AB Calculus – summer work

NO calculators *are permitted in doing this worksheet. It is best if the work is done individually as its intent is to review concepts and skills that are necessary for calculus. This worksheet should be ready to be handed in the FIRST FULL CLASS MEETING. Clearly show all your work on a separate sheet of paper OR re-space the document.*

You are also expected to know the following better than you know your email password:
THE UNIT CIRCLE LOG RULES DOMAINS and RANGES OF COMMON PARENT FUNCTIONS

1. Solve the following for *x* in terms of *y*.

(a) $y = \ln(x+1)$

(b) $y = \sin(x-1)$

(c) $y = e^x - 2$

(d) $y = \dfrac{1}{x^2} + 2$

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2. Evaluate the following limits:

(a) $\lim\limits_{x \to 3} (2x+1)$

(b) $\lim\limits_{x \to 2} \dfrac{x-2}{x^2-4}$

(c) $\lim\limits_{x \to \infty} \dfrac{\sin x}{x}$

(d) $\lim\limits_{x \to -\infty} (e^x - 3)$

(e) $\lim\limits_{x \to \infty} \dfrac{x^4+3}{2x^2-5}$

(f) $\lim\limits_{x \to \infty} \cos\left(\dfrac{3\pi}{x}\right)$

#2 continued

(g) $\lim_{x \to 3^+} \frac{x-5}{x-3}$

(h) $\lim_{x \to 7} \frac{\sqrt{x+2}-3}{x-7}$

3. Consider the piecewise function defined below.

$$f(x) = \begin{cases} x^2+1 & x<2 \\ k & x=2 \\ 3x-1 & x>2 \end{cases}$$

(a) Evaluate : $f(0) =$ $f(1) =$ $f(3) =$

(b) Determine $\lim_{x \to 2} f(x)$. Show the analysis that leads to your answer.

(c) Determine the value of k so that $f(x)$ is defined and continuous at $x = 2$.

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4. Consider the piecewise function defined below

$$g(x) = \begin{cases} \dfrac{3x^2-5x-2}{x^2-4} & x \neq 2,\ -2 \\ \dfrac{1}{2} & x = 2 \end{cases}$$

(a) Determine if $g(x)$ is continuous at $x = 2$. Justify your answer using the definition of continuity.

(b) It is not possible for $g(x)$ to be continuous at $x = -2$. Why? Explain your answer using the definition of continuity.

5. Solve the following inequalities using an interval (number) line analysis. Show the work that leads to your answer.

(a) $(x-3)^2(x+2)<0$

(b) $\dfrac{(2x+1)(x-1)}{(x-2)(x+5)^2}>0$

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6. (a) Use the definition of the derivative to determine an expression for $f'(x)$, the derivative of $f(x)$, for $f(x)=3x^2-5x+4$.

(b) Determine the slope of the line tangent to $f(x)$ at $x=-2$.

(c) Determine the average rate of change of $f(x)$ over the interval $[1, 4]$.

(d) Determine the x –coordinate of the point on $f(x)$ where the instantaneous rate of change of $f(x)$ is equal to the average rate of change over $[1, 4]$.

(e) Write an equation of the line tangent to $f(x)$ at $x=1$.

7. Fully simplify the following complex fraction: $$\frac{\sqrt{x+3}-\frac{x}{\sqrt{x+3}}}{(x+3)}$$

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8. Consider the function $f(x)=\sqrt{4x+1}$.

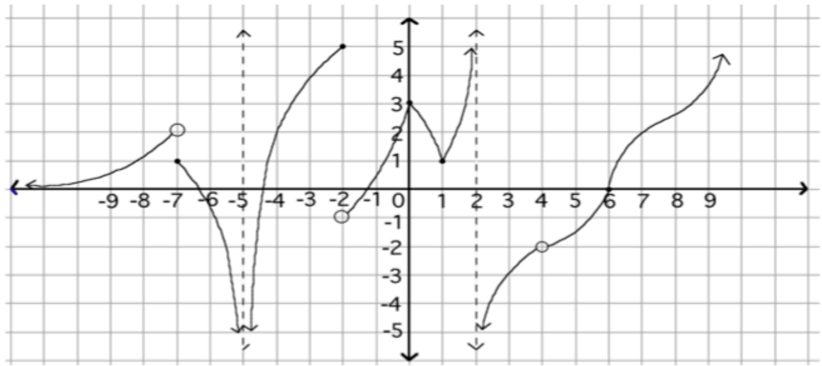
a. Determine an expression for $f'(x)$ using the definition of the derivative.

b. Find the slope of the tangent to the curve of $f(x)$ when $x=2$.

c. Write an equation of the line tangent to $f(x)$ at $x=2$.

d. Determine the point on the graph of *f* where the instantaneous rate of change is -5.

g. Determine the *x*-coordinate on the curve where the tangent line is perpendicular to $x+2y=7$.



9. The graph of a function $f(x)$ is shown above. Use the graph to determine the value of each of the following limits. If the limit does not exist, write *dne and justify* why the limit fails to exist.

(a) $\lim_{x\to\infty} f(x) =$

(b) $\lim_{x\to-\infty} f(x) =$

(c) $\lim_{x\to-7} f(x) =$

(d) $\lim_{x\to-5} f(x) =$

(e) $\lim_{x\to-3} f(x) =$

(f) $\lim_{x\to-2^-} f(x) =$

(g) $\lim_{x\to0} f(x) =$

(h) $\lim_{x\to2^+} f(x) =$

(j) $\lim_{x\to4} f(x) =$

(k) State the domain and range of $f(x)$.

(m) State the values of *x* for which $f(x)$ is not continuous and *justify* your answers using the definition of continuity.

10. Solve the following equations over the real number system – use $[0,\ 2\pi)$ for trig equations.

(a) $2\sin x + 1 = 0$

(b) $\sec 2x = 0$

(c) $\ln 3x - 1 = 0$

(d) $3e^{2x} - 4 = 0$

(e) $\arctan x + \sqrt{3} = 0$

(f) $x^4 - 3x^2 - 10 = 0$

11. Consider the functions $f(x) = x^2 - 3x + 2$ and $g(x) = 4 - x^2$.

(a) Find the points of intersection between the two functions and draw a sketch of their graphs, labeling the points of intersection and their zeros.

(b) Determine the interval over which $g(x) \geq f(x)$.

(c) Write a simplified algebraic expression for $h(x)$ that expresses the vertical distance between the functions over that interval.