Calculus - SUMMER PACKET

NAME: Solutions

Summer + Math = (Best Summer Ever)^2

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

$f(-2) = (-2)^2 - 2(-2) + 5$

$f(-2) = 4 + 4 + 5$

$\boxed{f(-2) = 13}$

2. $f(x+2) =$

$\boxed{x^2 + 2x + 5}$

3. $f(x+h) =$

$(x+h)^2 - 2(x+h) + 5$

$(x+h)(x+h) - 2x - 2h + 5$

$x^2 + xh + xh + h^2 - 2x - 2h + 5$

$\boxed{x^2 + 2xh + h^2 - 2x - 2h + 5}$

Use the graph $f(x)$ to answer the following.

4. $f(0) = -4$

$f(-1) = -3.5$

$f(2) =$ DNE (Does not exist) or undefined

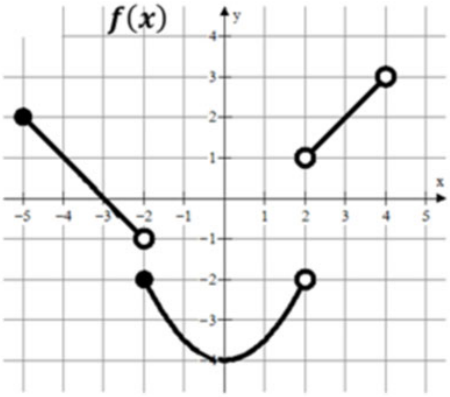
$f(x) = 2$ when $x = ?$ -5 and 3

$f(4) =$ DNE (Does not exist) or undefined

$f(-2) = -2$

$f(3) = 2$

$f(x) = -3$ when $x = ?$ -1.5 and 1.5



Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

5. slope $= 3$ and $(4, -2)$

$y - -2 = 3(x - 4)$

$\boxed{y + 2 = 3(x - 4)}$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

$\boxed{y - 7 = -\frac{3}{2}(x + 5)}$

7. $f(4) = -8$ and $f(-3) = 12$

$m = \frac{12 - -8}{-3 - 4} = \frac{20}{-7}$

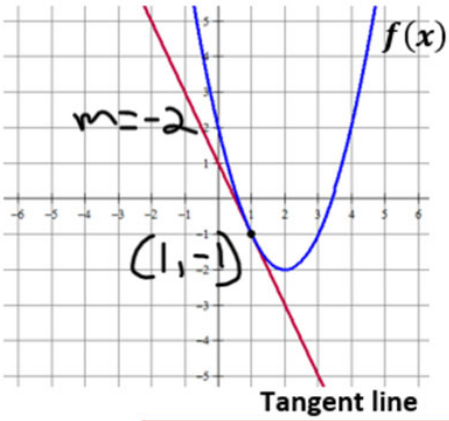
$y - 12 = -\frac{20}{7}(x + 3)$

or

$y + 8 = -\frac{20}{7}(x - 4)$

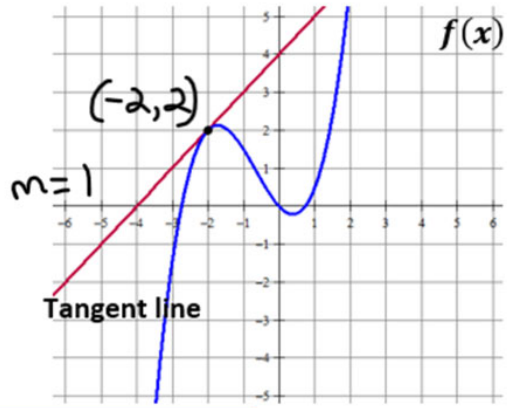
Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



$y+1=-2(x-1)$

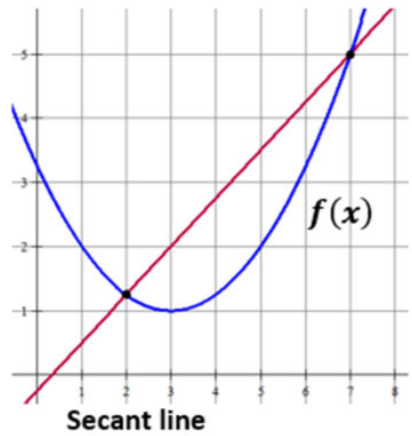
9. The line tangent to $f(x)$ at $x = -2$



$y-2=1(x+2)$

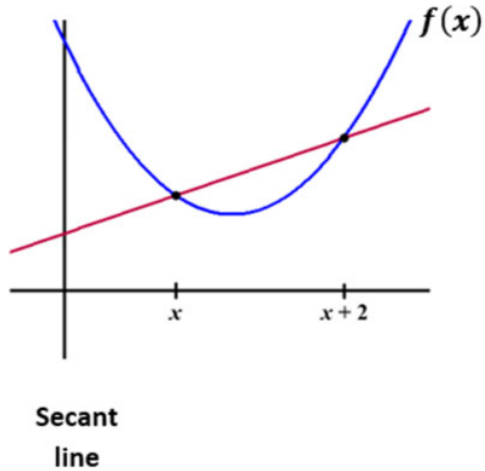
MULTIPLE CHOICE! Remember slope $= \frac{y_2-y_1}{x_2-x_1}$

10. Which choice represents the slope of the secant line shown?



A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$

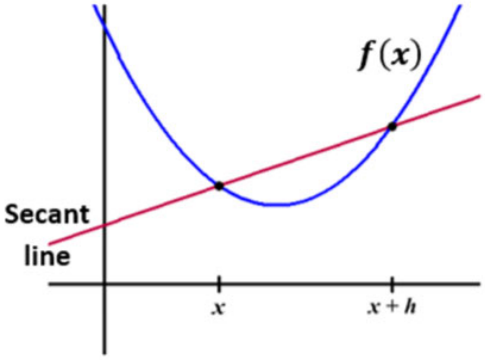
11. Which choice represents the slope of the secant line shown?



A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$

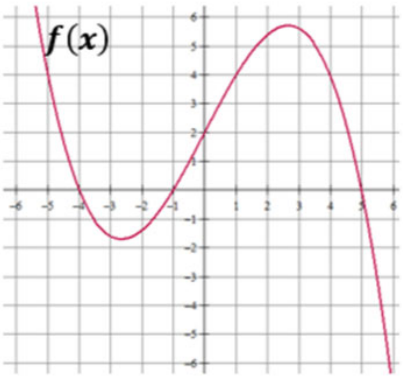
12. Which choice represents the slope of the secant line shown?



A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function $f(x)$ is true?

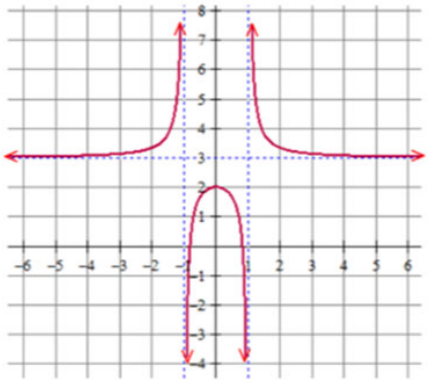


I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



Domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

Range: $(-\infty,2]\cup(3,\infty)$

Horizontal Asymptote(s): $y=3$

Vertical Asymptotes(s): $x=1$ $x=-1$

15.

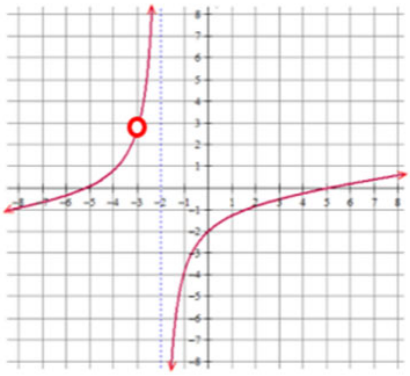
Domain: $(-\infty,-1)\cup(-1,3)\cup(3,\infty)$

Range: $(-\infty,\infty)$

Horizontal Asymptote(s): $y=0$

Vertical Asymptotes(s): $x=-1$ $x=3$

16.



Domain: $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$

Range: $(-\infty,\infty)$

Horizontal Asymptote(s): none

Vertical Asymptotes(s): $x=-2$

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $= 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

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18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$ $x^{\frac{3}{5}} + (2x)^{\frac{1}{5}}$	20. $\sqrt{x+1}$ $(x+1)^{\frac{1}{2}}$	21. $\frac{1}{\sqrt{x+1}}$ $(x+1)^{-\frac{1}{2}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$ $x^{-\frac{1}{2}} - 2x^{-1}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$ $\frac{1}{4}x^{-3} + \frac{1}{2}x^{\frac{3}{4}}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$ $\frac{1}{4}x^{-\frac{1}{2}} - 2(x+1)^{\frac{1}{2}}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$ $\frac{1}{\sqrt{x}} - \sqrt{x^3}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$ $\frac{1}{2\sqrt{x}} + \frac{1}{x}$	27. $3x^{-\frac{1}{2}}$ $\frac{3}{\sqrt{x}}$
28. $(x+4)^{-\frac{1}{2}}$ $\frac{1}{\sqrt{x+4}}$	29. $x^{-2} + x^{\frac{1}{2}}$ $\frac{1}{x^2} + \sqrt{x}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$ $\frac{2}{x^2} + \frac{3}{2x}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$ $\frac{1}{2}$	32. $\cos\frac{\pi}{4}$ $\frac{\sqrt{2}}{2}$	33. $\sin 2\pi$ 0
34. $\tan\pi$ 0	35. $\sec\frac{\pi}{2}$ undefined	36. $\cos\frac{\pi}{6}$ $\frac{\sqrt{3}}{2}$
37. $\sin\frac{\pi}{3}$ $\frac{\sqrt{3}}{2}$	38. $\sin\frac{3\pi}{2}$ -1	39. $\tan\frac{\pi}{4}$ 1
40. $\csc\frac{\pi}{2}$ 1	41. $\sin\pi$ 0	42. $\cos\frac{\pi}{3}$ $\frac{1}{2}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$ $\frac{\pi}{6}$ and $\frac{5\pi}{6}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$ 0, π, and 2π	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$ π

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$ $e^x = 1$ $\ln(e^x) = \ln(1)$ $\boxed{x=0}$	47. $3e^x + 5 = 8$ $3e^x = 3$ $e^x = 1$ $\ln e^x = \ln 1$ $\boxed{x=0}$	48. $e^{2x} = 1$ $\ln e^{2x} = \ln(1)$ $2x = 0$ $\boxed{x=0}$
49. $\ln x = 0$ $e \quad e$ $\boxed{x=1}$	50. $3 - \ln x = 3$ $-\ln x = 0$ $\ln x = 0$ $e \quad e$ $\boxed{x=1}$	51. $\ln(3x) = 0$ $e \quad e$ $3x = 1$ $\boxed{x=\frac{1}{3}}$
52. $x^2 - 3x = 0$ $x(x-3) = 0$ $\boxed{x=0 \quad x=3}$	53. $e^x + xe^x = 0$ $e^x(1+x) = 0$ $e^x = 0$ $1+x=0$ not possible $\boxed{x=-1}$	54. $e^{2x} - e^x = 0$ $e^x(e^x - 1) = 0$ $e^x = 0$ $e^x - 1 = 0$ not possible $e^x = 1$ $\boxed{x=0}$

Solve the following trig equations where $0 \le x \le 2\pi$.

55. $\sin x = \frac{1}{2}$ $x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$	56. $\cos x = -1$ $x = \pi$	57. $\cos x = \frac{\sqrt{3}}{2}$ $x = \frac{\pi}{6}$ and $\frac{11\pi}{6}$
58. $2\sin x = -1$ $\sin x = -\frac{1}{2}$ $\boxed{x = \frac{7\pi}{6} \text{ and } \frac{11\pi}{6}}$	59. $\cos x = \frac{\sqrt{2}}{2}$ $x = \frac{\pi}{4}$ and $\frac{7\pi}{4}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$ $\frac{x}{2} = \frac{\pi}{6}$ $\quad \frac{x}{2} = \frac{11\pi}{6}$ $\boxed{x = \frac{\pi}{3}}$ $\quad x = \frac{11\pi}{3}$ not in the domain interval
61. $\tan x = 0$ $\frac{\sin x}{\cos x} = 0 \rightarrow \sin x = 0$ $\boxed{x = 0, \pi, 2\pi}$	62. $\sin(2x) = 1$ $2x = \frac{\pi}{2}$ and $2x = \frac{5\pi}{2}$ $\boxed{x = \frac{\pi}{4} \text{ and } x = \frac{5\pi}{4}}$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$ $\frac{x}{4} = \frac{\pi}{3}$ $\quad \frac{x}{4} = \frac{2\pi}{3}$ $\boxed{x = \frac{4\pi}{3} \text{ and } x = \frac{4\pi}{3}}$

For each function, determine its domain and range.

Function	Domain	Range
64. $y = \sqrt{x-4}$	$x \ge 4$	$y \ge 0$
65. $y = (x-3)^2$	$\mathbb{R}$ all real numbers	$y \ge 0$
66. $y = \ln x$	$x > 0$	$\mathbb{R}$
67. $y = e^x$	$\mathbb{R}$	$y > 0$
68. $y = \sqrt{4-x^2}$	$-2 \le x \le 2$	$0 \le y \le 2$

Simplify.

69. $\frac{\sqrt{x}}{x}$ $x^{\frac{1}{2}-1}$ $x^{-\frac{1}{2}}$ $\boxed{\frac{1}{\sqrt{x}}}$	70. $e^{\ln x}$ $\boxed{x}$	71. $e^{1+\ln x}$ $e^1 \cdot e^{\ln x}$ $\boxed{ex}$

72. $\ln 1$

$\boxed{0}$

73. $\ln e^7$

$\boxed{7}$

74. $\log_3 \frac{1}{3}$

$\log_3 3^{-1}$

$\boxed{-1}$

75. $\log_{1/2} 8$

$\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{-3}$

$\boxed{-3}$

76. $\ln \frac{1}{2}$ Calculator needed

$\approx \boxed{-0.693}$

77. $27^{\frac{2}{3}}$

$\sqrt[3]{27}^{\,2}$

3^2

$\boxed{9}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

$20a^{\frac{2}{3}+\frac{3}{2}}$

$\boxed{20a^{\frac{13}{6}}}$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

$\frac{1}{3}x^{1--\frac{1}{3}}y^{-2--5}$

$\boxed{\frac{1}{3}x^{\frac{4}{3}}y^{3}}$

80. $\left(4a^{5/3}\right)^{3/2}$

$\sqrt[3]{4}\,a^{\frac{5}{3}\cdot\frac{3}{2}}$

$\boxed{8a^{\frac{5}{2}}}$

If $f(x) = \{(3,5),(2,4),(1,7)\}$ $g(x) = \sqrt{x-3}$
$h(x) = \{(3,2),(4,3),(1,6)\}$ $k(x) = x^2 + 5$ **, then determine each of the following.**

81. $(f+h)(1)$

$f(1)+h(1)$

$7+6$

$\boxed{13}$

82. $(k-g)(5)$

$k(5)-g(5)$

$(25+5)-(\sqrt{2})$

$\boxed{30-\sqrt{2}}$

83. $f(h(3))$

$f(2)$

$\boxed{4}$

84. $g(k(7))$

$g(7^2+5)$

$g(54)$

$\boxed{\sqrt{54-3} = \sqrt{51}}$

85. $h(3)$

$\boxed{2}$

86. $g(g(9))$

$g(\sqrt{6})$

$\boxed{\sqrt{\sqrt{6}-3}}$

87. $f^{-1}(4)$

$\boxed{2}$

88. $k^{-1}(x)$

$x = y^2+5$

$x-5 = y^2$

$\boxed{y = \sqrt{x-5}}$

89. $k(g(x)) = (\sqrt{x-3})^2+5$

$x-3+5$

$\boxed{x+2}$

90. $g(f(2))$

$g(4) = \sqrt{4-3}$

$\boxed{1}$