

12th Summer Review Math Applications and Interpretations

Please work all of the following problems completely. Show all work and put answers in blanks provided.

A formula book and calculator are permitted on this assignment.

Problems are due on the first day of class. Problems will be reviewed, and solutions provided.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. A group of 20 students travelled to a gymnastics tournament together. Their ages, in years, are given in the following table.

<b>Age (years)</b>	14	15	16	17	18	19	20	22
<b>Frequency</b>	1	2	7	1	4	1	1	3

- (a) For the students in this group

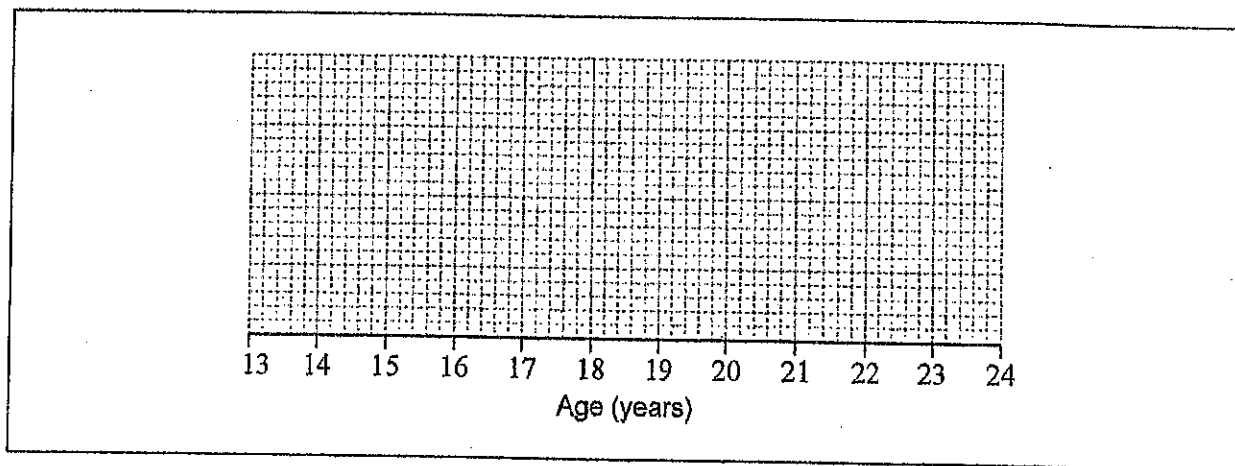
- (i) find the mean age;
- (ii) write down the median age.

[3]

The lower quartile of the ages is 16 and the upper quartile is 18.5.

- (b) Draw a box-and-whisker diagram, for these students' ages, on the following grid.

[3]



(This question continues on the following page)

**Working:**

**Answers:**

(a) (i) .....

(ii) .....

2. A class of 13 Mathematics students received the following grades in their final IB examination.

3 5 3 4 7 3 2 7 5 6 5 3 4

For these grades, find

- (a) the mode; [1]
- (b) the median; [2]
- (c) the upper quartile; [1]
- (d) the interquartile range. [2]

*Working:*

*Answers:*

- (a) .....
- (b) .....
- (c) .....
- (d) .....

3. In a particular week, the number of eggs laid by each hen on a farm was counted. The results are summarized in the following table.

Number of eggs	1	2	3	4	5	6
Frequency	4	7	12	10	14	13

- (a) State whether these data are discrete or continuous. [1]
- (b) Write down
- (i) the number of hens on the farm;
- (ii) the modal number of eggs laid. [2]
- (c) Calculate
- (i) the mean number of eggs laid;
- (ii) the standard deviation. [3]

*Working:*

*Answers:*

- (a) .....
- (b) (i) .....
- (ii) .....
- (c) (i) .....
- (ii) .....

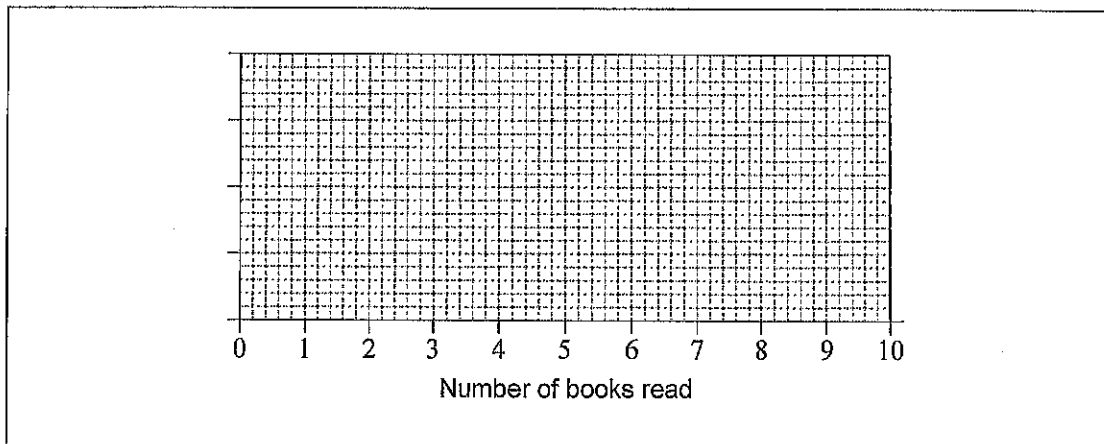
4. Two groups of 40 students were asked how many books they have read in the last two months. The results for the first group are shown in the following table.

Number of books read	Frequency
2	5
3	8
4	13
5	7
6	4
7	2
8	1

The quartiles for these results are 3 and 5.

(a) Write down the value of the median for these results. [1]

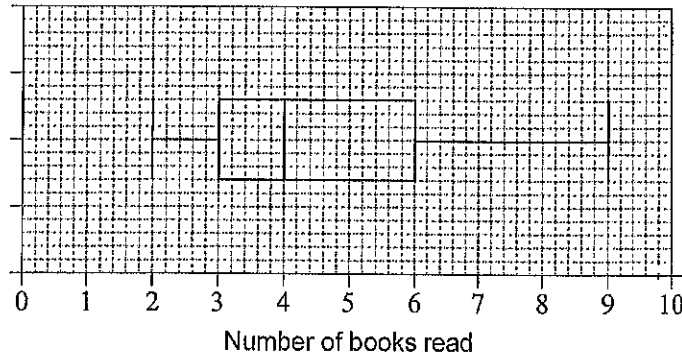
(b) Draw a box-and-whisker diagram for these results on the following grid. [3]



(This question continues on the following page)

(Question 4 continued)

The results for the **second group** of 40 students are shown in the following box-and-whisker diagram.



(c) Estimate the number of students in the **second group** who have read at least 6 books. [2]

Working:

Answers:

(a) .....

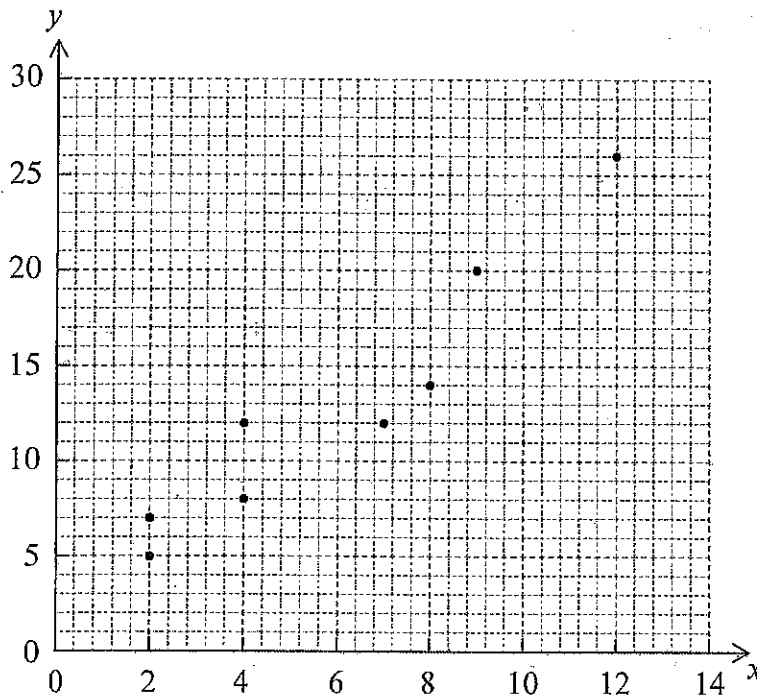
(c) .....





Consider the following set of data which is plotted on the scatter diagram below.

$x$	2	4	7	12	4	8	9	2
$y$	5	8	12	26	12	14	20	7



- (a) Write down the coordinates of the mean point  $(\bar{x}, \bar{y})$ . [2 marks]
- (b) Write down the value of  $r$ , the Pearson's product-moment correlation coefficient for this set of data. [2 marks]
- (c) Draw the regression line for  $y$  on  $x$  on the set of axes above. [2 marks]

Working:

Answers:

- (a) .....
- (b) .....

7. [Maximum mark: 7]

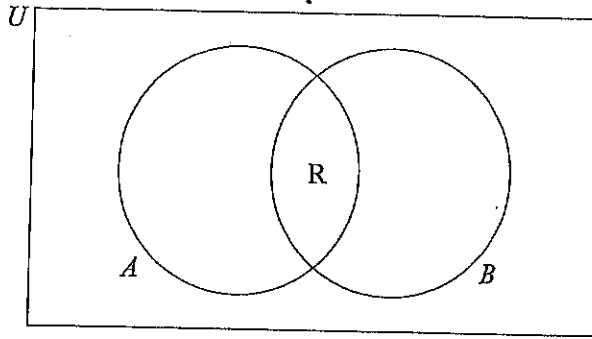
In an arithmetic sequence, the third term is 10 and the fifth term is 16.

- (a) Find the common difference. [2]
- (b) Find the first term. [2]
- (c) Find the sum of the first 20 terms of the sequence. [3]

A large rectangular box containing horizontal dotted lines for writing the answer.

8. Tuti has the following polygons to classify: rectangle (R), rhombus (H), isosceles triangle (I), regular pentagon (P), and scalene triangle (T).

In the Venn diagram below, set  $A$  consists of the polygons that have at least one pair of parallel sides, and set  $B$  consists of the polygons that have at least one pair of equal sides.



- (a) Complete the Venn diagram by placing the letter corresponding to each polygon in the appropriate region. For example, R has already been placed, and represents the rectangle. [3]

- (b) State which polygons from Tuti's list are elements of

(i)  $A \cap B$ ;

(ii)  $(A \cup B)'$ .

[3]

*Working:*

*Answers:*

- (b) (i) .....
- .....
- .....
- (ii) .....
- .....
- .....

9.

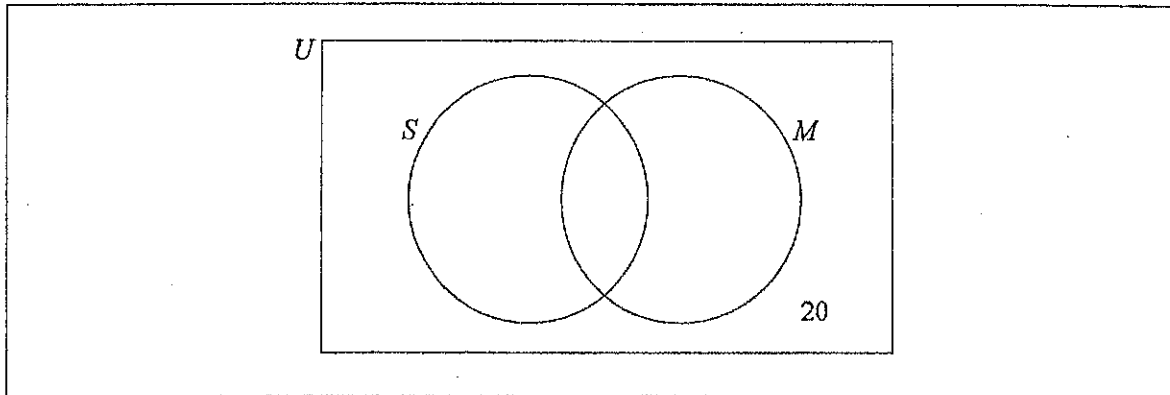
Rosewood College has 120 students. The students can join the sports club ( $S$ ) and the music club ( $M$ ).

For a student chosen at random from these 120, the probability that they joined both clubs is  $\frac{1}{4}$  and the probability that they joined the music club is  $\frac{1}{3}$ .

There are 20 students that did not join either club.

(a) Complete the Venn diagram for these students.

[2]



(b) One of the students who joined the sports club is chosen at random. Find the probability that this student joined both clubs.

[2]

(c) Determine whether the events  $S$  and  $M$  are independent.

[2]

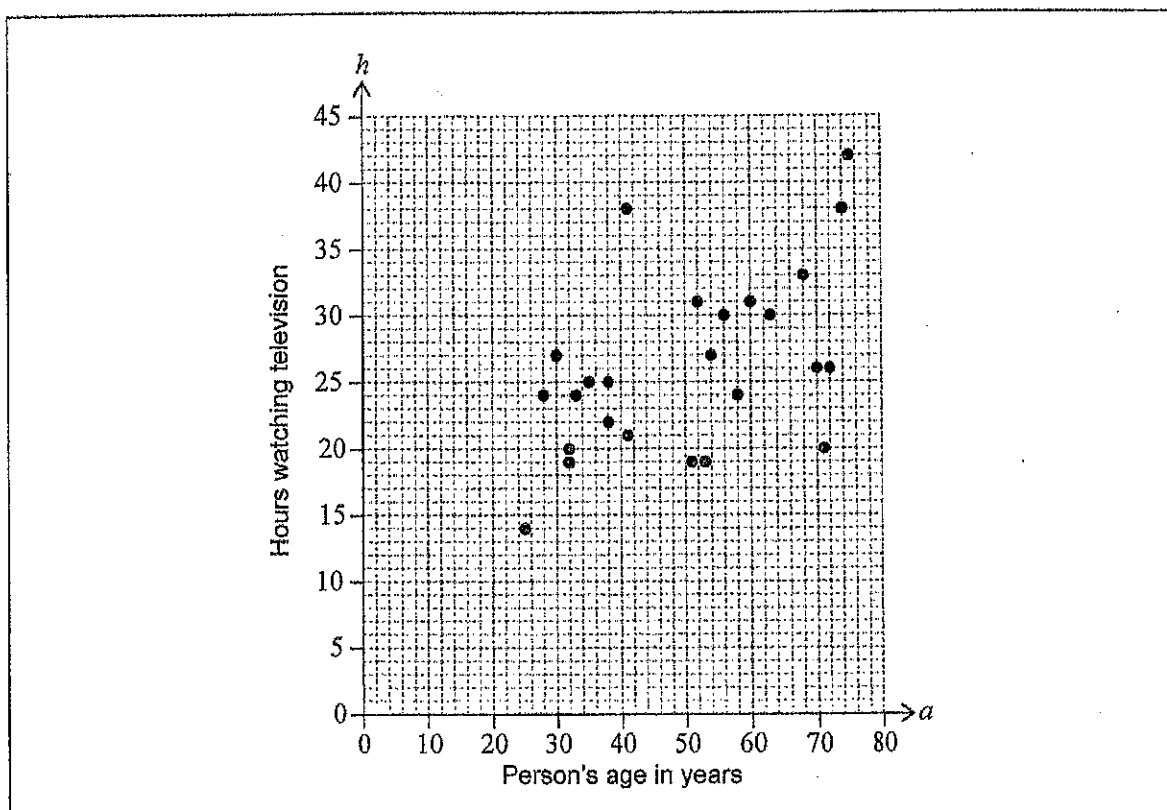
**Working:**

**Answers:**

(b) .....

(c) .....

10. A survey was carried out to investigate the relationship between a person's age in years ( $a$ ) and the number of hours they watch television per week ( $h$ ). The scatter diagram represents the results of the survey.



The mean age of the people surveyed was 50.

For these results, the equation of the regression line  $h$  on  $a$  is  $h = 0.22a + 15$ .

- (a) Find the mean number of hours that the people surveyed watch television per week. [2]
- (b) Draw the regression line on the scatter diagram. [2]
- (c) By placing a tick ( $\checkmark$ ) in the correct box, determine which of the following statements is true:

The correlation between $h$ and $a$ is positive.	<input type="checkbox"/>
The correlation between $h$ and $a$ is negative.	<input type="checkbox"/>
There is no correlation between $h$ and $a$ .	<input type="checkbox"/>

[1]

- (d) Diogo is 18 years old. Give a reason why the regression line should not be used to estimate the number of hours Diogo watches television per week. [1]

(This question continues on the following page)

(Question continued)

**Working:**

**Answers:**

(a) .....

(d) .....

.....

11. Yun Bin invests 5000 euros in an account which pays a nominal annual interest rate of 6.25 % , **compounded monthly**.  
Give all answers correct to two decimal places.

Find

- (a) the value of the investment after 3 years; [3 marks]
- (b) the difference in the final value of the investment if the interest was compounded quarterly at the same nominal rate. [3 marks]

*Working:*

*Answers:*

(a) .....

(b) .....

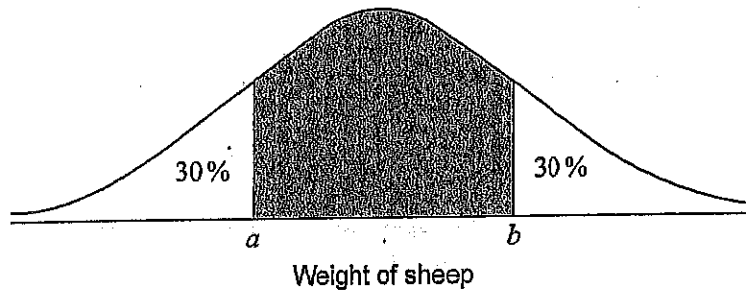
12.

[Maximum mark: 13]

The weights of sheep on a farm are normally distributed with a mean of 110 kg and a standard deviation of 8 kg.

- (a) Sketch a diagram of the distribution of the weights of these sheep. On your diagram, label the mean and label one standard deviation above and below the mean. [2]
- (b) (i) A sheep has a weight of 94 kg. Write down the number of standard deviations that this weight is below the mean. [3]
- (ii) Find the probability that a sheep, chosen at random, weighs more than 94 kg. [3]
- (c) (i) Find the probability that a sheep, chosen at random, weighs between 88 kg and 116 kg. [4]
- (ii) The farmer weighs 160 sheep. Find the number of sheep that he would expect to weigh between 88 kg and 116 kg. [4]
- (d) Given that 75% of the sheep weigh less than  $w$  kg, find the value of  $w$ . [2]

A sheep is chosen at random. Its weight is within the central shaded region of the following diagram.



- (e) Find the value of  $a$  and of  $b$ . [2]

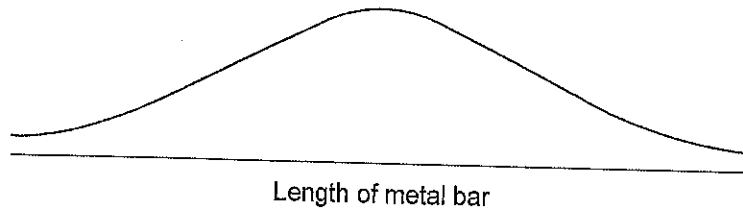




13. A factory makes metal bars. Their lengths are assumed to be normally distributed with a mean of 180 cm and a standard deviation of 5 cm.

- (a) On the following diagram, shade the region representing the probability that a metal bar, chosen at random, will have a length less than 175 cm.

[2]



A metal bar is chosen at random.

- (b) (i) The probability that the length of the metal bar is less than 175 cm is equal to the probability that the length is greater than  $h$  cm. Write down the value of  $h$ .
- (ii) Find the probability that the length of the metal bar is greater than one standard deviation above the mean.

[4]

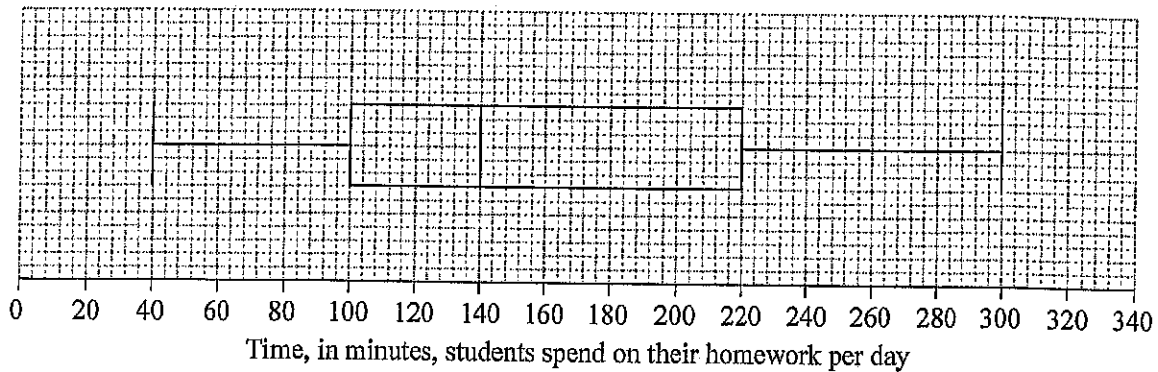
Working:

Answers:

- (b) (i) .....
- (ii) .....

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

- 14 The time, in minutes, that students in a school spend on their homework per day is presented in the following box-and-whisker diagram.



- (a) Find
- (i) the longest amount of time spent on homework per day;
  - (ii) the interquartile range. [3]
- (b) State the statistical term corresponding to the value of 140 minutes. [1]
- (c) Find the percentage of students who spend
- (i) between 100 and 140 minutes per day on their homework;
  - (ii) more than 100 minutes per day on their homework. [2]

*(This question continues on the following page)*

(Question : continued)

*Working:*

*Answers:*

- (a) (i) .....
- (ii) .....
- (b) .....
- (c) (i) .....
- (ii) .....



16. [Maximum mark: 18]

A group of students at Dune Canyon High School were surveyed. They were asked which of the following products: books (B), music (M) or films (F), they downloaded from the internet.

The following results were obtained:

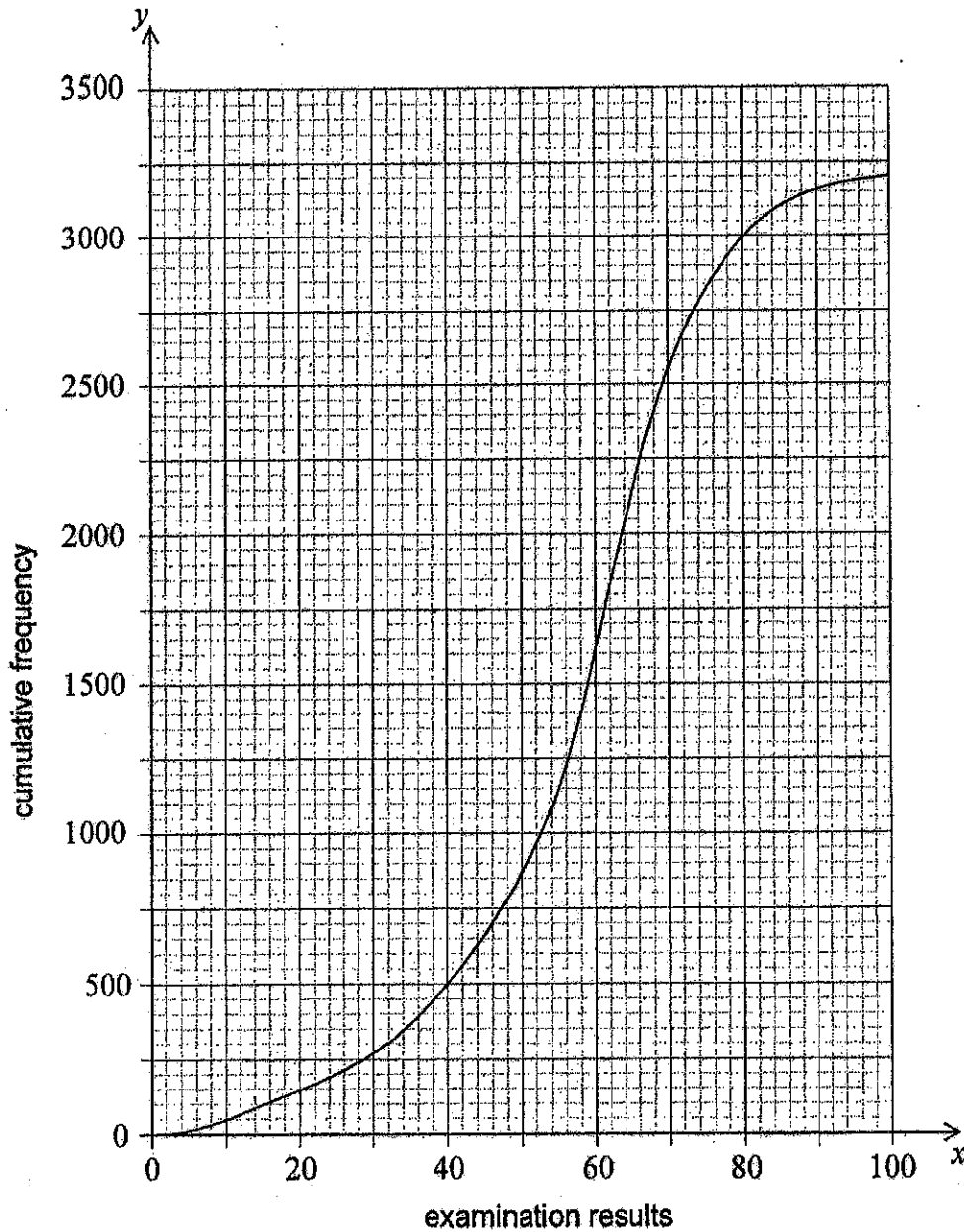
100 students downloaded music;  
95 students downloaded films;  
68 students downloaded films and music;  
52 students downloaded books and music;  
50 students downloaded films and books;  
40 students downloaded all three products;  
8 students downloaded books **only**;  
25 students downloaded none of the three products.

- (a) Use the above information to complete a Venn diagram. [5]
- (b) Calculate the number of students who were surveyed. [2]
- (c) (i) On your Venn diagram, shade the set  $(F \cup M) \cap B'$ . Do not shade any labels or values on the diagram.
- (ii) Find  $n((F \cup M) \cap B')$ . [3]
- (d) A student who was surveyed is chosen at random.  
Find the probability that
- (i) the student downloaded music;
- (ii) the student downloaded books, given that they had not downloaded films;
- (iii) the student downloaded at least two of the products. [6]
- Dune Canyon High School has 850 students.
- (e) Find the expected number of students at Dune Canyon High School that downloaded music. [2]



17. [Maximum mark: 16]

The final examination results obtained by a group of 3200 Biology students are summarized on the cumulative frequency graph.



(a) Find

- (i) the median of the examination results;
- (ii) the interquartile range.

[5]

350 of the group obtained the highest possible grade in the examination.

(b) Find the final examination result required to obtain the highest possible grade.

[2]



Question continued)

The grouped frequency table summarizes the examination results of this group of students.

Examination result ( $x$ )	$0 < x \leq 20$	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$
Frequency	150	350	1100	1400	200

(c) Write down

(i) the modal class;

(ii) the mid-interval value of the modal class. [3]

(d) Calculate an estimate of

(i) the mean examination result;

(ii) the standard deviation, giving your answer correct to **three decimal places**. [3]

The teacher sets a grade boundary that is one standard deviation below the mean.

(e) Use the cumulative frequency graph to estimate the number of students whose final examination result was below this grade boundary. [3]



18. [Maximum mark: 14]

The Brahma chicken produces eggs with weights in grams that are normally distributed about a mean of 55 g with a standard deviation of 7 g. The eggs are classified as small, medium, large or extra large according to their weight, as shown in the table below.

Size	Weight (g)
Small	Weight < 53
Medium	$53 \leq \text{Weight} < 63$
Large	$63 \leq \text{Weight} < 73$
Extra Large	Weight $\geq 73$

- (a) Sketch a diagram of the distribution of the weight of Brahma chicken eggs. On your diagram, show clearly the boundaries for the classification of the eggs.

[3 marks]

An egg is chosen at random.

- (b) Find the probability that the egg is

(i) medium;

(ii) extra large.

[4 marks]

There is a probability of 0.3 that a randomly chosen egg weighs more than  $w$  grams.

- (c) Find  $w$ .

[2 marks]

The probability that a Brahma chicken produces a large size egg is 0.121. Frank's Brahma chickens produce 2000 eggs each month.

- (d) Calculate an estimate of the number of large size eggs produced by Frank's chickens each month.

[2 marks]

*(Question continued)*

The selling price, in US dollars (USD), of each size is shown in the table below.

Size	Selling price (USD)
Small	0.30
Medium	0.50
Large	0.65
Extra Large	0.80

The probability that a Brahma chicken produces a small size egg is 0.388.

- (e) Estimate the monthly income, in USD, earned by selling the 2000 eggs.  
Give your answer correct to two decimal places.

*[3 marks]*

**Turn over**



19. [Maximum mark: 14]

John purchases a new bicycle for 880 US dollars (USD) and pays for it with a Canadian credit card. There is a transaction fee of 4.2% charged to John by the credit card company to convert this purchase into Canadian dollars (CAD).

The exchange rate is  $1 \text{ USD} = 1.25 \text{ CAD}$ .

(a) Calculate, in CAD, the total amount John pays for the bicycle.

[3]

John insures his bicycle with a US company. The insurance company produces the following table for the bicycle's value during each year.

Year	Value of the bicycle (USD)
1st	880
2nd	704
3rd	563.20
...	...

The values of the bicycle form a geometric sequence.

(b) Find the value of the bicycle during the 5th year. **Give your answer to two decimal places.**

[3]

(c) Calculate, in years, when the bicycle value will be less than 50 USD.

[2]

During the 1st year John pays 120 USD to insure his bicycle. Each year the amount he pays to insure his bicycle is reduced by 3.50 USD.

(d) Find the total amount John has paid to insure his bicycle for the first 5 years.

[3]

John purchased the bicycle in 2008.

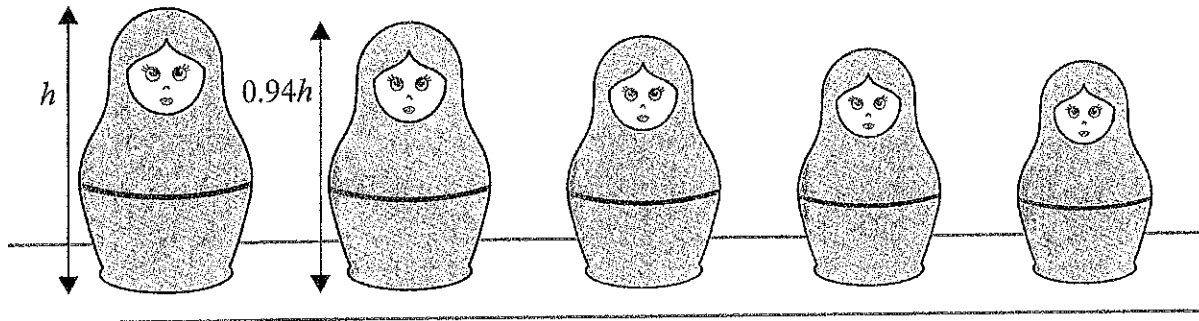
(e) Justify why John should not insure his bicycle in 2019.

[3]



20. Matryoshka dolls, or Russian dolls, are similarly designed dolls which open up and fit inside each other.

The largest set of these type of dolls is a 51 piece set which was completed in 2003. The height of the largest piece in this set is 54 cm. The heights of successive smaller dolls are 94% of the preceding larger doll, as shown.



(a) Find the height of the smallest doll in this set. [3]

(b) Find the **total** height if all 51 dolls were placed one on top of another. [3]

Working:

Answers:

- (a) .....  
(b) .....