary in Algebra I anyway). The Algebra I curriculum is well aligned with NJSLS expectations. The units, however, are very

large and could be broken down into more manageable portions like the PreCalculus curriculum document. Note that much of the content in unit 1 (linear equations and inequalities) are 8th grade standards in NJ. So why is it necessary to spend 10 weeks on this material?

- Modeling with Linear Equations and Inequalities - 10 weeks
- Modeling with Linear & Exponential Functions & Systems - 10 weeks
- Quadratic Equations, Functions and Polynomials - 13 weeks
- Modeling with Statistics - 4 weeks

Geometry - The Geometry curriculum is again well aligned with NJSLS expectations but once again has very large units and could be broken down to drive instruction more carefully. The modeling unit is more accurately titled Volume and Surface Area.

- Congruence & Constructions - 10 weeks
- Congruence, Similarity and Proof - 10 weeks
- Trigonometric Ratios & Geometric Equations - 11 weeks
- Geometric Modeling - 6 weeks

Algebra II - Once again, the Algebra II curriculum is again well aligned with NJSLS expectations and contains large units. This is the first introduction to probability in high school for students so the district might consider if probability should be integrated in an earlier course. Note that this curriculum document lists Algebra II/Plus/ACC but does not in any way distinguish between them.

- Complex Solutions and Modeling with Rational Exponents - 8 weeks
- Polynomials & Analysis of Nonlinear Functions - 10 weeks
- Periodic Models and the Unit Circle - 9 weeks
- Inferences, Conclusions and Probability - 10 weeks

Appl of Adv Algebra/Modeling of Math - The course description notes that Applications on Advanced Algebra provides a review and extension of the concepts taught in Algebra 2. This course is PreCalc "lite" and the district should consider what is the rationale behind such? Why create a PreCalc "lite"? If students are not being adequately prepared for PreCalculus, then the district should examine why and not create a watered down course. I would suggest the elimination of this course.

- Overarching Essential Question - 1 week
- Prerequisite Skills - 3 weeks
- Functions & Graphs - 5 weeks
- Polynomials Functions - 5 weeks
- Rational and Radical Functions - 5 weeks
- Exponential & Logarithms Functions - 6 weeks
- Trigonometry - 7 weeks
- Graphing Trigonometric Functions - 6 weeks

PreCalculus. The curriculum here is exemplary with manageable sized units. It should be noted that more than half the curriculum is trigonometry.

- Basic Concepts of Algebra - 2 weeks
- Graphs, Functions and Models - 1 week
- Polynomials and Rational Functions - 2 weeks
- Exponential and Logarithmic Functions - 2 weeks
- Sequences, Series and Combinations - 5 weeks
- Trigonometric Functions - 7 weeks
- Trigonometric Identities and Inverse Functions - 8 weeks
- Applications of Trigonometry - 6 weeks
- System of Equations and Matrices - 4 weeks

Calculus. The curriculum is well aligned with expectations for the AP assessment.

- PreCalculus - 2 weeks
- Limits - 4 weeks
- Differentiation - 6 weeks
- Applications of Derivatives - 5 weeks
- Integrals - 9 weeks
- Applications of Integrals - 4 weeks
- Differential Equations - 2 weeks
- Infinite Series - 4 weeks
- Parametric, Polar and Vector functions - 3 weeks

AP statistics. A very detailed and thorough curriculum which is well aligned with AP expectations.

- Organizing Data: Exploring Data - 1 Week
- Organizing Data: Introduction to Probability – 1 week
- Organizing Data: The Normal Distribution – 2 weeks
- Organizing Data: Examining Relationships - 2 Weeks
- Organizing Data: More on Two-Variable Data – 2 weeks
- Producing Data - 2 Weeks
- Probability: The Study of Randomness - 3 Weeks
- Probability: Sampling Distribution - 4 Weeks
- Inference: Introduction to Inference – 3 weeks
- Inference: Inference for Distributions - 1 Week
- Inference: Inference for Proportions - 2 Weeks
- Inference: Inference for Tables: Chi-Square Proced - 2 Weeks
- Inference: Inference for Regression – 3 weeks
- AP Exam Review - 4 Weeks

Intro to Stats. The content here only covers 17 weeks. A full year introduction to statistics course is certainly warranted. The district should expand the content for a full year. A sample curriculum developed by the NSF and taught in Los Angeles Unified Public Schools can be found here.

- Unit 1: Exploring and Understanding Data - 5 Weeks
- Unit 2: Exploring Relationships Between Variables - 5 Weeks
- Unit 3: Gathering Data - 4 Weeks
- Unit 4: Randomness and Probability - 3 Weeks

Discrete Mathematics. The content here is 20 weeks - thus I'm assuming this is a one semester course.

- Unit 1: Set Theory - 4 Weeks
- Unit 2: Election Theory - 5 Weeks
- Unit 3: Matrix Theory - 4 Weeks
- Unit 4: Graph Theory - 4 Weeks
- Unit 5: Cartography - 3 Weeks

Multi Calc/Lin Alg (D) – There is no course description for this curriculum and the content only spans 14 weeks.

- Vectors and Motion in Space - 5 Weeks
- Multivariable Functions and Their Derivatives - 5 Weeks
- Multiple Integrals - 4 Weeks

MIDDLE SCHOOL - While the high school curriculum is well aligned with the NJSLS in almost all courses, the middle school curriculum combines standards from grades 6, 7 and 8 in various misnamed algebra courses without careful regard for scope and sequence. For all practical purposes, the course labeled grade 7 algebra part 1 is a grade 7 NJSLS class and should be renamed appropriately. The grade 8 algebra part 2 course is nearly equivalent to the high school Algebra I course. The Grade 6 pre-algebra course is a combination of many NJSLS standards and needs a complete review of its scope and sequence which as currently written is not a coherent set. It should be carefully noted that there is extensive research evidence suggesting that students placed into less rigorous versions of algebra ultimately have lower achievement in mathematics, even if their performance in the less rigorous version of the course is stronger than that of students in more rigorous versions (Tyson and Roksa 2017). The result of these placement practices is the same: inequitable learning outcomes.

Moreover, some grade 8 students are enrolled in high school courses including Geometry and Algebra II. Due to the lack of cohesiveness of the 6th and 7th grade curriculum, this practice should be revisited. Moreover, acceleration does enable and encourage selected students to take the same coursework one, or in very rare cases, two years ahead of what is typical but in making these decisions, it is essential that appropriate accelerated opportunities ensure that no critical concepts are rushed or skipped. Moreover, any such acceleration should entail a deeper treatment of the *same* content a year earlier with room for additional enrichment and *not* a faster race to take more courses sooner. This push to skip key concepts is what has been called a "race to calculus." Such a race is often misguided. Mathematics learning is not a race, and evidence suggests that students who speed through content without developing deep understanding are the very ones who tend to drop out of mathematics when they have the chance (Boaler 2016, p. 192). Some evidence suggests that policies that accelerate all students into algebra or higher in grade 8 do not result in overall improved student learning, as measured by NAEP (Loveless 2008, 2013).

Acceleration should be seen as an option for selected, highly capable students and not a different track with distinctly different potential outcomes. Decisions on exactly which students benefit from acceleration options and effective placement into these courses earlier than normal need to be collaboratively made by the Princeton mathematics department and the guidance department and thoroughly understood by all of the members of the community.

Grade 6 Pre Algebra Accelerated - Even though only 6th grade NJSLS are listed in Rubicon, the content of this course (no course description) has combined NJSLS content from all the middle school grades without regard for scope/sequence . For instance equations and expressions are taught for 11 weeks to begin the year but students do **not** have understandings of fractions, decimals, integers nor order of operations at this point. I suggest that this curriculum be revisited with a careful attention to scope/sequence of math content.

- *Operations with Equations and Expressions -11 Weeks*
- *Order of Operations/Standard Algorithms -3 Weeks*
- *Integers and Inequalities - 5 Weeks*
- *Rational Numbers/Ratios/Rates/Proportions -13 Weeks*
- *Number Theory and Fractions -5 Weeks*
- *Graphing and Linear Functions -3 Weeks*
- *Decimals -4 Weeks*
- *Geometry -3 Weeks*
- *Ratios, Rates, Proportions, and Percent -5 Weeks*
- *Statistics - 4 Weeks*
- *Angle Relationships and Transformations -3 Weeks*
- *Simple Equations - 6 Weeks*

Grade 7 Algebra Part 1 - this course draws the majority of its content from the 7th & 8th grade NJSLS. Calling this course Algebra Part 1 is not in the best interest of students and parents.

- *Operations on Rational Numbers - 7 Weeks*
- *Equations and Ratio & Proportion - 12 Weeks*
- *Drawing Inferences about Populations & Probability - 5 Weeks*

- *Problem Solving with Geometry -11 Weeks*

Grade 8 Algebra Part 2 - this course draws its content from various NJSLS standards including 8th grade and high school algebra. It eliminates all content from the 8th grade Geometry standards (some of that content is taught in grade 7 Algebra Part 1)

- *Modeling with Linear Equations and Inequalities - 9 Weeks*
- *Modeling w/Equations,Systems,Exponential Functions - 11 Weeks*
- *Quadratic Equations, Functions, & Polynomials - 13 Weeks*
- *Modeling with Statistics - 4 Weeks*

Algebra I mixed grades - this course is equivalent to Grade 8 Algebra Part 2.

- *Modeling with Linear Equations & Inequalities - 10 Weeks*
- *Model w/ Linear & Exponential Functions, & Systems - 10 Weeks*
- *Quadratic Equations, Functions & Polynomials - 13 Weeks*
- *Modeling with Statistics - 4 Weeks*

ELEMENTARY (Grades K-5) - The elementary curriculum is well aligned with the NJSLS in most cases with the notable exception of grade 4 which has content from many other higher grades. In some grades as noted above, the NJSLS are not cited explicitly. Lastly, the district should consider the major standards of grades K-8 (see here) when writing and/or revising scope and sequence.

Kindergarten - the curriculum is well aligned with the NJSLS standards with an appropriate scope and sequence.

- *Overarching Essential Question*
- *Unit 1- Numbers 0 to 5 - 7 Weeks*
- *Unit 2 Numbers 6 to 10 - 7 Weeks*
- *Unit 3 Geometry - 5 Weeks*
- *Unit 4 Numbers Within 10 - 10 Weeks*
- *Unit 5 Numbers 11 to 100 - 5 Weeks*
- *Unit 6 Measurement - 4 Weeks*

Grade 1 - the curriculum is well aligned with the NJSLS standards but the curriculum units need more appropriate titles (not names of months). The measurement standards should be moved to later in the year.

- *Unit 1 September-October - 4 Weeks*
- *October-November unit 2-3 - 6 Weeks*
- *November-January unit 4-5 -9 Weeks*
- *January-March unit 6-7 - 8 Weeks*
- *March-May unit 8-9 - 7 Weeks*
- *May-June unit 10 -3 Weeks*

Grade 2 - the first three units do not list the 2nd grade NJSLS standards explicitly while subsequent units do so. Once again, unit titles should be revised and not be titled by months. Addition and subtraction (May/June unit) should be moved to earlier in the year.

- *Unit 1 Numbers Within 20:* - *4 Weeks*
- *Numbers Within 100* - *7 Weeks*
- *Numbers Within 1000* - *4 Weeks*
- *January-February Units 5-6 (Geometry) - 5 Weeks*
- *February-March Units 7-8* - *4 Weeks*
- *March-April Units 9-10* - *7 Weeks*
- *May-June Units 11-12 -6 Weeks*

Grade 3 - the curriculum is well aligned with the NJSLS standards (but the standards are not cited explicitly) with an appropriate scope and sequence.

- *Unit 1 Three Digit Numbers -5 Weeks*
- *Unit 2 Multiplication and Division - 10 Weeks*
- *Unit 3 Multiplication - 6 Weeks*
- *Unit 4 Fractions - 6 Weeks*
- *Unit 5 Measurement - 4 Weeks*
- *Unit 6 Shapes - 5 Weeks*

Grade 4 – The units does not cite the NJSLS at all in any unit. Moreover, some units are duplicates of others (Geometry/Measurement and Geometry I) and some units (probability and rates) are not appropriate for this grade level as per the NJSLS. I suggest this grade level be completely revised to align with the 4th grade NJSLS.

- *Unit 1-Whole Numbers - 6 Weeks*
- *Operations - 7 Weeks*
- *Multi-Digit Operations and Measurement - 7 Weeks*
- *Fractions, Decimals, and Measurement - 11 Weeks*
- *Geometry and Measurement - 6 Weeks*
- *Geometry I - 4 Weeks*
- *Place Value & Basic Operations - 6 Weeks*
- *Decimals & Metric Measurement - 4 Weeks*
- *Multiplication & Estimation - 4 Weeks*
- *Division & Angles - 3 Weeks*
- *Fractions & Probability - 4 Weeks*
- *Geometry II - 5 Weeks*
- *Fractions/Decimals/Percents - 4 Weeks*
- *3-D Geometry & Measurement - 2 Weeks*
- *Rates - 1 Week*

Grade 5 - the curriculum is well aligned with the NJSLS standards (but the standards are not cited explicitly) with an appropriate scope and sequence.

- *Unit 1 Whole Number Operations and Applications* - *6 Weeks*
- *Unit 2 Decimals and Fractions* - *9 Weeks*
- *Unit 3: More Decimals and Fractions* - *11 Weeks*
- *Unit 4: Measurement, Data, and Geometry* - *5 Weeks*
- *Unit 5: Algebraic Thinking & the Coordinate Plane* - *5 Weeks*

With respect to QSAC, the district math curriculum is lacking in many areas. All documents need to be updated to include:

- Differentiation section including accommodations and modifications for special education students, English language learners, students at risk of school failure, gifted and talented students, and students with 504 plans;
- Assessments, including, formative, summative, benchmark, and alternative assessments;
- Learning Activities section and a list of core instructional and supplemental materials, including various levels of texts at each grade level;
- A more detailed pacing guide;
- Ideas for service learning connections and interdisciplinary connections;
- Integration of 21st century skills and technology

Note that there are three additional courses (ESL Algebra 1, ESL Geometry, ESL Algebra II) that the district offers that I do NOT see any details in the curriculum documents. Moreover, the district should consider adding social justice cultural relevant content to its mathematics curriculum. See here for some examples by Benjamin Dickman.

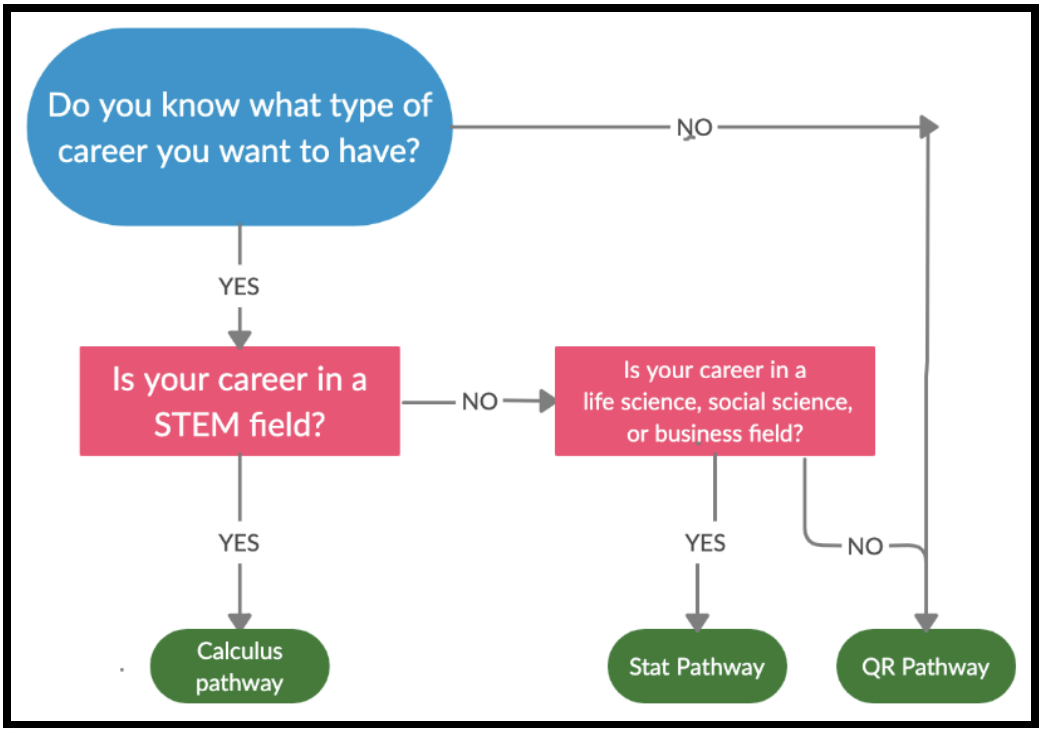
TRACKING

The district offers multiple tracks of the Algebra-Geometry-Algebra II sequence. It is recommended that the district reexamine its numerous tracks and its tracking policies. There is intensive research on the detrimental effects of tracking in mathematics. As far back as 1983, the report *A Nation at Risk* included urgent language around dismantling tracking in American high schools and called for a commitment to the "twin goals of equity and high-quality schooling." More than 40 years later, tracking is still prevalent as seen in the Princeton mathematics curriculum.

In 2018, the National Council of Teachers of Mathematics (NCTM) published *Catalyzing Change in High School Mathematics,* which again calls for high school mathematics to discontinue the practice of tracking, both of tracking students into qualitatively different or dead-end course pathways *and* also the tracking teachers in ways that usually ensure the most needy students get the least experienced and weakest teachers. This publication emphatically

states that: “Tracking is insidious because it places some students into qualitatively different or lower levels of a mathematics course and, in some cases, puts students into terminal mathematics course pathways that are not mathematically meaningful and do not prepare them for any continued study of fundamental mathematical concepts. Too often placement into different tracks is based on a variety of nonacademic factors, such as perceived (but not potential) academic ability, race, socioeconomic status, gender, language, or other expectations ascribed to students by adults. Princeton schools, instead, should develop mathematics pathways, and the structures and practices associated with them, which would be designed to eliminate tracking and both the implicit and explicit bias that comes with tracking students.

Mathematics pathways are where students are offered options based on their own aspirations and interests. With appropriate guidance and information, students implementing their own choices may work harder than students who have been simply assigned to a course. If the best way to ensure that students have genuine opportunities to prepare for their futures is by offering multiple math pathways, who decides which pathway a student pursues, and when should that choice be made? This is a difficult and important question. Teachers, parents and counselors will have to actively support students in developing STEM aspirations and recruiting students from racial, ethnic, gender, or social class groups not well represented in STEM. Moreover, students who initially opt for a non-STEM pathway must have the opportunity to switch to a STEM pathway during high school and vice versa, if and when their interests change.



Designing new math pathways can accomplish at least four important goals:

1. Students will be able to learn the mathematics that prepares them for STEM careers.
2. Students will be able to learn the mathematics that prepares them for other careers without being blocked by irrelevant requirements.
3. Latinx and African American students will have ample opportunities to thrive in college, including in STEM fields, as will female students of all ethnicities.
4. Students who initially choose a non-STEM pathway will be able to switch to a STEM pathway during high school or college, and vice versa, if their interests change.

Sample Math Pathways

<table>
<tr><th>Grade</th><th colspan="3">Courses</th></tr>
<tr><td>9th</td><td colspan="3">Algebra I (or Integrated Math I)</td></tr>
<tr><td>10th</td><td colspan="3">Geometry (or Integrated Math II)</td></tr>
<tr><td rowspan="2">11th</td><td>Quantitative Literacy Pathway</td><td>Statistics Pathway</td><td>Calculus Pathway</td></tr>
<tr><td>The mathematics of real world problem-solving, modeling, financial literacy and effective citizenship: Students intending to follow paths in technical fields, liberal arts, and communications.</td><td>The mathematics of data, uncertainty and chance: Students intending to enter health, social science, and business fields.</td><td>The mathematics of functions and change: Students intending to enter STEM and natural science fields.</td></tr>
<tr><td>12th</td><td colspan="2">A range of 4th year math options including: AP Statistics, Introduction to Data Science, Topics in Finite & Discrete Mathematics and Financial Algebra</td><td>Pre-Calculus or AP Statistics</td></tr>
</table>

In conclusion, this review is based only on the intended curriculum as written in Rubicon. The next step would to review the implemented curriculum taught in Princeton math classrooms. Thank you for the opportunity to provide this review of the Princeton K-12 mathematics offerings. I can reached for any questions or clarifications at milou@rowan.edu