Cincinnati Country Day School

AP Calculus (BC) Summer Packet

Directions: The AP Calculus BC curriculum consists of 10 total units, where several of these units were covered in Pre-AP Calculus BC. This packet, which is due on the first day of class of the 2022-2023 school year, is a review of the first 3 units. Copy each problem onto a separate piece of paper and solve. Show your procedure, not just your answer. If you use a graph, you should show a properly labeled sketch of that graph. These Skills are expected to be MASTERED to achieve maximum success in AP Calculus.

Skill 1: Calculate limits.

1. Find the limit:
1 pts.

$$\lim_{x \to -3} \frac{x^2 - 9}{x^2 + x - 6}$$

- A. 1
- B. 3/2
- C. 6/5
- D. -9
- E. 3

2. Find the limit:
1 pts.

$$\lim_{x \to -\infty} \frac{8x^4 + 2x - 1}{5 + 7x - 2x^4}$$

- A. 8
- B. 8/5
- C. 4
- D. -4
- E. 1/2

3. Find the limit:
1 pts.

$$\lim_{x \to \infty} \frac{(2x + 7)^2}{(3x - 1)(4 - x)}$$

- A. 4/3
- B. -4/3
- C. 2/3
- D. -2/3
- E. 1/3

4. Find the limit:
1 pts.

$$\lim_{x \to 4^+} \frac{3}{4 - x}$$

- A. Positive Infinity
- B. Negative Infinity
- C. 0
- D. 3/4
- E. -3

5. Find the limit:
1 pts.

$$\lim_{x \to 7^-} \frac{2}{(x - 7)^2}$$

- A. Positive Infinity
- B. Negative Infinity
- C. 2/49
- D. 0

11. Find the limit:
1 pts.

$$\lim_{x \to 2} \frac{x - 2}{x^2 - 4}$$

- A. 2
- B. 4
- C. 1/2
- D. 1/4
- E. 1

12. Find the limit:
1 pts.

$$\lim_{x \to \infty} \frac{5 - 3x^5}{2x^5 + 7x - 1}$$

- A. 5/2
- B. -3/2
- C. -5
- D. 3
- E. Infinity

13. Find the limit:
1 pts.

$$\lim_{x \to \infty} \frac{(5x - 1)(x + 2)(3x + 2)}{(2x - 3)^3}$$

- A. 5/2
- B. 15/2
- C. 15/8

14. Find:
1 pts.

$$\lim_{x \to 5} \frac{x - 5}{x^2 - 3x - 10}$$

- A. 1
- B. 1/2
- C. 7
- D. 1/7

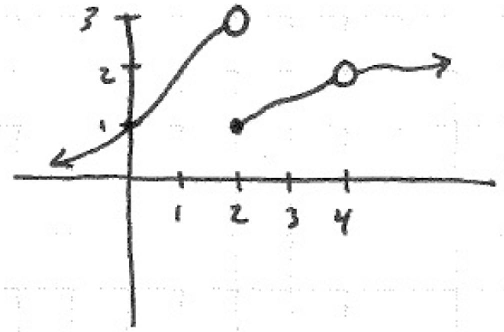
Skill 2: Find limits using a graph.

1. Consider the graph below. Which of the following statements are true?
1 pts.

I. $\lim_{x\to 4} f(x) = \lim_{x\to 2^+} f(x)$

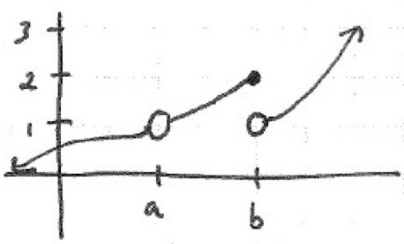
II. $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^-} f(x)$

III. $\lim_{x\to 2^-} f(x) = f(2)$



- A. II only
- B. III only
- C. I and II
- D. I and III
- E. II and III

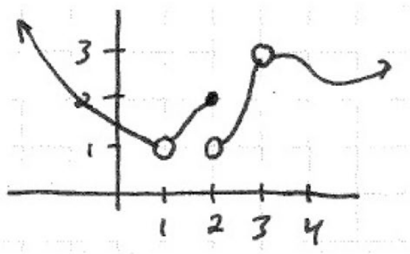
2. Consider the graph below. Which statement is false?
1 pts.



- A. $f(a)$ does not exist
- B. $\lim_{x\to a^-} f(x) = 1$
- C. $\lim_{x\to a} f(x)$ does not exist
- D. $\lim_{x\to b} f(x)$ does not exist
- E. $f(b) = 2$

3. .
1 pts.

Referring to the graph below, if $\lim_{x \to c} f(x) = 1$, then what must c equal?



- A. 1
- B. 2
- C. 3
- D. 4
- E. none of the above

<u>Skill 3</u>: Identify asymptotes of functions.

4. .
1 pts.

What kind of asymptote does $\lim_{x \to 5} f(x) = -\infty$ describe?

- A. Horizontal
- B. Vertical

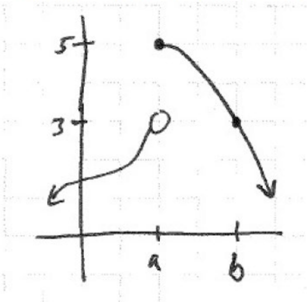
6. Which function has a horizontal asymptote of y = 3? (THERE MAY BE MORE THAN ONE CORRECT ANSWER - SELECT ALL THAT APPLY) (Choose all that Apply)
1 pts.

- A. $y = 3x$
- B. $y = e^x + 3$
- C. $y = \dfrac{-3x^2 + 5x - 1}{6 - x^2}$
- D. $y = \dfrac{x+3}{x+1}$
- E. $y = \dfrac{1}{x-3}$

Skill 4: Determine the continuity of a function.

1. Consider the graph below. Which statement is false?

1 pts.



- A. $\lim_{x \to a} f(x)$ does not exist
- B. $\lim_{x \to b} f(x) = 3$
- C. f is defined at $x = a$ (This means that $f(a)$ exists)
- D. f is continuious at $x = a$
- E. f is continuious at $x = b$

2. At what value(s) of x is the function below discontinuous?

1 pts.

$$f(x) = \frac{(x+1)^2(x-2)}{(x+1)(x-3)}$$

- A. -1 only
- B. 3 only
- C. -1 and 3 only
- D. -1, 2, and 3
- E. f is continuous for all values of x

3. .

1 pts.

For what value of c is $f(x) = \begin{cases} 3x - 7 & \text{if } x \leq 1 \\ 2x + c & \text{if } x > 1 \end{cases}$ continuous?

- A. -7
- B. -6
- C. 1
- D. 3
- E. 8

Skill 5: Evaluate derivatives using basic rules.

1. Given the function $f(x) = 2\sqrt[5]{x^6}$, find $f'(x)$. Express your answer in radical form without using negative exponents, simplifying all fractions.

2. For the following equation, find $f'(x)$.

$$f(x) = 9x^4 - x^3 - 2$$

3. For the following equation, evaluate $f'(-1)$.

$$f(x) = -4x^5 + x^3 + x^2$$

4. Given $f(x) = 2x^2 - x$, find the equation of the tangent line of f at the point where $x = -3$.

Skill 6: Evaluate derivatives using the product and quotient rules.

1. Given the function $f(x) = 3x^2 - x^2 \cos x$, find $f'(x)$ in any form.

2. Given the function $f(x) = \sqrt{25x} \sin x$, find $f'(x)$ in any form.

3.

Given the function $f(x) = \frac{3x^2}{4x^2+3}$, find $f'(x)$ in *simplified* form.

4.

Given the function $y = \frac{5-x^3}{1-x}$, find $\frac{dy}{dx}$ in *simplified* form.

Skill 7: Evaluate derivatives using the chain rule.

1.

Given the function $y = 4(x^2 + 9)^{\frac{3}{2}}$, find $\frac{dy}{dx}$

2.

Given the function $f(x) = -\sqrt{\cos x}$, find $f'(x)$.

3.

Given the function $f(x) = 3\cos\left[(2x^2 + 6)^5\right]$, find $f'(x)$.

4.

Given the function $y = \cos(2x^3)\sin^4(x)$, find $\frac{dy}{dx}$

5.

Given the function $y = \sqrt[3]{\frac{4x^3}{5+5x}}$, find $\frac{dy}{dx}$

<u>Skill 8</u>: Use implicit differentiation to find derivatives.

1.
If $-y^3 - 4y^2 + x^3 = -4y$ then find $\frac{dy}{dx}$ in terms of x and y.

2.
If $2x^3 + 4xy = -y^3 + 5$ then find $\frac{dy}{dx}$ in terms of x and y.

3.
Given $\sin\left(x + y\right) = x^3$, find $\frac{dy}{dx}$ in terms of x and y.

4.
If $-4x + y^2 - xy = 0$ then find the equations of all tangent lines to the curve when $x = 2$.

<u>Skill 9</u>: Find basic antiderivatives.

1. $\int (3x^5 - 4x^2 + 11)dx$

2. $\int (cosx - sinx)dx$

3. $\int \frac{3x^2+6x}{x} dx$

4. $\int (2x - 1)(3x + 8)dx$

5. $\int (4cosx + x^5 - 10sinx)dx$

Skill 10: Find antiderivatives using u-substitution.

1. $\int 3cos(2x-5)dx$

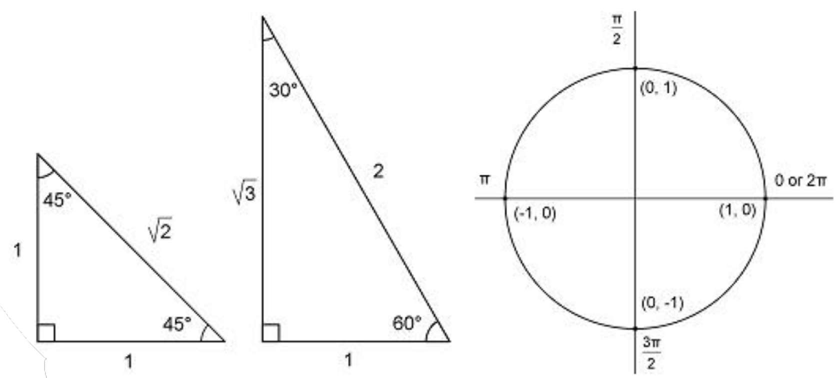
2. $\int 2x(x^2+4)^7 dx$

3. $\int \frac{3x^2+3}{(x^3+3x-5)^4} dx$

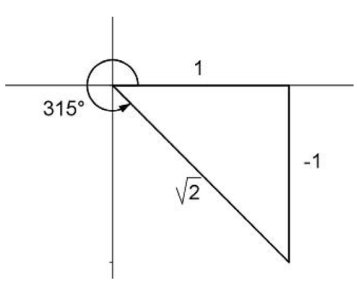
Skill 11: Trigonometry Review

Find basic trig values using the reference triangles and the unit circle.

You should know the 2 reference triangles and the unit circle like the back of your hand. Remember on the unit circle, the cosine is the x-coordinate and the sine is the y-coordinate (it's alphabetical).



Example: To find $\sin \frac{7\pi}{4}$, we first realize that the angle is in the 4th quadrant since $\frac{7\pi}{4}$ is just short of $\frac{8\pi}{4} = 2\pi$. We draw the reference triangle with a positive adjacent side since it is to the right of the origin and a negative opposite side since it is below the origin. Of course, the hypotenuse is always positive. From the triangle, we see that $\sin \frac{7\pi}{4} = -\frac{1}{\sqrt{2}}$



1. Without a calculator, evaluate the following.

(a) cos 210°   (b) $\sin\frac{5\pi}{4}$ (c) $\tan^{-1}(-1)$ (d) $\sin^{-1}(-1)$

(e) $\cos\frac{9\pi}{4}$ (f) $\sin^{-1}\frac{\sqrt{3}}{2}$ (g) $\tan\frac{7\pi}{6}$ (h) $\cos^{-1}(-1)$

(i) $\sin\frac{\pi}{6}$ (j) $\tan\frac{7\pi}{6}$ (k) $\cos 0$ (l) $\cos\frac{\pi}{4}$

(m) $\csc\left(\frac{-5\pi}{6}\right)$ (n) $\sec\pi$ (o) $\cot\left(\frac{-\pi}{2}\right)$ (p) $\tan\frac{\pi}{2}$

(q) $\sin\frac{5\pi}{6}$ (r) $\cot\frac{2\pi}{3}$ (s) $\sin\frac{\pi}{2}$ (t) $\sec\frac{3\pi}{4}$

(u) $\csc\pi$ (v) $\sec\frac{11\pi}{6}$ (w) $\cot\frac{4\pi}{3}$ (x) $\cos^{-1}\frac{\sqrt{3}}{2}$

(y) $\cot^{-1}(-1)$ (z) $\tan^{-1}(-1)$ (aa) $\sin^{-1}\left(-\frac{1}{2}\right)$ (bb) $\sin\left(\csc^{-1}(-2)\right)$

(cc) $\cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right)$

<u>**Skill 12**</u>: Solve Trigonometric Equations

1. Solve the following trigonometric equations on the interval $0 \le x \le 2\pi$

   a. $2\sin x = 1$

   b. $2\cos^2 x - 3\cos x + 1 = 0$

   c. $3\sin^2 x = \cos^2 x$