

Radicals

What does $\sqrt{\quad}$ mean?

$$\sqrt{25} = 5$$

You should be familiar with the perfect squares up to 20.

$$\sqrt{1} = 1$$

$$\sqrt{64} = 8$$

$$\sqrt{225} = 15$$

$$\sqrt{4} = 2$$

$$\sqrt{81} = 9$$

$$\sqrt{256} = 16$$

$$\sqrt{9} = 3$$

$$\sqrt{100} = 10$$

$$\sqrt{289} = 17$$

$$\sqrt{16} = 4$$

$$\sqrt{121} = 11$$

$$\sqrt{324} = 18$$

$$\sqrt{25} = 5$$

$$\sqrt{144} = 12$$

$$\sqrt{361} = 19$$

$$\sqrt{36} = 6$$

$$\sqrt{169} = 13$$

$$\sqrt{400} = 20$$

$$\sqrt{49} = 7$$

$$\sqrt{196} = 14$$

What do you do when the number under the radical is not a perfect square?

Most Basic Way

Step 1 – Find the prime factorization of the number.

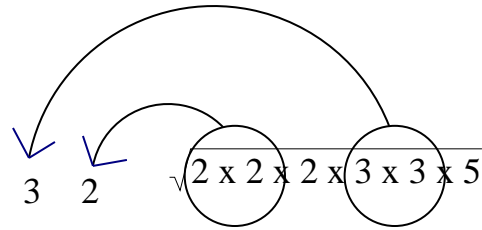
example: $\sqrt{360}$

$$2 \times 2 \times 2 \times 3 \times 3 \times 5$$

Step 2 – Put the numbers in order under the radical.

$$\sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 5}$$

Step 3 – Circle any pairs of numbers – they come out as ONE of that number.



Step 4 – Multiply outside numbers together and left over inside numbers together.

$$6\sqrt{10}$$

More Advanced Way

Step 1 – Think of a perfect square factor of the number under the radical.

example: $\sqrt{360}$

$$\sqrt{36 \times 10} = \sqrt{36} \times \sqrt{10}$$

Step 2 – Take the square root of the number from Step 1 - Bring it out. Leave the rest under the radical.

$$6\sqrt{10}$$

Exercises – Simplify

1. $\sqrt{24}$

2. $\sqrt{12}$

3. $\sqrt{32}$

4. $\sqrt{48}$

5. $\sqrt{250}$

6. $\sqrt{98}$

7. $6\sqrt{20}$

8. $\sqrt{44}$

Multiplying Radicals – think of them as variables – multiply radicands by radicands and coefficients by coefficients.

Example- $3\sqrt{2} \times 4\sqrt{5} = (3 \times 4) \sqrt{(2 \times 5)} = 12\sqrt{10}$

Exercises – Multiply

9. $4\sqrt{2} \times 8\sqrt{3} =$

10. $\sqrt{99} \times \sqrt{3} =$

11. $5\sqrt{2} \times 3\sqrt{6} =$

12. $\sqrt{18} \times \sqrt{2} =$

13. $\sqrt{12} \times \sqrt{6} =$

14. $\sqrt{8} \times \sqrt{6} =$

Dividing Radicals – Sometimes it is easier to reduce the fraction first – sometimes it is easier to take the square root first.

Example - $\frac{\sqrt{24}}{\sqrt{2}} = \sqrt{\frac{24}{2}} = \sqrt{12} = 2\sqrt{3}$

Example - $\sqrt{\frac{5}{25}} = \frac{\sqrt{5}}{\sqrt{25}} = \frac{\sqrt{5}}{5}$

Exercises – Divide

15. $\frac{\sqrt{18}}{\sqrt{6}}$

16. $\frac{\sqrt{72}}{\sqrt{36}}$

17. $\frac{\sqrt{75}}{\sqrt{3}}$

18. $\frac{\sqrt{3}}{\sqrt{4}}$

Rationalize the denominator- You may never have a radical in the denominator of a fraction or a fraction under a radical. The process to eliminate this is called “Rationalizing the Denominator.”

Step 1 – Find the smallest number that you would have to multiply the radicand in the denominator by to get a perfect square.

$$\frac{3}{\sqrt{2}}$$

Step 2 – Multiply the numerator and the denominator by one in the form of $\frac{\sqrt{x}}{\sqrt{x}}$, where x is the number found in Step 1.

$$\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{4}}$$

Step 3 – Simplify – this will eliminate the radical in the denominator.

$$\frac{3\sqrt{2}}{2}$$

Exercises – Rationalize the denominator

19. $\frac{\sqrt{5}}{\sqrt{3}}$

21. $\frac{\sqrt{3}}{\sqrt{8}}$

20. $\frac{\sqrt{12}}{\sqrt{5}}$

22. $\sqrt{\frac{5}{12}}$

Adding and Subtracting Radical Expressions – Again, think of variables, the radicands must be the same in order to add or subtract.

Example- $2\sqrt{3} + 8\sqrt{3} = (2 + 8)\sqrt{3} = 10\sqrt{3}$

$$15\sqrt{10} - 2\sqrt{10} = (15 - 2)\sqrt{10} = 13\sqrt{10}$$

$$4\sqrt{2} + 6\sqrt{7} - 11\sqrt{7} = 4\sqrt{2} + (6\sqrt{7} - 11\sqrt{7}) = 4\sqrt{2} - 5\sqrt{7}$$

(only the $\sqrt{7}$'s can be combined)

Sometimes you have to simplify first to add or subtract.

Example- $3\sqrt{8} + 5\sqrt{2} = 6\sqrt{2} + 5\sqrt{2} = 11\sqrt{2}$

Exercises – Add or subtract.

23. $6\sqrt{2} + 8\sqrt{2} - \sqrt{2} =$

25. $4\sqrt{20} + 6\sqrt{45} - 3\sqrt{5} =$

24. $5\sqrt{3} - 2\sqrt{12} =$

26. $-3\sqrt{7} + 5\sqrt{28} =$

Multi-Step Equations. Solve each equation.

27. $-20 = -4x - 6x$

28. $8x - 2 = -9 + 7x$

29. $-8 = -(x + 4)$

30. $-18 - 6x = 6(1 + 3x)$

31. $2(4x - 3) - 8 = 4 + 2x$

32. $-(1 + 7x) - 6(-7 - x) = 36$

33. $-3(4x + 3) + 4(6x + 1) = 43$

34. $-5(1 - 5x) + 5(-8x - 2) = -4x - 8x$

Find the slope of the line that passes through the following points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

35. $(-3, 5), (8, -2)$

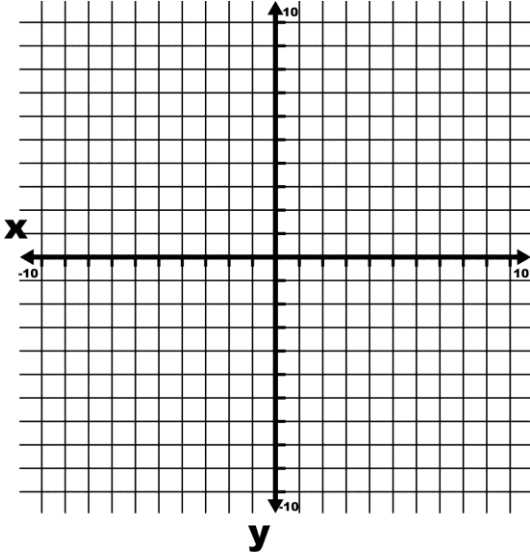
37. $(2, -7), (6, -7)$

36. $(0, 4), (-8, 1)$

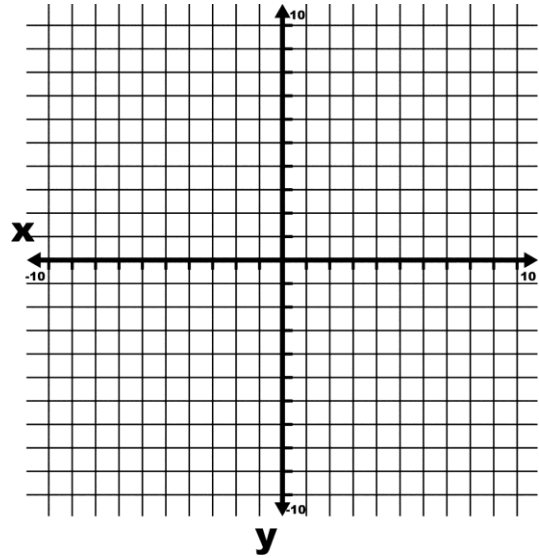
38. $(9, 4), (9, -2)$

Graph each of the following lines.

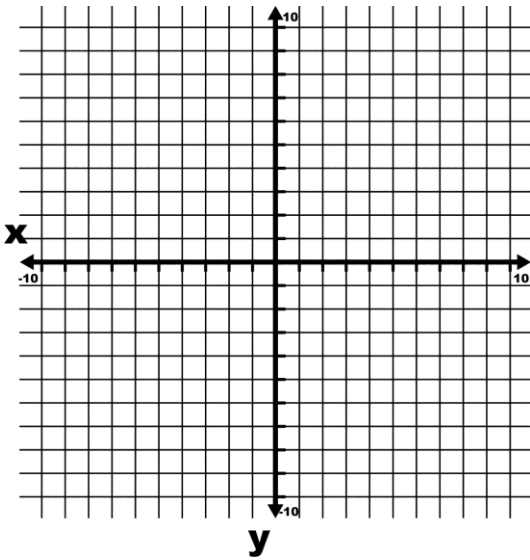
39. $y = \frac{4}{3}x - 2$



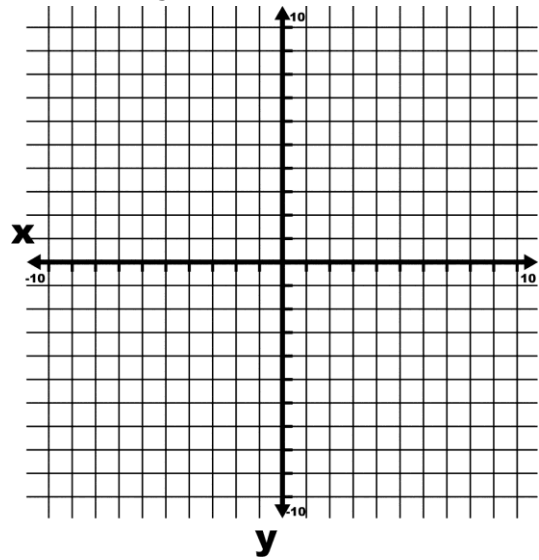
41. $y = -5$



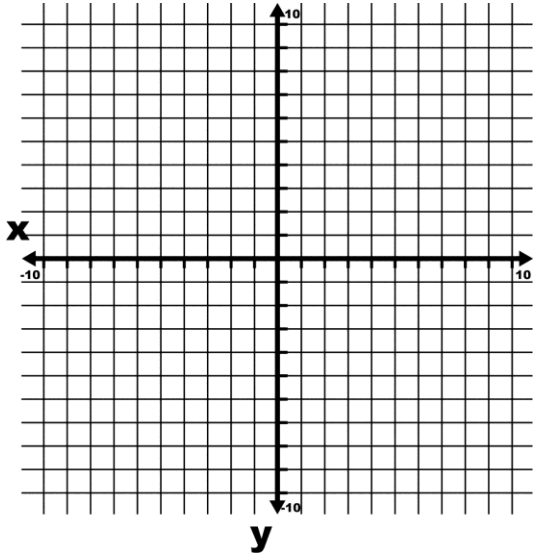
40. $y = -6x + 3$



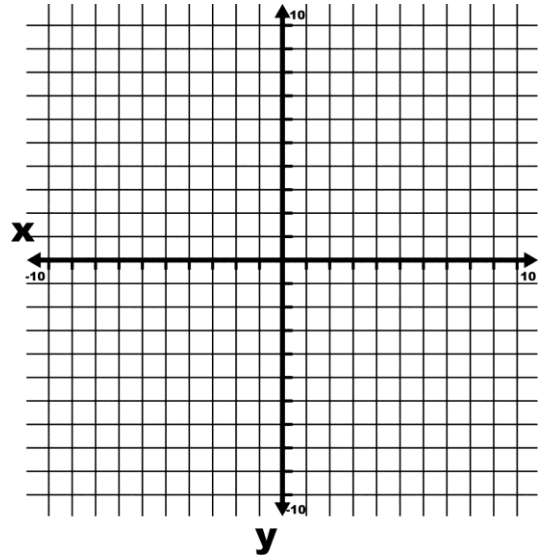
42. $y = \frac{-3}{5}x + 6$



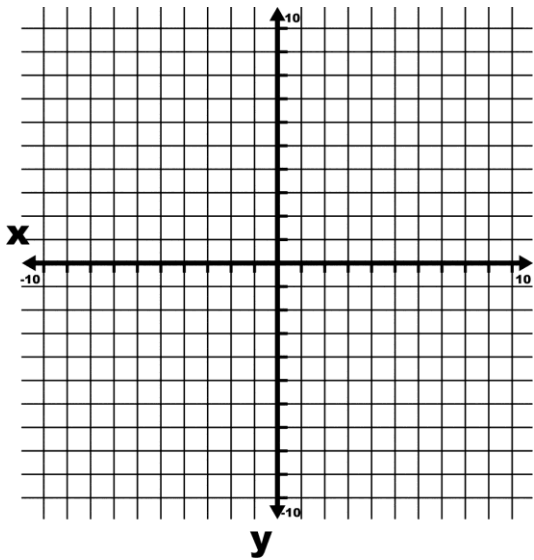
43. $y = \frac{1}{4}x + 2$



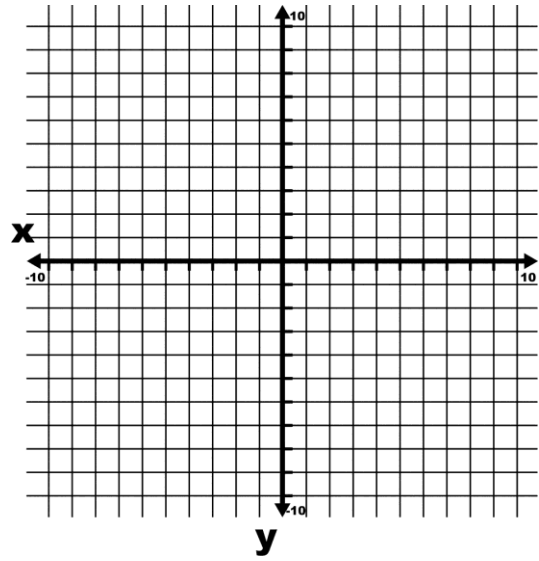
45. $y = x$



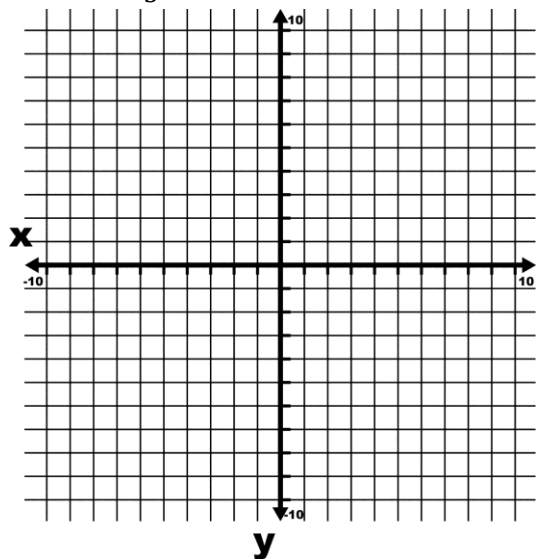
44. $x = 4$



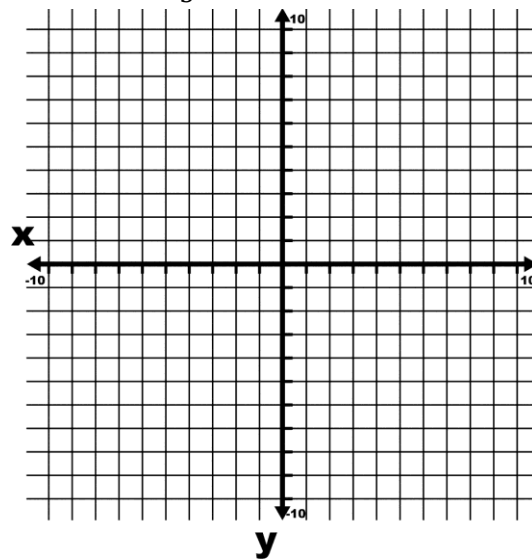
46. $y = -x$



$$47. y = \frac{5}{3}x$$



$$48. y = -\frac{1}{3}x + 3$$



Write the slope-intercept form of the equation of each line.

$$49. 3x - 2y = -16$$

$$52. x - 3y = 6$$

$$50. 13x - 11y = -12$$

$$53. 11x - 4y = 32$$

$$51. 9x - 7y = -7$$

$$54. 11x - 8y = -48$$

Write the slope-intercept form of the equation of the line described.

$$55. \text{ through: } (1, 2), \text{ slope } = 7$$

$$56. \text{ through: } (3, -1), \text{ slope } = -1$$

$$57. \text{ through: } (-2, 5), \text{ slope } = -4$$

$$58. \text{ through: } (3, 5), \text{ slope } = \frac{7}{3}$$

$$59. \text{ through: } (2, 5), \text{ slope } = \text{undefined}$$

$$60. \text{ through: } (-1, 2), \text{ slope } = 0$$

61. through: $(4, 2)$, parallel to $y = -\frac{3}{4}x - 5$

62. through: $(-3, -3)$, parallel to $y = \frac{7}{3}x + 3$

63. through: $(2, 0)$, parallel to $y = \frac{1}{3}x + 3$

64. through: $(-2, 4)$, parallel to $y = -\frac{5}{2}x + 5$