

AP Calculus AB Summer Assignment

This assignment is designed to make the transition to AP Calculus AB a smooth one. You will be practicing skills you have acquired in earlier math classes. It is imperative that you are able to do most of these problems with ease, **without the use of a calculator.** **The entire assignment is due on the first day of class.** There will be a test on this material during the first week of school in the Fall.

Directions: In order to receive credit all work must be completed in pencil. Remember that we care about process, so show your work carefully on lined paper. **This should include: problem numbers, calculations done neatly, sketches drawn carefully, and labeled answers (circled, underlined, or boxed).** Graphs should be done on graph paper. Organize your work into columns and work down, not across the paper. **No Calculators! No Calculator, unless directed to do so! When appropriate, answers should be in simplified rational or fractional form. Last thing: ABSOLUTELY NO CALCULATORS, unless directed to do so!**

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Evaluate:

1. $f[g(x-1)]$

2. $g[h(x^3)]$

3. $g(f(a+h))$

Find $\frac{f(x+h) - f(x)}{h}$ for each given function $f(x)$. Let h represent some constant.

4. $f(x) = 9x + 3$

5. $f(x) = x^2 - 10$

6. $f(x) = x^2 - 3x + 2$

Find the x and y intercepts for each equation:

7. $y = x\sqrt{16 - x^2}$

8. $y = x^2(x-1)^2$

9. $y = \frac{x - \frac{1}{2}}{x + 2}$

The domain is the set of all the possible values for x (everything you can input), and the range is the set of all possible values for y (every possible output.)

Find the domain and range of each function. Write your answer in interval notation.

10. $f(x) = x^2 - 5$

11. $f(x) = -\sqrt{x+3}$

12. $f(x) = 3\sin x$

13. $f(x) = \frac{x}{x^2 - 9}$

14. $f(x) = \frac{x-1}{x^2 - x - 6}$

15. $f(x) = \frac{3x^2}{x^2 + 1}$

16. $f(x) = \frac{2}{x-1}$

17. Write the equation of a line with a slope of 3 and a y -intercept of 5 in slope-intercept form.

18. Write the equation of a line passing through the point $(0, 5)$ with a slope of $\frac{2}{3}$.

19. Write the equation of a line passing through the points $(-3, 6)$ and $(1, 2)$.

20. Write the equation of a line passing through the points $(-4, 2)$ with a slope of zero.

21. Write the equation of a line passing through the points $(-4, 2)$ with an undefined slope.

22. Write the equation of a line passing through the points $(2, 8)$ and parallel to the line $y = \frac{5}{6}x - 1$.

23. Write the equation of a line passing through the point $(2, -3)$ normal to $y = \frac{5}{6}x - 1$.

Find the value without a calculator.

24. $\tan\left(\arccos\frac{2}{3}\right)$ 25. $\sec\left(\sin^{-1}\frac{12}{13}\right)$ 26. $\sin\left(\arctan\frac{12}{5}\right)$ 27. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

The vertical asymptote occurs where the function is undefined (where the denominator is equal to zero).

Determine the vertical asymptote(s) for the function.

28. $f(x) = \frac{1}{x^2}$ 29. $f(x) = \frac{x^2}{x^2 - 4}$ 30. $f(x) = \frac{2+x}{x^2(1-x)}$

Determining the horizontal asymptote.

Case 1: The degree of the numerator is less than the degree of the denominator. The function has a horizontal asymptote at $y = 0$.

Case 2: The degree of the numerator is equal to the degree of the denominator. The function has a horizontal asymptote equal to the ration of the leading coefficients.

Case 3: The degree of the numerator is greater than the degree of the denominator. The function has no horizontal asymptote.

Determine the horizontal asymptote.

31. $f(x) = \frac{x^2 - 4x + 10}{x^3 + x - 5}$ 32. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$ 33. $f(x) = \frac{4x^5}{x^2 - 7}$

Solve:

34. $x^2 - 6x + 5 = 0$ 35. $3x^3 - 4x = 1$ 36. $2e^{-t} - 2te^{-t} = 0$

37. $\sin^2 x - \sin x = 0$, for $0 \leq x \leq \frac{3\pi}{2}$

Simplify:

38. $\frac{x^2 - a^2}{x^4 - a^4}$ 39. $\sqrt{x}(x-1)^2$ 40. $\frac{x+h}{x+h+2} - \frac{x}{x^2}$ 41. $\frac{1}{(x+h)^2} - \frac{1}{x^2}$

42. $\left(\frac{1}{x} + \frac{1}{y}\right)(x+y)^{-1}$ 43. $(x^{-1} + y^{-1})^{-1}$ 44. $\frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$

Simplify completely. Leave no variables in the denominator.

45. $\frac{(2a^2)^3}{b}$ 46. $\frac{a\left(\frac{2}{b}\right)}{\frac{3}{a}}$ 47. $\frac{ab-a}{b^2-b}$ 48. $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$ 49. $\left(\frac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^2 \left(\frac{b^{\frac{3}{2}}}{a^2}\right)$

Solve for the indicated variable:

50. Solve for z in terms of x and y : $3xy^2z + y^3 + 2x^2yz + 2xy^2 = 0$

51. Solve for z in terms of y : $z - z \sin y = 1$

52. Solve for y in terms of x : $\frac{1}{2}e^{2y} = x^3 + 2$

53. If $f(x) = \frac{1}{3}x^3$, then find k so that $f(k) - f(-3) = 0$.

54. If $f(x) = 3x^2 - 6x + 12$, then find the minimum value of f .

55. If $f(x) = \frac{2}{9}(x^3 + 1)^{\frac{3}{2}}$, then find the value of $\frac{1}{2-0}[f(2) - f(0)]$.

56. If $f(x) = 3 \ln x + C$, and $f(e) = 2$, then find the value of C .

57. If $f(x) = \frac{x^2}{2} \ln x - \frac{3}{4}x^2 + C$, and $f(1) = 6$, then find the value of C .

58. If $f(x) = \sec^2(2x)$, then find $f\left(\frac{\pi}{6}\right)$.

59. If $f(x) = e^x - \frac{x^2}{2}$, then find the value of $f(4) - f(0)$.

60. If $f(x) = \frac{3}{2}\sqrt{1 + \tan x} \cdot \sec^2 x$, then find $f(0)$.

61. If $f(x) = \frac{3}{5}(2x)^{\frac{5}{3}}$, then find $f(4)$.

62. If $f(x) = -x - \frac{x^2}{2} + \frac{x^3}{3}$, then find $f(3)$.

63. If $f(x) = -\frac{x}{2} - x^2 + \frac{x^3}{3}$, then find $f(4) - f(2)$.

64. If $f(x) = \begin{cases} 2x^2, & \text{when } x < 0 \\ 3x, & \text{when } 0 \leq x \leq 1 \\ |6-x|, & \text{when } x > 1 \end{cases}$, then find $f(-3) + f(1)$.

Multiply:

65. $2^3 \cdot 2^m$

66. $(a^k + 3)(a^k - 2)$

67. $x^{\frac{5}{8}} \left(x^{\frac{3}{8}} - 10x^{\frac{11}{8}} \right)$

Without using a calculator, evaluate the following:

68. $\cos 210^\circ$

69. $\sin \frac{5\pi}{4}$

70. $\tan^{-1}(-1)$

71. $\sin^{-1}(-1)$

72. $\cos \frac{9\pi}{4}$

73. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

74. $\tan \frac{7\pi}{6}$

75. $\cos^{-1}(-1)$

76. $\sec\left(\frac{\pi}{6}\right)$

77. Divide $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$.

78. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.

79. Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$.

Write as a single equation in x and y :

80. $\begin{cases} x = t + 1 \\ y = t^2 - t \end{cases}$

81. $\begin{cases} x = \sqrt[3]{t} - 1 \\ y = t^2 - t \end{cases}$

82. $\begin{cases} x = \sin t \\ y = \cos t \end{cases}$

Graph the following. You should recall how they are graphed from Pre-Cal. Use a calculator only if you absolutely need to:

83. $y = \ln x$

84. $y = \frac{|x|}{x}$

85. $y = e^x$

86. $y = e^{-x}$

87. $y = 2^x$

88. $y = \sqrt{x}$

89. $y = \begin{cases} \sqrt{x+1}, & 0 \leq x \leq 3 \\ 5-x, & 3 < x \leq 5 \end{cases}$

Using the Graphing Calculator: These are the only problems that you should use a calculator for!!!!!!!!!!!!

90. Solve $6e^{2x} = 18x^2$.

91. How many solutions exist for the equation $\frac{\cos^2 x}{x} - \frac{1}{5} = 0$ on the interval $0 < x < 10$?

92. If $f(x) = 2xe^{2x}$, then find the absolute minimum of f and the range of f .

93. If $f(x) = 80 - 10\cos\left(\frac{\pi x}{12}\right)$, then (a) graph f over the interval $0 \leq x \leq 24$, and (b) find x -values such that $f(x) \geq 78$.

94. For $0 \leq t \leq 10$, how many values of t satisfy the equation $\cos t = -2e^{-2t}$? Do not find specific values.

95. Solve: $1 - 3e^{-0.2\sqrt{x}} = 0$.

96. Find the range of $f(x) = xe^{1-x}$.

97. Find the domain of $y = \ln\left(\frac{x}{x-1}\right)$.