BC Summer Work

Name _____

1) Do all work on these sheets
2) No calculator
3) To be done in **August**
4) Due the **very first** time class meets
5) Provide a **detailed** interval line for #6, #16, #25

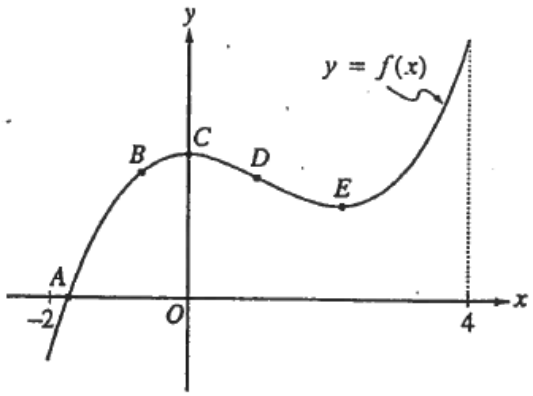
_____1. If $f(x) = \sin^2(3-x)$, then $f'(0) =$

(A) $-2\cos 3$ (B) $-2\sin(3)\cos(3)$ (C) $6\cos(3)$
(D) $2\sin(3)\cos(3)$ (E) $6\sin(3)\cos(3)$

_____2. What is the average rate of change of the function f defined by $f(x) = 100 \cdot 2^x$ on the interval [0,4] ?
(A) 100 (B) 375 (C) 400 (D) 1500 (E) 1600

_____3. $\lim_{h \to 0} \dfrac{\tan 3(x+h) - \tan(3x)}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent



_____4. The function f is shown in the figure above. At which of the following points could the derivative of f be equal to the average rate of change of f over the closed interval [-2,4]?
(A) A (B) B (C) C (D) D (E) E

_____ 5. $\dfrac{d}{dx}\left(\sin(\cos x)\right) =$

(A) $\cos(\cos x)$
(B) $\sin(-\sin x)$
(C) $\left(\sin(-\sin x)\right)\cos x$
(D) $-\left(\cos(\cos x)\right)\sin x$
(E) $-\left(\sin(\cos x)\right)\sin x$

_____ 6. If $f(x) = \dfrac{\ln x}{x}$ for $x > 0$, which of the following is true?

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1
(C) f is decreasing for all x between 0 and 1
(D) f is decreasing for all x between 1 and e
(E) f is decreasing for all x greater than e

____ 7.

Let f be the function defined by

$$f(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5} & \text{for } x \neq 5 \\ 0 & \text{for } x = 5. \end{cases}$$

Which of the following statements about f are true?

I. $\lim_{x \to 5} f(x)$ exists.
II. $f(5)$ exists.
III. $f(x)$ is continuous at $x = 5$.

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

____ 8.

If a particle moves in the xy-plane so that at time $t > 0$ its position vector is $\left(\ln(t^2 + 2t), 2t^2\right)$, then at time $t = 2$, its velocity vector is

(A) $\left(\frac{3}{4}, 8\right)$ (B) $\left(\frac{3}{4}, 4\right)$ (C) $\left(\frac{1}{8}, 8\right)$ (D) $\left(\frac{1}{8}, 4\right)$ (E) $\left(-\frac{5}{16}, 4\right)$

____ 9.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
10	35	15	6	4
20	8	5	12	10
30	24	25	20	10

Selected values of the functions f and g and their derivatives, f' and g', are given in the table above. If $h(x) = f(g(x))$, what is $h'(30)$?

(A) 5 (B) 15 (C) 35 (D) 50 (E) 250

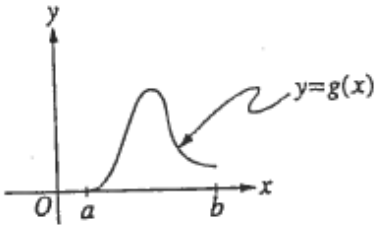
_____ 10. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$
(ii) $f(0) = 1$

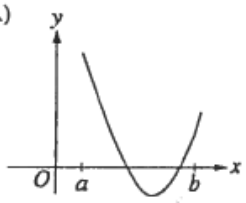
If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

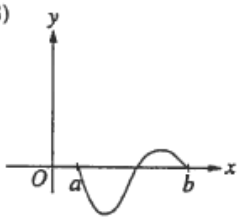
(A) $f'(x)$ (C) $g(x)$ (C) e^x (D) 0 (E) 1

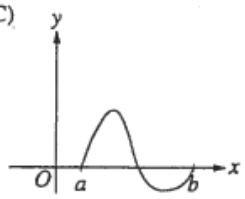
____ 11.

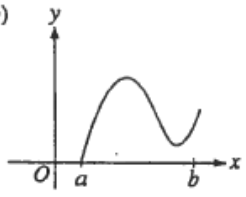


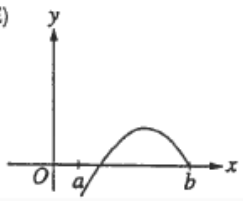
The figure above shows the graph of g on $[a,b]$. If $g'(x) = f(x)$, which of the following could be the graph of f on $[a,b]$?

(A)


(B)


(C)


(D)


(E)


____ 12.

If $f(x) = 1 + x^{\frac{2}{3}}$, which of the following is NOT true?

(A) f is continuous for all real numbers.

(B) f has a minimum at $x = 0$.

(C) f is increasing for $x > 0$.

(D) $f'(x)$ exists for all x.

(E) $f''(x)$ is negative for $x > 0$.

____ 13. If $f(x)=\dfrac{\sin x}{2x}$, then $f'(x)=$

(A) $\dfrac{\cos x}{2}$

(B) $\dfrac{x\cos x-\sin x}{2x^2}$

(C) $\dfrac{x\cos x-\sin x}{4x^2}$

(D) $\dfrac{\sin x-x\cos x}{2x^2}$

(E) $\dfrac{\sin x-x\cos x}{4x^2}$

14. Suppose $f(1) = 2, f'(1) = 3$, and $f'(2) = 4$. Then $(f^{-1})'(2)$

(A) equals $-\frac{1}{3}$ (B) equals $-\frac{1}{4}$ (C) equals $\frac{1}{4}$

(D) equals $\frac{1}{3}$ (E) cannot be determined

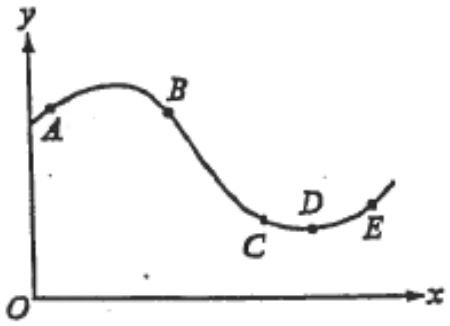
15. A spherical balloon is being inflated at a constant rate of 25 cm^3/sec. At what rate, in cm/sec, is the radius of the balloon changing when the radius is 2 cm? (The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) $\frac{25}{16\pi}$ (B) $\frac{25}{8\pi}$ (C) $\frac{75}{16\pi}$ (D) $\frac{32\pi}{25}$ (E) $\frac{32\pi}{3}$

16. _____

Which of the following statements about the curve $y = x^4 - 2x^3$ is true?

(A) The curve has no relative extremum.
(B) The curve has one point of inflection and two relative extrema.
(C) The curve has two points of inflection and one relative extremum.
(D) The curve has two points of inflection and two relative extrema.
(E) The curve has two points of inflection and three relative extrema.



17. _____

At which of the five points on the graph in the figure above are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

(A) A (B) B (C) C (D) D (E) E

18. $\lim_{x \to 3} \frac{1}{x - 3}$ is

(A) –3 (B) 0 (C) 1 (D) 3 (E) nonexistent

19. If $x^2 + y^3 = x^3y^2$, then $\frac{dy}{dx} =$

(A) $\frac{2x + 3y^2 - 3x^2y^2}{2x^3y}$

(B) $\frac{2x^3y + 3x^2y^2 - 2x}{3y^2}$

(C) $\frac{3x^2y^2 - 2x}{3y^2 - 2x^3y}$

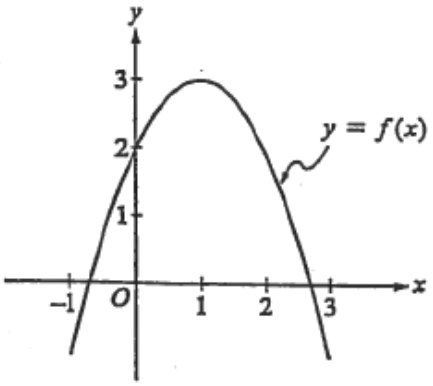
(D) $\frac{3y^2 - 2x^3y}{3x^2y^2 - 2x}$

(E) $\frac{6x^2y - 2x}{3y^2}$

20. If the function f is continuous for all real numbers and $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = 7$, then which of the following statements must be true?

(A) $f(a) = 7$
(B) f is differentiable at $x = a$.
(C) f is differentiable for all real numbers.
(D) f is increasing for $x > 0$.
(E) f is increasing for all real numbers.

21.



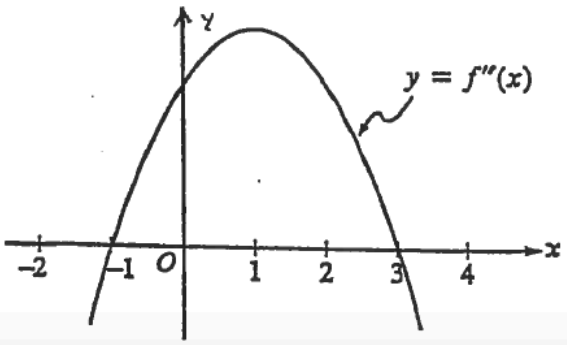
The function f has a relative maximum value of 3 at $x = 1$, as shown in the figure above. If $h(x) = x^2 f(x)$, then $h'(1) =$

(A) –6 (B) –3 (C) 0 (D) 3 (E) 6

22.

The graph of f'', the second derivative of the function f, is shown in the figure. On what intervals is f concave up?

(A) $(-\infty, \infty)$
(B) $(-\infty, -1)$ and $(3, \infty)$
(C) $(-\infty, 1)$
(D) $(-1, 3)$
(E) $(1, \infty)$



23. Let f(x) and g(x) be differentiable functions and let the values of f(x), g(x), and the derivatives f ’(x) and g ’(x) at x = 1 and x = 2 be given by the table below:

x	f (x)	g (x)	f ’(x)	g ’(x)
1	3	2	5	4
2	2	π	6	7

Determine the value of each of the following.

a) The derivative of f + 3g at x = 2

b) The derivative of f · g at x = 2

c) The derivative of f / g at x = 2

d) h ’(1) where h(x) = f (g (2x))

e) Write an equation for the line tangent to f at x = 1

24.

Let f be a function defined by $f(x) = \begin{cases} 2x - x^2 & \text{for } x \le 1, \\ x^2 + kx + p & \text{for } x > 1. \end{cases}$

(a) For what values of k and p will f be continuous and differentiable at $x = 1$?

(b) For the values of k and p found in part (a), on what interval or intervals is f increasing?

(c) Using the values of k and p found in part (a), find all points of inflection of the graph of f. Support your conclusion.

25.

Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

(a) Write an equation of the horizontal asymptote for the graph of f.

(b) Find the x-coordinate of each critical point of f. For each such x, determine whether $f(x)$ is a relative maximum, a relative minimum, or neither.

(c) For what values of x is the graph of f concave down?

(d) Using the results found in parts (a), (b), and (c), sketch the graph of $y = f(x)$ on the axes provided below.