Algebra II w/Trig
Summer Work

Your task is to get through the first chapter of Algebra II w/Trig this summer. You will read in the book, use Khan Academy and other resources to make your way through problems from each section in the chapter. You will submit this work to your teacher at the beginning of the second week of classes. There will be an assessment given in the second week on this Chapter 1 material. In this packet, you will find Chapter 1 of the Algebra II w/Trig textbook. It is recommended that you do not print out the entire chapter as it is many pages long! Solutions to the odd problems are included. It is expected that you check your answers to the odd problems against the solutions and correct any that are incorrect.

- 1.1 Pg. 7 #15-47 every other odd, 61-63 (example of every other odd: 15,19,23,27, etc.)
- 1.2 Pg. 14 #15,21,22,26,27,31,33,37,45,47,49,53,55,59
- 1.3 Pg. 22 # 23, 25, 31, 33, 35, 39, 41, 45-49 odd, 51, 53
- 1.4 Pg. 30 #30-34, 40
- 1.5 Pg. 38 #18,22,23,28,29
- 1.6 Pg. 45 #13-18 all, 31,34,41,43,44
- 1.7 Pg. 53 #1, 2, 4, 5, 17, 33, 37, 39, 41, 42, 47, 56, 65, 69, 76, 80, 81

Khan Academy videos:

1.4 Manipulating Formulas

1.6: Solving Linear Equations

1.7 Absolute Value Equations

Absolute Value Inequalities

1.1 Real Numbers and Number Operations

What *you should learn*

GOAL 1 Use a number line to graph and order real numbers.

GOAL 2 Identify properties of and use operations with real numbers, as applied in **Exs. 64 and 65**.

Why *you should learn it*

▼ To solve **real-life** problems, such as how to exchange money in **Example 7**.

REAL LIFE

GOAL 1 Using the Real Number Line

The numbers used most often in algebra are the *real numbers*. Some important subsets of the real numbers are listed below.

SUBSETS OF THE REAL NUMBERS

WHOLE NUMBERS 0, 1, 2, 3, . . .

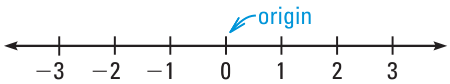
INTEGERS . . . , −3, −2, −1, 0, 1, 2, 3, . . .

RATIONAL NUMBERS Numbers such as $\frac{3}{4}$, $\frac{1}{3}$, and $\frac{-4}{1}$ (or −4) that can be written as the ratio of two integers. When written as decimals, rational numbers terminate or repeat. For example, $\frac{3}{4} = 0.75$ and $\frac{1}{3} = 0.333\ldots$

IRRATIONAL NUMBERS Real numbers that are not rational, such as $\sqrt{2}$ and π. When written as decimals, irrational numbers neither terminate nor repeat.

The three dots in the lists of the whole numbers and the integers above indicate that the lists continue without end.

Real numbers can be pictured as points on a line called a *real number line*. The numbers increase from left to right, and the point labeled 0 is the **origin**.



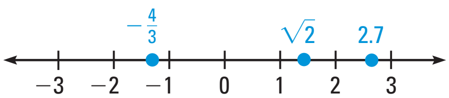
The point on a number line that corresponds to a real number is the **graph** of the number. Drawing the point is called *graphing* the number or *plotting* the point. The number that corresponds to a point on a number line is the **coordinate** of the point.

EXAMPLE 1 *Graphing Numbers on a Number Line*

Graph the real numbers $-\frac{4}{3}$, $\sqrt{2}$, and 2.7.

SOLUTION

First, recall that $-\frac{4}{3}$ is $-1\frac{1}{3}$, so $-\frac{4}{3}$ is between −2 and −1. Then, approximate $\sqrt{2}$ as a decimal to the nearest tenth: $\sqrt{2} \approx 1.4$. (The symbol ≈ means *is approximately equal to.*) Finally, graph the numbers.



A number line can be used to order real numbers. The *inequality symbols* $<$, $\leq$, $>$, and $\geq$ can be used to show the order of two numbers.

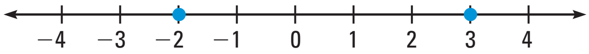
EXAMPLE 2 *Ordering Real Numbers*

Use a number line to order the real numbers.

a. -2 and 3 **b.** -1 and -3

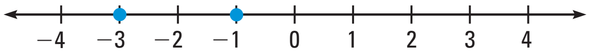
SOLUTION

a. Begin by graphing both numbers.



Because -2 is to the left of 3, it follows that -2 *is less than* 3, which can be written as $-2 < 3$. This relationship can also be written as $3 > -2$, which is read as "3 *is greater than* -2."

b. Begin by graphing both numbers.



Because -3 is to the left of -1, it follows that -3 *is less than* -1, which can be written as $-3 < -1$. (You can also write $-1 > -3$.)

REAL LIFE *Geography*

EXAMPLE 3 *Ordering Elevations*

Here are the elevations of five locations in Imperial Valley, California.

Alamorio: -135 feet

Curlew: -93 feet

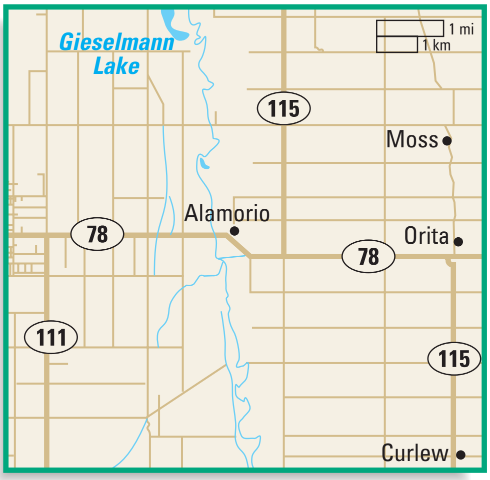
Gieselmann Lake: -162 feet

Moss: -100 feet

Orita: -92 feet

a. Order the elevations from lowest to highest.

b. Which locations have elevations below -100 feet?



SOLUTION

a. From lowest to highest, the elevations are as follows.

Location	Gieselmann Lake	Alamorio	Moss	Curlew	Orita
Elevation (ft)	-162	-135	-100	-93	-92

b. Gieselmann Lake and Alamorio have elevations below -100 feet.

GOAL 2 USING PROPERTIES OF REAL NUMBERS

When you add or multiply real numbers, there are several properties to remember.

CONCEPT SUMMARY — PROPERTIES OF ADDITION AND MULTIPLICATION

Let a, b, and c be real numbers.

Property	Addition	Multiplication
CLOSURE	$a + b$ is a real number.	ab is a real number.
COMMUTATIVE	$a + b = b + a$	$ab = ba$
ASSOCIATIVE	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
IDENTITY	$a + 0 = a, 0 + a = a$	$a \cdot 1 = a, 1 \cdot a = a$
INVERSE	$a + (-a) = 0$	$a \cdot \frac{1}{a} = 1, a \neq 0$

The following property involves both addition and multiplication.

DISTRIBUTIVE $a(b + c) = ab + ac$

EXAMPLE 4 *Identifying Properties of Real Numbers*

Identify the property shown.

a. $(3 + 9) + 8 = 3 + (9 + 8)$ **b.** $14 \cdot 1 = 14$

SOLUTION

a. Associative property of addition **b.** Identity property of multiplication

STUDENT HELP

Study Tip

- If a is positive, then its opposite, $-a$, is negative.
- The opposite of 0 is 0.
- If a is negative, then its opposite, $-a$, is positive.

The **opposite**, or *additive inverse*, of any number a is $-a$. The **reciprocal**, or *multiplicative inverse*, of any nonzero number a is $\frac{1}{a}$. Subtraction is defined as *adding the opposite*, and division is defined as *multiplying by the reciprocal.*

$a - b = a + (-b)$ **Definition of subtraction**

$\frac{a}{b} = a \cdot \frac{1}{b}, b \neq 0$ **Definition of division**

EXAMPLE 5 *Operations with Real Numbers*

a. The difference of 7 and -10 is:

$7 - (-10) = 7 + 10$ **Add 10, the opposite of −10.**

$= 17$ **Simplify.**

b. The quotient of -24 and $\frac{1}{3}$ is:

$\frac{-24}{\frac{1}{3}} = -24 \cdot 3$ **Multiply by 3, the reciprocal of $\frac{1}{3}$.**

$= -72$ **Simplify.**

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

When you use the operations of addition, subtraction, multiplication, and division in real life, you should use *unit analysis* to check that your units make sense.

EXAMPLE 6 *Using Unit Analysis*

Perform the given operation. Give the answer with the appropriate unit of measure.

a. 345 miles − 187 miles = 158 miles

b. $(1.5\ \cancel{\text{hours}})\left(\dfrac{50\text{ miles}}{1\ \cancel{\text{hour}}}\right) = 75\text{ miles}$

c. $\dfrac{24\text{ dollars}}{3\text{ hours}} = 8\text{ dollars per hour}$

d. $\left(\dfrac{88\ \cancel{\text{feet}}}{1\ \cancel{\text{second}}}\right)\left(\dfrac{3600\ \cancel{\text{seconds}}}{1\text{ hour}}\right)\left(\dfrac{1\text{ mile}}{5280\ \cancel{\text{feet}}}\right) = 60\text{ miles per hour}$

FOCUS ON APPLICATIONS

REAL LIFE MONEY EXCHANGE In 1997, 17,700,000 United States citizens visited Mexico and spent \$7,200,000,000. That same year 8,433,000 Mexican citizens visited the United States and spent \$4,289,000,000.

EXAMPLE 7 *Operations with Real Numbers in Real Life*

MONEY EXCHANGE You are exchanging \$400 for Mexican pesos. The exchange rate is 8.5 pesos per dollar, and the bank charges a 1% fee to make the exchange.

a. How much money should you take to the bank if you do not want to use part of the \$400 to pay the exchange fee?

b. How much will you receive in pesos?

c. When you return from Mexico you have 425 pesos left. How much can you get in dollars? Assume that you use other money to pay the exchange fee.

SOLUTION

a. To find 1% of \$400, multiply to get:

$$1\% \times \$400 = 0.01 \times \$400 \qquad \text{Rewrite 1\% as 0.01.}$$
$$= \$4 \qquad \text{Simplify.}$$

▶ You need to take \$400 + \$4 = \$404 to the bank.

b. To find the amount you will receive in pesos, multiply \$400 by the exchange rate.

$$(400\ \cancel{\text{dollars}})\left(\frac{8.5\text{ pesos}}{1\ \cancel{\text{dollar}}}\right) = (400 \times 8.5)\text{ pesos}$$
$$= 3400\text{ pesos}$$

▶ You receive 3400 pesos for \$400.

c. To find the amount in dollars, divide 425 pesos by the exchange rate.

$$\frac{425\text{ pesos}}{8.5\text{ pesos per dollar}} = (425\ \cancel{\text{pesos}})\left(\frac{1\text{ dollar}}{8.5\ \cancel{\text{pesos}}}\right)$$
$$= \frac{425}{8.5}\text{ dollars}$$
$$= \$50$$

▶ You receive \$50 for 425 pesos.

STUDENT HELP

INTERNET DATA UPDATE Visit our Web site www.mcdougallittell.com

Guided Practice

Vocabulary Check ✓

1. What is a rational number? What is an irrational number?

Concept Check ✓

2. Give an example of each of the following: a whole number, an integer, a rational number, and an irrational number.
3. Which of the following is false? Explain.

   A. No integer is an irrational number.

   B. Every integer is a rational number.

   C. Every integer is a whole number.

Skill Check ✓

Graph the numbers on a number line. Then decide which number is the greatest.

4. $-3, 4, 0, -8, -10$
5. $\frac{3}{2}, -1, -\frac{5}{2}, 3, -5$
6. $1, -2.5, 4.5, -0.5, 6$
7. $3.2, -0.7, \frac{3}{4}, -\frac{3}{2}, 0$

Identify the property shown.

8. $5 + 2 = 2 + 5$
9. $6 + (-6) = 0$
10. $24 \cdot 1 = 24$
11. $8 \cdot 10 = 10 \cdot 8$
12. $13 + 0 = 13$
13. $7\left(\frac{1}{7}\right) = 1$
14. Find the product. Give the answer with the appropriate unit of measure. Explain your reasoning.

$$\left(\frac{90 \text{ miles}}{1 \text{ hour}}\right)\left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right)\left(\frac{1 \text{ hour}}{60 \text{ minutes}}\right)\left(\frac{1 \text{ minute}}{60 \text{ seconds}}\right)$$

Practice and Applications

Student Help

Extra Practice to help you master skills is on p. 940.

Using a Number Line **Graph the numbers on a number line. Then decide which number is greater and use the symbol < or > to show the relationship.**

15. $\frac{1}{2}, -5$
16. $4, \frac{3}{4}$
17. $2.3, -0.6$
18. $0.3, -2.1$
19. $-\frac{5}{3}, \sqrt{3}$
20. $0, -\sqrt{10}$
21. $-\frac{9}{4}, -3$
22. $-\frac{3}{2}, -\frac{11}{3}$
23. $\sqrt{5}, 2$
24. $-2, \sqrt{2}$
25. $\sqrt{8}, 2.5$
26. $-4.5, -\sqrt{24}$

Student Help

Homework Help

Examples 1, 2: Exs. 15–32
Example 3: Exs. 55, 56
Example 4: Exs. 33–42
Example 5: Exs. 43–50
Example 6: Exs. 51–54
Example 7: Exs. 57–65

Ordering Numbers **Graph the numbers on a number line. Then write the numbers in increasing order.**

27. $-\frac{1}{2}, 2, \frac{13}{4}, -3, -6$
28. $\sqrt{15}, -4, -\frac{2}{9}, -1, 6$
29. $-\sqrt{5}, -\frac{5}{2}, 0, 3, -\frac{1}{3}$
30. $\frac{1}{6}, 2.7, -1.5, -8, -\sqrt{7}$
31. $0, -\frac{12}{5}, -\sqrt{12}, 0.3, -1.5$
32. $0.8, \sqrt{10}, -2.4, -\sqrt{6}, \frac{9}{2}$

IDENTIFYING PROPERTIES **Identify the property shown.**

33. $-8 + 8 = 0$

34. $(3 \cdot 5) \cdot 10 = 3 \cdot (5 \cdot 10)$

35. $7 \cdot 9 = 9 \cdot 7$

36. $(9 + 2) + 4 = 9 + (2 + 4)$

37. $12(1) = 12$

38. $2(5 + 11) = 2 \cdot 5 + 2 \cdot 11$

LOGICAL REASONING **Tell whether the statement is true for all real numbers *a*, *b*, and *c*. Explain your answers.**

39. $(a + b) + c = a + (b + c)$

40. $(a - b) - c = a - (b - c)$

41. $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

42. $(a \div b) \div c = a \div (b \div c)$

OPERATIONS **Select and perform an operation to answer the question.**

43. What is the sum of 32 and -7?

44. What is the sum of -9 and -6?

45. What is the difference of -5 and 8?

46. What is the difference of -1 and -10?

47. What is the product of 9 and -4?

48. What is the product of -7 and -3?

49. What is the quotient of -5 and $-\frac{1}{2}$?

50. What is the quotient of -14 and $\frac{7}{4}$?

UNIT ANALYSIS **Give the answer with the appropriate unit of measure.**

51. $8\frac{1}{6}$ feet $+ 4\frac{5}{6}$ feet

52. $27\frac{1}{2}$ liters $- 18\frac{5}{8}$ liters

53. $(8.75 \text{ yards})\left(\frac{\$70}{1 \text{ yard}}\right)$

54. $\left(\frac{50 \text{ feet}}{1 \text{ second}}\right)\left(\frac{1 \text{ mile}}{5280 \text{ feet}}\right)\left(\frac{3600 \text{ seconds}}{1 \text{ hour}}\right)$

55. **STATISTICS CONNECTION** The lowest temperatures ever recorded in various cities are shown. List the cities in decreasing order based on their lowest temperatures. How many of these cities have a record low temperature below $-25°F$? ▶ Source: National Climatic Data Center

City	Low temp.	City	Low temp.
Albany, NY	$-28°F$	Jackson, MS	$2°F$
Atlanta, GA	$-8°F$	Milwaukee, WI	$-26°F$
Detroit, MI	$-21°F$	New Orleans, LA	$11°F$
Helena, MT	$-42°F$	Norfolk, VA	$-3°F$
Honolulu, HI	$53°F$	Seattle-Tacoma, WA	$0°F$

56. **MASTERS GOLF** The table shows the final scores of 10 competitors in the 1998 Masters Golf Tournament. List the players in increasing order based on their golf scores. ▶ Source: *Sports Illustrated*

Player	Score	Player	Score
Paul Azinger	-6	Lee Janzen	$+6$
Tiger Woods	-3	Jeff Maggert	$+1$
Jay Haas	-2	Mark O'Meara	-9
Jim Furyk	-7	Corey Pavin	$+9$
Vijay Singh	$+12$	Jumbo Ozaki	$+8$

FOCUS ON PEOPLE

REAL LIFE **VIJAY SINGH,** a world-class golfer from Fiji, won the 1998 PGA Championship. His final score of –9 was 2 better than that of the second-place finisher. Singh, whose first name means "victory" in Hindi, has won several PGA Tour events.

FOCUS ON APPLICATIONS

0 25215 04658 2
first digit
check digit

REAL LIFE

BAR CODES Using the operations on this bar code produces $(0 + 5 + 1 + 0 + 6 + 8)(3) + (2 + 2 + 5 + 4 + 5) = 78$. The next highest multiple of 10 is 80, and $80 - 78 = 2$, which is the check digit.

BAR CODES In Exercises 57 and 58, use the following information.
All packaged products sold in the United States have a Universal Product Code (UPC), or bar code, such as the one shown at the left. The following operations are performed on the first eleven digits, and the result should equal the twelfth digit, called the *check digit*.

- Add the digits in the odd-numbered positions. Multiply by 3.
- Add the digits in the even-numbered positions.
- Add the results of the first two steps.
- Subtract the result of the previous step from the next highest multiple of 10.

57. Does a UPC of 0 76737 20012 9 check? Explain.

58. Does a UPC of 0 41800 48700 3 check? Explain.

59. **SOCIAL STUDIES CONNECTION** Two of the tallest buildings in the world are the Sky Central Plaza in Guangzhou, China, which reaches a height of 1056 feet, and the Petronas Tower I in Kuala Lumpur, Malaysia, which reaches a height of 1483 feet. Find the heights of both buildings in yards, in inches, and in miles. Give your answers to four significant digits.
▶ Source: Council on Tall Buildings and Urban Habitat

Petronas Tower I

STUDENT HELP

Skills Review
For help with significant digits, see p. 911.

60. **ELEVATOR SPEED** The elevator in the Washington Monument takes 75 seconds to travel 500 feet to the top floor. What is the speed of the elevator in miles per hour? Give your answer to two significant digits.
▶ Source: National Park Service

TRAVEL In Exercises 61–63, use the following information.
You are taking a trip to Switzerland. You are at the bank exchanging \$600 for Swiss francs. The exchange rate is 1.5 francs per dollar, and the bank charges a 1.5% fee to make the exchange.

61. You brought \$10 extra with you to pay the exchange fee. Do you have enough to pay the fee?

62. How much will you receive in Swiss francs for your \$600?

63. After your trip, you have 321 Swiss francs left. How much is this amount in dollars? Assume that you use other money to pay the exchange fee.

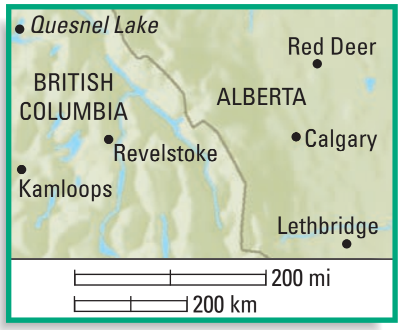
HISTORY CONNECTION **In Exercises 64 and 65, use the following information.**
In 1862, James Glaisher and Henry Coxwell went up too high in a hot-air balloon. At 25,000 feet, Glaisher passed out. To get the balloon to descend, Coxwell grasped a valve, but his hands were too numb to pull the cord. He was able to pull the cord with his teeth. The balloon descended, and both men made it safely back. The temperature of air drops about 3°F for each 1000 foot increase in altitude.

64. How much had the temperature dropped from the sea level temperature when Glaisher and Coxwell reached an altitude of 25,000 feet?

65. If the temperature at sea level was 60°F, what was the temperature at 25,000 feet?

Test Preparation

66. MULTI-STEP PROBLEM You are taking a trip through the provinces of Alberta and British Columbia in Canada. You are at Quesnel Lake when you decide to visit some of the national parks. You visit the following places in order: Kamloops, Revelstoke, Lethbridge, and Red Deer. After you visit Red Deer, you return to Quesnel Lake.



a. Using the scale on the map, estimate the distance traveled (in kilometers) for the entire trip. Approximately where was the "halfway point" of your trip?

b. Your car gets 12 kilometers per liter of gasoline. If your gas tank holds 60 liters and the cost of gasoline is \$.29 per liter, about how much will you spend on gasoline for the entire trip? How many times will you have to stop for gasoline if you begin the trip with a full tank?

c. If you drive at an average speed of 88 kilometers per hour, how many hours will you spend driving on your trip?

★ **Challenge**

67. LOGICAL REASONING Show that $a + (a + 2) = 2(a + 1)$ for all values of a by justifying the steps using the properties of addition and multiplication.

$a + (a + 2) = (a + a) + 2$	**a.** ?
$= (1 \cdot a + 1 \cdot a) + 2$	**b.** ?
$= (1 + 1)a + 2$	**c.** ?
$= 2a + 2 \cdot 1$	**d.** ?
$= 2(a + 1)$	**e.** ?

EXTRA CHALLENGE
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MIXED REVIEW

OPERATIONS WITH SIGNED NUMBERS **Perform the operation.** (Skills Review, p. 905)

68. $4 - 12$ **69.** $(-7)(-9)$ **70.** $-20 \div 5$ **71.** $6(-5)$

72. $-14 + 9$ **73.** $6 - (-13)$ **74.** $56 \div (-7)$ **75.** $-16 + (-18)$

ALGEBRAIC EXPRESSIONS **Write the given phrase as an algebraic expression.** (Skills Review, p. 929 for 1.2)

76. 7 more than a number **77.** 3 less than a number

78. 6 times a number **79.** $\frac{1}{4}$ of a number

GEOMETRY CONNECTION **Find the area of the figure.** (Skills Review, p. 914)

80. Triangle with base 6 inches and height 4 inches

81. Triangle with base 7 inches and height 3 inches

82. Rectangle with sides 5 inches and 7 inches

83. Rectangle with sides 25 inches and 30 inches

1.2 Algebraic Expressions and Models

What *you should learn*

GOAL 1 Evaluate algebraic expressions.

GOAL 2 Simplify algebraic expressions by combining like terms, as applied in **Example 6**.

Why *you should learn it*

▼ To solve **real-life** problems, such as finding the population of Hawaii in **Ex. 57**.

GOAL 1 EVALUATING ALGEBRAIC EXPRESSIONS

A **numerical expression** consists of numbers, operations, and grouping symbols. In Lesson 1.1 you worked with addition, subtraction, multiplication, and division. In this lesson you will work with *exponentiation*, or raising to a power.

Exponents are used to represent repeated factors in multiplication. For instance, the expression 2^5 represents the number that you obtain when 2 is used as a factor 5 times.

$$2^5 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{5 factors of 2}} \qquad \text{2 to the fifth power}$$

The number 2 is the **base**, the number 5 is the **exponent**, and the expression 2^5 is a **power**. The exponent in a power represents the number of times the base is used as a factor. For a number raised to the first power, you do not usually write the exponent 1. For instance, you usually write 2^1 simply as 2.

EXAMPLE 1 *Evaluating Powers*

a. $(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$

b. $-3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$

In Example 1, notice how parentheses are used in part (a) to indicate that the base is -3. In the expression -3^4, however, the base is 3, not -3. An **order of operations** helps avoid confusion when evaluating expressions.

ORDER OF OPERATIONS

1. First, do operations that occur within grouping symbols.
2. Next, evaluate powers.
3. Then, do multiplications and divisions from left to right.
4. Finally, do additions and subtractions from left to right.

EXAMPLE 2 *Using Order of Operations*

$$\begin{aligned} -4 + 2(-2 + 5)^2 &= -4 + 2(3)^2 && \text{Add within parentheses.} \\ &= -4 + 2(9) && \text{Evaluate power.} \\ &= -4 + 18 && \text{Multiply.} \\ &= 14 && \text{Add.} \end{aligned}$$

A **variable** is a letter that is used to represent one or more numbers. Any number used to replace a variable is a **value of the variable**. An expression involving variables is called an **algebraic expression**.

When the variables in an algebraic expression are replaced by numbers, you are *evaluating* the expression, and the result is called the **value of the expression**. To evaluate an algebraic expression, use the following flow chart.

Write algebraic expression. → **Substitute values of variables.** → **Simplify.**

EXAMPLE 3 *Evaluating an Algebraic Expression*

STUDENT HELP

Skills Review
For help with operations with signed numbers, see p. 905.

Evaluate $-3x^2 - 5x + 7$ when $x = -2$.

$$\begin{aligned} -3x^2 - 5x + 7 &= -3(-2)^2 - 5(-2) + 7 && \text{Substitute } -2 \text{ for } x. \\ &= -3(4) - 5(-2) + 7 && \text{Evaluate power.} \\ &= -12 + 10 + 7 && \text{Multiply.} \\ &= 5 && \text{Add.} \end{aligned}$$

An expression that represents a real-life situation is a **mathematical model**. When you create the expression, you are *modeling* the real-life situation.

REAL LIFE *Movies*

EXAMPLE 4 *Writing and Evaluating a Real-Life Model*

You have \$50 and are buying some movies on videocassettes that cost \$15 each. Write an expression that shows how much money you have left after buying n movies. Evaluate the expression when $n = 2$ and $n = 3$.

SOLUTION

PROBLEM SOLVING STRATEGY

VERBAL MODEL | Original amount − Price per movie · Number of movies bought

LABELS

Original amount	= **50**	(dollars)
Price per movie	= **15**	(dollars per movie)
Number of movies bought	= ***n***	(movies)

ALGEBRAIC MODEL | $\mathbf{50 - 15}\,\boldsymbol{n}$

When you buy **2** movies, you have $50 - 15(\mathbf{2}) = \$20$ left.

When you buy **3** movies, you have $50 - 15(\mathbf{3}) = \$5$ left.

UNIT ANALYSIS You can use unit analysis to check your verbal model.

$$\text{dollars} - \left(\frac{\text{dollars}}{\cancel{\text{movie}}}\right)(\cancel{\text{movies}}) = \text{dollars} - \text{dollars} = \text{dollars}$$

GOAL 2 SIMPLIFYING ALGEBRAIC EXPRESSIONS

For an expression such as $2x + 3$, the parts that are added together, $2x$ and 3, are called **terms**. When a term is the product of a number and a power of a variable, such as $2x$ or $4x^3$, the number is the **coefficient** of the power.

Terms such as $3x^2$ and $-5x^2$ are **like terms** because they have the same variable part. **Constant terms** such as -4 and 2 are also like terms. The distributive property lets you *combine like terms* that have variables by adding the coefficients.

STUDENT HELP

Study Tip
For an expression like $2x - 3$, think of the expression as $2x + (-3)$, so the terms are $2x$ and -3.

EXAMPLE 5 *Simplifying by Combining Like Terms*

STUDENT HELP

Skills Review
For help with opposites, see p. 936.

a. $7x + 4x = (7 + 4)x$ — Distributive property

$= 11x$ — Add coefficients.

b. $3n^2 + n - n^2 = (3n^2 - n^2) + n$ — Group like terms.

$= 2n^2 + n$ — Combine like terms.

c. $2(x + 1) - 3(x - 4) = 2x + 2 - 3x + 12$ — Distributive property

$= (2x - 3x) + (2 + 12)$ — Group like terms.

$= -x + 14$ — Combine like terms.

Two algebraic expressions are **equivalent** if they have the same value for all values of their variable(s). For instance, the expressions $7x + 4x$ and $11x$ are equivalent, as are the expressions $5x - (6x + y)$ and $-x - y$. A statement such as $7x + 4x = 11x$ that equates two equivalent expressions is called an **identity**.

EXAMPLE 6 *Using a Real-Life Model*

FOCUS ON PEOPLE

REAL LIFE **HEITARO NAKAJIMA** could be called the inventor of the compact disc (CD). He was head of the research division of the company that developed the first CDs in 1982. A CD usually has a diameter of 12 centimeters, just the right size to hold Beethoven's Ninth Symphony.

MUSIC You want to buy either a CD or a cassette as a gift for each of 10 people. CDs cost \$13 each and cassettes cost \$8 each. Write an expression for the total amount you must spend. Then evaluate the expression when 4 of the people get CDs.

SOLUTION

VERBAL MODEL: Price per CD · Number of CDs + Price per cassette · Number of cassettes

LABELS:

CD price $= 13$ (dollars per CD)

Number of CDs $= n$ (CDs)

Cassette price $= 8$ (dollars per cassette)

Number of cassettes $= 10 - n$ (cassettes)

ALGEBRAIC MODEL:

$13\,n + 8\,(10 - n) = 13n + 80 - 8n$

$= 5n + 80$

When $n = 4$, the total cost is $5(4) + 80 = 20 + 80 = \$100$.

GUIDED PRACTICE

Vocabulary Check ✓

1. Copy 8^4 and label the base and the exponent. What does each number represent?

Concept Check ✓

2. Identify the terms of $6x^3 - 17x + 5$.

3. Explain how the order of operations is used to evaluate $3 - 8^2 \div 4 + 1$.

ERROR ANALYSIS **Find the error. Then write the correct steps.**

4.

$$\begin{aligned} 5 + 2(16 \div 2)^2 &= 5 + 2(16 \div 4) \\ &= 5 + 2(4) \\ &= 5 + 8 \\ &= 13 \end{aligned}$$

5.

$$\begin{aligned} 4x - (3y + 7x) &= 4x - 3y + 7x \\ &= (4x + 7x) - 3y \\ &= (4 + 7)x - 3y \\ &= 11x - 3y \end{aligned}$$

Skill Check ✓

Evaluate the expression for the given value of *x*.

6. $x - 8$ when $x = 2$

7. $3x + 14$ when $x = -3$

8. $x(x + 4)$ when $x = 5$

9. $x^2 - 9$ when $x = 6$

Simplify the expression.

10. $9y - 14y$

11. $11x + 6y - 2x + 3y$

12. $3(x + 4) - (6 + 2x)$

13. $3x^2 - 5x + 5x^2 - 3x$

14. **RETAIL BUYING** When you arrive at the music store to buy the CDs and cassettes for the 10 people mentioned in Example 6, you find that the store is having a sale. CDs now cost \$11 each and cassettes now cost \$7 each. Write an expression for the new total amount you will spend. Then evaluate the expression when 6 of the people get CDs.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 940.

WRITING WITH EXPONENTS **Write the expression using exponents.**

15. eight to the third power

16. x to the fifth power

17. 5 to the nth power

18. $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$

EVALUATING POWERS **Evaluate the power.**

19. 4^4

20. $(-4)^4$

21. -2^5

22. $(-2)^5$

23. 5^3

24. 3^5

25. 2^8

26. 8^2

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 15–26
Example 2: Exs. 27–32
Example 3: Exs. 33–46
Example 4: Exs. 56–61
Example 5: Exs. 47–52
Example 6: Exs. 56–61

USING ORDER OF OPERATIONS **Evaluate the expression.**

27. $13 + 20 - 9$

28. $14 \cdot 3 - 2$

29. $6 \cdot 2 + 35 \div 5$

30. $-6 + 3(-3 + 7)^2$

31. $24 - 8 \cdot 12 \div 4$

32. $16 \div (2 + 6) \cdot 10$

EVALUATING EXPRESSIONS **Evaluate the expression for the given value of *x*.**

33. $x - 12$ when $x = 7$

34. $6x + 9$ when $x = 4$

35. $25x(x - 4)$ when $x = -1$

36. $x^2 + 5 - x$ when $x = 5$

EVALUATING EXPRESSIONS **Evaluate the expression for the given values of *x* and *y*.**

37. $x^4 + 3y$ when $x = 2$ and $y = -8$

38. $(3x)^2 - 7y^2$ when $x = 3$ and $y = 2$

39. $9x + 8y$ when $x = 4$ and $y = 5$

40. $5\left(\frac{x}{y}\right) - x$ when $x = 6$ and $y = \frac{2}{3}$

41. $\frac{x^2}{2y + 1}$ when $x = -3$ and $y = 2$

42. $\frac{(x + 3)^2}{3y - 2}$ when $x = 2$ and $y = 4$

43. $\frac{x + y}{x - y}$ when $x = -4$ and $y = 9$

44. $\frac{2x + y}{3y + x}$ when $x = 10$ and $y = 6$

45. $\frac{4(x - 2y)}{x + y}$ when $x = 4$ and $y = -2$

46. $\frac{4y - x}{3(2x + y)}$ when $x = -3$ and $y = 3$

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

47. $7x^2 + 12x - x^2 - 40x$

48. $4x^2 + x - 3x - 6x^2$

49. $12(n - 3) + 4(n - 13)$

50. $5(n^2 + n) - 3(n^2 - 2n)$

51. $4x - 2y + y - 9x$

52. $8(y - x) - 2(x - y)$

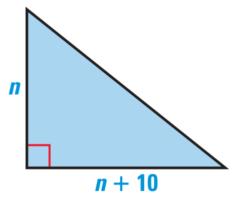
STUDENT HELP

Skills Review
For help with area, see p. 914.

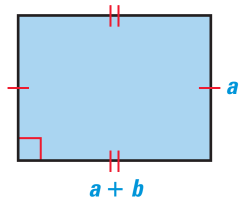
INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with problem solving in Ex. 56.

GEOMETRY CONNECTION **Write an expression for the area of the figure. Then evaluate the expression for the given value(s) of the variable(s).**

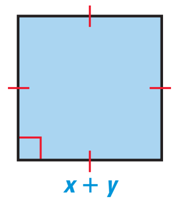
53. $n = 40$



54. $a = 8, b = 3$

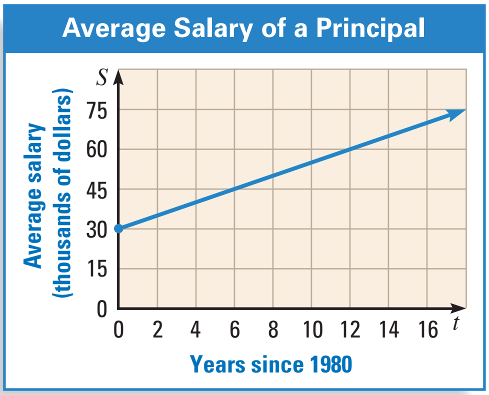


55. $x = 12, y = 5$



56. **AVERAGE SALARIES** In 1980, a public high school principal's salary was approximately \$30,000. From 1980 through 1996, the average salary of principals at public high schools increased by an average of \$2500 per year. Use the verbal model and labels below to write an algebraic model that gives a public high school principal's average salary t years after 1980. Evaluate the expression when $t = 5$, 10, and 15.

▶ Source: Educational Research Service



VERBAL MODEL

$\boxed{\text{Salary in 1980}} + \boxed{\text{Average increase per year}} \cdot \boxed{\text{Years since 1980}}$

LABELS

Salary in 1980 = **30** (thousands of dollars)

Average increase per year = **2.5** (thousands of dollars per year)

Years since 1980 = $\boldsymbol{t}$ (years)

FOCUS ON CAREERS

REAL LIFE **PHYSICAL THERAPIST**

Physical therapists help restore function, improve mobility, and relieve pain in patients with injuries or disease.

INTERNET CAREER LINK
www.mcdougallittell.com

57. **SOCIAL STUDIES CONNECTION** For 1980 through 1998, the population (in thousands) of Hawaii can be modeled by $13.2t + 965$ where t is the number of years since 1980. What was the population of Hawaii in 1998? What was the population increase from 1980 to 1998? ▶ Source: U.S. Bureau of the Census

58. **PHYSICAL THERAPY** In 1996 there were approximately 115,000 physical therapy jobs in the United States. The number of jobs is expected to increase by 8100 each year. Write an expression that gives the total number of physical therapy jobs each year since 1996. Evaluate the expression for the year 2010.
INTERNET DATA UPDATE of U.S. Bureau of Labor Statistics data at www.mcdougallittell.com

59. **MOVIE RENTALS** You buy a VCR for \$149 and plan to rent movies each month. Each rental costs \$3.85. Write an expression that gives the total amount you spend during the first twelve months that you own the VCR, including the price of the VCR. Evaluate the expression if you rent 6 movies each month.

60. **USED CARS** You buy a used car with 37,148 miles on the odometer. Based on your regular driving habits, you plan to drive the car 15,000 miles each year that you own it. Write an expression for the number of miles that appears on the odometer at the end of each year. Evaluate the expression to find the number of miles that will appear on the odometer after you have owned the car for 4 years.

61. **WALK-A-THON** You are taking part in a charity walk-a-thon where you can either walk or run. You walk at 4 kilometers per hour and run at 8 kilometers per hour. The walk-a-thon lasts 3 hours. Money is raised based on the total distance you travel in the 3 hours. Your sponsors donate \$15 for each kilometer you travel. Write an expression that gives the total amount of money you raise. Evaluate the expression if you walk for 2 hours and run for 1 hour.

Test Preparation

QUANTITATIVE COMPARISON In Exercises 62–67, choose the statement that is true about the given quantities.

Ⓐ The quantity in column A is greater.

Ⓑ The quantity in column B is greater.

Ⓒ The two quantities are equal.

Ⓓ The relationship cannot be determined from the given information.

	Column A	Column B
62.	2^6	$(-2)^6$
63.	-4^4	$(-4)^4$
64.	x^4	x^5
65.	$3(x - 2)$ when $x = 4$	$3x - 6$ when $x = 4$
66.	$x + 10(x^2 - 3)$ when $x = 3$	x^6 when $x = 2$
67.	$2(x^2 - 1)$	$2x^2 - 1$

★ Challenge

68. **MATH CLUB SHIRTS** The math club is ordering shirts for its 8 members. The club members have a choice of either a \$15 T-shirt or a \$25 sweatshirt. Make a table showing the total amount of money needed for each possible combination of T-shirts and sweatshirts that the math club can order. Describe any patterns you see. Write an expression that gives the total cost of the shirts. Explain what each term in the expression represents.

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

LEAST COMMON DENOMINATOR Find the least common denominator. (Skills Review, p. 908)

69. $\frac{1}{2}, \frac{3}{4}, \frac{4}{5}$ **70.** $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}$ **71.** $\frac{1}{3}, \frac{2}{5}, -\frac{14}{15}$

72. $\frac{1}{4}, -\frac{3}{8}, \frac{7}{12}$ **73.** $\frac{1}{3}, \frac{4}{5}, \frac{6}{7}$ **74.** $\frac{1}{2}, \frac{3}{4}, -\frac{1}{16}$

USING A NUMBER LINE Graph the numbers on a number line. Then decide which number is greater and use the symbol $<$ or $>$ to show the relationship. (Review 1.1)

75. $-\sqrt{3}, -3$ **76.** $-\frac{1}{2}, -\frac{11}{2}$ **77.** $2.75, \frac{7}{2}$

IDENTIFYING PROPERTIES Identify the property shown. (Review 1.1)

78. $(7 \cdot 9)8 = 7(9 \cdot 8)$ **79.** $-13 + 13 = 0$

80. $27 + 6 = 6 + 27$ **81.** $19 \cdot 1 = 19$

FINDING RECIPROCALS Give the reciprocal of the number. (Review 1.1 for 1.3)

82. -22 **83.** $\frac{7}{8}$ **84.** 12 **85.** $-\frac{5}{4}$

86. $\frac{11}{16}$ **87.** $-\frac{1}{9}$ **88.** 37 **89.** -14

QUIZ 1

Self-Test for Lessons 1.1 and 1.2

Graph the numbers on a number line. Then write the numbers in increasing order. (Lesson 1.1)

1. $\frac{9}{2}, -2.5, 0, -\frac{3}{4}, 1$ **2.** $\frac{10}{3}, 0.8, \frac{15}{8}, -1.5, -0.25$

Identify the property shown. (Lesson 1.1)

3. $5(3 - 7) = 5 \cdot 3 - 5 \cdot 7$ **4.** $(8 + 6) + 4 = 8 + (6 + 4)$

Evaluate the expression for the given value(s) of the variable(s). (Lesson 1.2)

5. $12x - 21$ when $x = 3$ **6.** $7x - (9x + 5)$ when $x = \frac{1}{3}$

7. $x^2 + 5x - 8$ when $x = -3$ **8.** $x^3 + 4(x - 1)$ when $x = 4$

9. $x^2 - 11x + 40y - 14$ when $x = 5$ and $y = -2$

Simplify the expression. (Lesson 1.2)

10. $3x - 2y - 9y + 4 + 5x$ **11.** $3(x - 2) - (4 + x)$

12. $5x^2 - 3x + 8x - 6 - 7x^2$ **13.** $4(x + 2x) - 2(x^2 - x)$

14. **COMPUTER DISKS** You are buying a total of 15 regular floppy disks and high capacity storage disks for your computer. Regular floppy disks cost \$.35 each and high capacity disks cost \$13.95 each. Write an expression for the total amount you spend on computer disks. (Lesson 1.2)

1.3 Solving Linear Equations

GOAL 1 SOLVING A LINEAR EQUATION

What you should learn

GOAL 1 Solve linear equations.

GOAL 2 Use linear equations to solve **real-life** problems, such as finding how much a broker must sell in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems, such as finding the temperature at which dry ice changes to a gas in **Ex. 43**.

REAL LIFE

An **equation** is a statement in which two expressions are equal. A **linear equation** in one variable is an equation that can be written in the form $ax = b$ where a and b are constants and $a \neq 0$. A number is a **solution** of an equation if the statement is true when the number is substituted for the variable.

Two equations are **equivalent** if they have the same solutions. For instance, the equations $x - 4 = 1$ and $x = 5$ are equivalent because both have the number 5 as their only solution. The following *transformations*, or changes, produce equivalent equations and can be used to solve an equation.

TRANSFORMATIONS THAT PRODUCE EQUIVALENT EQUATIONS	
ADDITION PROPERTY OF EQUALITY	*Add* the same number to both sides: If $a = b$, then $a + c = b + c$.
SUBTRACTION PROPERTY OF EQUALITY	*Subtract* the same number from both sides: If $a = b$, then $a - c = b - c$.
MULTIPLICATION PROPERTY OF EQUALITY	*Multiply* both sides by the same nonzero number: If $a = b$ and $c \neq 0$, then $ac = bc$.
DIVISION PROPERTY OF EQUALITY	*Divide* both sides by the same nonzero number: If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.

EXAMPLE 1 *Solving an Equation with a Variable on One Side*

Solve $\frac{3}{7}x + 9 = 15$.

SOLUTION

Your goal is to isolate the variable on one side of the equation.

$\frac{3}{7}x + 9 = 15$ **Write original equation.**

$\frac{3}{7}x = 6$ **Subtract 9 from each side.**

$x = \frac{7}{3}(6)$ **Multiply each side by $\frac{7}{3}$, the reciprocal of $\frac{3}{7}$.**

$x = 14$ **Simplify.**

▶ The solution is 14.

✓**CHECK** Check $x = 14$ in the original equation.

$\frac{3}{7}(14) + 9 \stackrel{?}{=} 15$ **Substitute 14 for *x*.**

$15 = 15$ ✓ **Solution checks.**

EXAMPLE 2 *Solving an Equation with a Variable on Both Sides*

STUDENT HELP

INTERNET

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

Solve $5n + 11 = 7n - 9$.

SOLUTION

$5n + 11 = 7n - 9$	Write original equation.
$11 = 2n - 9$	Subtract $5n$ from each side.
$20 = 2n$	Add 9 to each side.
$10 = n$	Divide each side by 2.

▶ The solution is 10. Check this in the original equation.

EXAMPLE 3 *Using the Distributive Property*

Solve $4(3x - 5) = -2(-x + 8) - 6x$.

SOLUTION

$4(3x - 5) = -2(-x + 8) - 6x$	Write original equation.
$12x - 20 = 2x - 16 - 6x$	Distributive property
$12x - 20 = -4x - 16$	Combine like terms.
$16x - 20 = -16$	Add $4x$ to each side.
$16x = 4$	Add 20 to each side.
$x = \frac{1}{4}$	Divide each side by 16.

▶ The solution is $\frac{1}{4}$. Check this in the original equation.

EXAMPLE 4 *Solving an Equation with Fractions*

STUDENT HELP

Skills Review
For help with finding the LCD, see p. 939.

Solve $\frac{1}{3}x + \frac{1}{4} = x - \frac{1}{6}$.

SOLUTION

$\frac{1}{3}x + \frac{1}{4} = x - \frac{1}{6}$	Write original equation.
$12\left(\frac{1}{3}x + \frac{1}{4}\right) = 12\left(x - \frac{1}{6}\right)$	Multiply each side by the LCD, 12.
$4x + 3 = 12x - 2$	Distributive property
$3 = 8x - 2$	Subtract $4x$ from each side.
$5 = 8x$	Add 2 to each side.
$\frac{5}{8} = x$	Divide each side by 8.

▶ The solution is $\frac{5}{8}$. Check this in the original equation.

FOCUS ON CAREERS

REAL LIFE

REAL ESTATE BROKER

Real estate brokers must have a thorough knowledge not only of the real estate market, but of mathematics as well. Brokers often provide buyers with information about loans, loan rates, and monthly payments.

INTERNET CAREER LINK
www.mcdougallittell.com

GOAL 2 USING LINEAR EQUATIONS IN REAL LIFE

EXAMPLE 5 *Writing and Using a Linear Equation*

REAL ESTATE A real estate broker's base salary is \$18,000. She earns a 4% commission on total sales. How much must she sell to earn \$55,000 total?

SOLUTION

VERBAL MODEL

$$\boxed{\text{Total income}} = \boxed{\text{Base salary}} + \boxed{\text{Commission rate}} \cdot \boxed{\text{Total sales}}$$

LABELS

Total income $= \mathbf{55{,}000}$ (dollars)

Base salary $= \mathbf{18{,}000}$ (dollars)

Commission rate $= \mathbf{0.04}$ (percent in decimal form)

Total sales $= x$ (dollars)

ALGEBRAIC MODEL

$\mathbf{55{,}000} = \mathbf{18{,}000} + \mathbf{0.04}\,x$ — Write linear equation.

$37{,}000 = 0.04x$ — Subtract 18,000 from each side.

$925{,}000 = x$ — Divide each side by 0.04.

▶ The broker must sell real estate worth a total of \$925,000 to earn \$55,000.

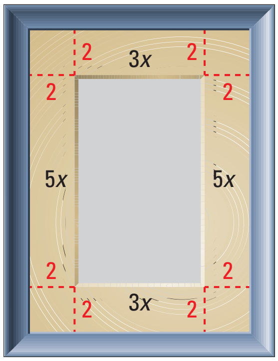
REAL LIFE
Photo Framing

EXAMPLE 6 *Writing and Using a Geometric Formula*

You have a 3 inch by 5 inch photo that you want to enlarge, mat, and frame. You want the width of the mat to be 2 inches on all sides. You want the perimeter of the framed photo to be 44 inches. By what percent should you enlarge the photo?

SOLUTION

Let x be the percent (in decimal form) of enlargement relative to the original photo. So, the dimensions of the enlarged photo (in inches) are $3x$ by $5x$. Draw a diagram.



PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\boxed{\text{Perimeter}} = 2 \cdot \boxed{\text{Width}} + 2 \cdot \boxed{\text{Length}}$$

LABELS

Perimeter $= \mathbf{44}$ (inches)

Width $= \mathbf{4 + 3x}$ (inches)

Length $= \mathbf{4 + 5x}$ (inches)

ALGEBRAIC MODEL

$\mathbf{44} = \mathbf{2}\,(\mathbf{4 + 3x}) + \mathbf{2}\,(\mathbf{4 + 5x})$ — Write linear equation.

$44 = 16 + 16x$ — Distribute and combine like terms.

$28 = 16x$ — Subtract 16 from each side.

$1.75 = x$ — Divide each side by 16.

▶ You should enlarge the photo to 175% of its original size.

Guided Practice

Vocabulary Check ✓

1. What is an equation?

Concept Check ✓

2. What does it mean for two equations to be equivalent? Give an example of two equivalent equations.
3. How does an equation such as $2(x + 3) = 10$ differ from an identity such as $2(x + 3) = 2x + 6$?

ERROR ANALYSIS **Describe the error(s). Then write the correct steps.**

4.

$$\frac{1}{5}x + \frac{1}{6} = -2$$
$$30\left(\frac{1}{5}x + \frac{1}{6}\right) = -2$$
$$6x + 5 = -2$$
$$6x = -7$$
$$x = -\frac{7}{6}$$

5.

$$2(x + 3) = -3(-x + 1)$$
$$2x + 6 = 3x - 3$$
$$5x + 6 = -3$$
$$5x = -9$$
$$x = -\frac{5}{9}$$

6. Describe the transformation(s) you would use to solve $2x - 8 = 14$.

Skill Check ✓

Solve the equation.

7. $x + 4 = 9$ **8.** $4x = 24$ **9.** $2x - 3 = 7$

10. $0.2x - 8 = 0.6$ **11.** $\frac{1}{3}x + \frac{1}{2} = \frac{11}{12}$ **12.** $\frac{3}{4}x - \frac{2}{3} = \frac{5}{6}$

13. $1.5x + 9 = 4.5$ **14.** $6x - 4 = 2x + 10$ **15.** $2(x + 2) = 3(x - 8)$

16. **REAL ESTATE SALES** The real estate broker's base salary from Example 5 has been raised to \$21,000 and the commission rate has been increased to 5%. How much real estate does the broker have to sell now to earn \$70,000?

Practice and Applications

STUDENT HELP

Extra Practice to help you master skills is on p. 940.

DESCRIBING TRANSFORMATIONS **Describe the transformation(s) you would use to solve the equation.**

17. $x + 5 = -7$ **18.** $\frac{1}{6}x = 3$ **19.** $-\frac{4}{7}x = 6$

20. $2x - 9 = 0$ **21.** $\frac{x}{3} + 2 = 89$ **22.** $3 = -x - 5$

SOLVING EQUATIONS **Solve the equation. Check your solution.**

23. $4x + 7 = 27$ **24.** $7s - 29 = -15$

25. $3a + 13 = 9a - 8$ **26.** $m - 30 = 6 - 2m$

27. $15n + 9 = 21$ **28.** $2b + 11 = 15 - 6b$

29. $2(x + 6) = -2(x - 4)$ **30.** $4(-3x + 1) = -10(x - 4) - 14x$

31. $-(x + 2) - 2x = -2(x + 1)$ **32.** $-4(3 + x) + 5 = 4(x + 3)$

STUDENT HELP

HOMEWORK HELP
Examples 1–4: Exs. 17–40
Examples 5, 6: Exs. 43–49

SOLVING EQUATIONS **Solve the equation. Check your solution.**

33. $\frac{7}{2}x - 1 = 2x + 5$

34. $\frac{1}{2}x - \frac{5}{3} = -\frac{1}{2}x + \frac{19}{4}$

35. $\frac{3}{4}\left(\frac{4}{5}x - 2\right) = \frac{11}{4}$

36. $-\frac{2}{3}\left(\frac{6}{5}x - \frac{7}{10}\right) = \frac{17}{20}$

37. $2.7n + 4.3 = 12.94$

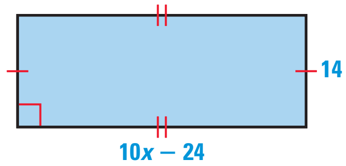
38. $-4.2n - 6.5 = -14.06$

39. $3.1(x + 2) - 1.5x = 5.2(x - 4)$

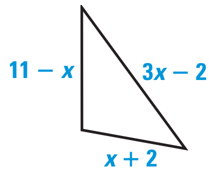
40. $2.5(x - 3) + 1.7x = 10.8(x + 1.5)$

GEOMETRY CONNECTION **Find the dimensions of the figure.**

41. Area = 504



42. Perimeter = 23



In Exercises 43 and 44, use the following formula.

$$\text{degrees Fahrenheit} = \frac{9}{5}(\text{degrees Celsius}) + 32$$

43. **DRY ICE** Dry ice is solid carbon dioxide. Dry ice does not melt — it changes directly from a solid to a gas. Dry ice changes to a gas at −109.3°F. What is this temperature in degrees Celsius?

44. **VETERINARY MEDICINE** The normal body temperature of a dog is 38.6°C. Your dog's temperature is 101.1°F. Does your dog have a fever? Explain.

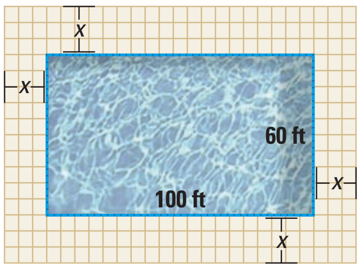
45. **CAR REPAIR** The bill for the repair of your car was \$390. The cost for parts was \$215. The cost for labor was \$35 per hour. How many hours did the repair work take?

46. **SUMMER JOBS** You have two summer jobs. In the first job, you work 28 hours per week and earn \$7.25 per hour. In the second job, you earn \$6.50 per hour and can work as many hours as you want. If you want to earn \$255 per week, how many hours must you work at your second job?

47. **STOCKBROKER** A stockbroker earns a base salary of \$40,000 plus 5% of the total value of the stocks, mutual funds, and other investments that the stockbroker sells. Last year, the stockbroker earned \$71,750. What was the total value of the investments the stockbroker sold?

48. **WORD PROCESSING** You are writing a term paper. You want to include a table that has 5 columns and is 360 points wide. $\left(\text{A point is } \frac{1}{72} \text{ of an inch.}\right)$ You want the first column to be 200 points wide and the remaining columns to be equal in width. How wide should each of the remaining columns be?

49. **WALKWAY CONSTRUCTION** You are building a walkway of uniform width around a 100 foot by 60 foot swimming pool. After completing the walkway, you want to put a fence along the outer edge of the walkway. You have 450 feet of fencing to enclose the walkway. What is the maximum width of the walkway?



FOCUS ON CAREERS

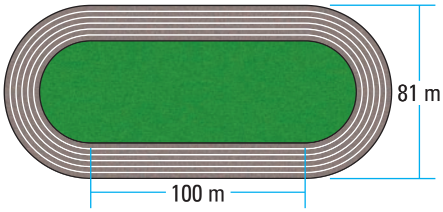
REAL LIFE

STOCKBROKER
Stockbrokers buy and sell stocks, bonds, and other securities for clients as discussed in Ex. 47. Stockbrokers typically study economics in college.

INTERNET **CAREER LINK**
www.mcdougallittell.com

Test Preparation

50. MULTI-STEP PROBLEM You are in charge of constructing a fence around the running track at a high school. The fence is to be built around the track so that there is a uniform gap between the outside edge of the track and the fence.



a. What is the maximum width of the gap between the track and the fence if no more than 630 meters of fencing is used? (*Hint:* Use the equation for the circumference of a circle, $C = 2\pi r$, to help you.)

b. You are charging the school \$10.50 for each meter of fencing. The school has \$5250 in its budget to spend on the fence. How many meters of fencing can you use with this budget?

c. CRITICAL THINKING Explain whether or not it is geometrically reasonable to put up the new fence with the given budget.

★ **Challenge**

SOLVING EQUATIONS Solve the equation. If there is no solution, write *no solution*. If the equation is an identity, write *all real numbers*.

51. $5(x - 4) = 5x + 12$

52. $3(x + 5) = 3x + 15$

53. $7x + 14 - 3x = 4x + 14$

54. $11x - 3 + 2x = 6(x + 4) + 7x$

55. $-2(4 - 3x) + 7 = -2x + 6 + 8x$

56. $5(2 - x) = 3 - 2x + 7 - 3x$

EXTRA CHALLENGE
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MIXED REVIEW

GEOMETRY CONNECTION Find the area of the figure. (Skills Review, p. 914)

57. Circle with radius 5 inches

58. Square with side 4 inches

59. Circle with radius 7 inches

60. Square with side 9 inches

EVALUATING EXPRESSIONS Evaluate the expression. (Review 1.2 for 1.4)

61. $24 - (9 + 7)$

62. $-16 + 3(8 - 4)$

63. $-3 + 6(1 - 3)^2$

64. $2(3 - 5)^3 + 4(-4 + 7)$

65. $2x + 3$ when $x = 4$

66. $8(x - 2) + 3x$ when $x = 6$

67. $5x - 7 + 2x$ when $x = -3$

68. $6x - 3(2x + 4)$ when $x = 5$

SIMPLIFYING EXPRESSIONS Simplify the expression. (Review 1.2)

69. $3(7 + x) - 8x$

70. $2(8 + x) + 2x - x$

71. $4x - (6 - 3x)$

72. $2x - 3(4x + 7)$

73. $3(x + 9) + 2(4 - x)$

74. $-4(x - 3) - 2(x + 7)$

75. $2(x^2 + 2) - x + x^2 + 7$

76. $2(x^2 - 81) - 3x^2$

77. $x^2 - 5x + 3(x^2 + 7x)$

78. $4x^2 - 2(x^2 - 3x) + 6x + 8$

1.4

Rewriting Equations and Formulas

What *you should learn*

GOAL 1 Rewrite equations with more than one variable.

GOAL 2 Rewrite common formulas, as applied in **Example 5**.

Why *you should learn it*

▼ To solve **real-life** problems, such as finding how much you should charge for tickets to a benefit concert in **Example 4**.

REAL LIFE

GOAL 1 EQUATIONS WITH MORE THAN ONE VARIABLE

In Lesson 1.3 you solved equations with one variable. Many equations involve more than one variable. You can solve such an equation for one of its variables.

EXAMPLE 1 *Rewriting an Equation with More Than One Variable*

Solve $7x - 3y = 8$ for y.

SOLUTION

$7x - 3y = 8$	Write original equation.
$-3y = -7x + 8$	Subtract $7x$ from each side.
$y = \frac{7}{3}x - \frac{8}{3}$	Divide each side by -3.

ACTIVITY Developing Concepts

Equations with More Than One Variable

Given the equation $2x + 5y = 4$, use each method below to find y when $x = -3, -1, 2,$ and 6. Tell which method is more efficient.

Method 1 Substitute $x = -3$ into $2x + 5y = 4$ and solve for y. Repeat this process for the other values of x.

Method 2 Solve $2x + 5y = 4$ for y. Then evaluate the resulting expression for y using each of the given values of x.

EXAMPLE 2 *Calculating the Value of a Variable*

Given the equation $x + xy = 1$, find the value of y when $x = -1$ and $x = 3$.

SOLUTION

Solve the equation for y.

$x + xy = 1$	Write original equation.
$xy = 1 - x$	Subtract x from each side.
$y = \frac{1 - x}{x}$	Divide each side by x.

Then calculate the value of y for each value of x.

When $x = -1$: $y = \frac{1 - (-1)}{-1} = -2$ **When $x = 3$:** $y = \frac{1 - 3}{3} = -\frac{2}{3}$

Benefit Concert

EXAMPLE 3 *Writing an Equation with More Than One Variable*

You are organizing a benefit concert. You plan on having only two types of tickets: adult and child. Write an equation with more than one variable that represents the revenue from the concert. How many variables are in your equation?

SOLUTION

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Total revenue = Adult ticket price · Number of adults + Child ticket price · Number of children

LABELS

Total revenue	$= R$	(dollars)
Adult ticket price	$= p_1$	(dollars per adult)
Number of adults	$= A$	(adults)
Child ticket price	$= p_2$	(dollars per child)
Number of children	$= C$	(children)

ALGEBRAIC MODEL

$$R = p_1 A + p_2 C$$

This equation has five variables. The variables p_1 and p_2 are read as "p sub one" and "p sub two." The small lowered numbers 1 and 2 are subscripts used to indicate the two different price variables.

EXAMPLE 4 *Using an Equation with More Than One Variable*

BENEFIT CONCERT For the concert in Example 3, your goal is to sell \$25,000 in tickets. You plan to charge \$25.25 per adult and expect to sell 800 adult tickets. You need to determine what to charge for child tickets. How much should you charge per child if you expect to sell 200 child tickets? 300 child tickets? 400 child tickets?

SOLUTION

First solve the equation $R = p_1A + p_2C$ from Example 3 for p_2.

$R = p_1A + p_2C$ Write original equation.

$R - p_1A = p_2C$ Subtract p_1A from each side.

$\dfrac{R - p_1A}{C} = p_2$ Divide each side by C.

Now substitute the known values of the variables into the equation.

If $C = 200$, the child ticket price is $p_2 = \dfrac{25{,}000 - 25.25(800)}{200} = \$24.$

If $C = 300$, the child ticket price is $p_2 = \dfrac{25{,}000 - 25.25(800)}{300} = \$16.$

If $C = 400$, the child ticket price is $p_2 = \dfrac{25{,}000 - 25.25(800)}{400} = \$12.$

FOCUS ON APPLICATIONS

BENEFIT CONCERT
Farm Aid, a type of benefit concert, began in 1985. Since that time Farm Aid has distributed more than \$13,000,000 to family farms throughout the United States.

GOAL 2 REWRITING COMMON FORMULAS

Throughout this course you will be using many formulas. Several are listed below.

COMMON FORMULAS

	FORMULA	VARIABLES
Distance	$d = rt$	d = distance, r = rate, t = time
Simple Interest	$I = Prt$	I = interest, P = principal, r = rate, t = time
Temperature	$F = \frac{9}{5}C + 32$	F = degrees Fahrenheit, C = degrees Celsius
Area of Triangle	$A = \frac{1}{2}bh$	A = area, b = base, h = height
Area of Rectangle	$A = \ell w$	A = area, ℓ = length, w = width
Perimeter of Rectangle	$P = 2\ell + 2w$	P = perimeter, ℓ = length, w = width
Area of Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	A = area, b_1 = one base, b_2 = other base, h = height
Area of Circle	$A = \pi r^2$	A = area, r = radius
Circumference of Circle	$C = 2\pi r$	C = circumference, r = radius

EXAMPLE 5 *Rewriting a Common Formula*

STUDENT HELP

Skills Review
For help with perimeter, see p. 914.

The formula for the perimeter of a rectangle is $P = 2\ell + 2w$. Solve for w.

SOLUTION

$P = 2\ell + 2w$ — **Write perimeter formula.**

$P - 2\ell = 2w$ — **Subtract 2ℓ from each side.**

$\frac{P - 2\ell}{2} = w$ — **Divide each side by 2.**

EXAMPLE 6 *Applying a Common Formula*

REAL LIFE
Gardening

You have 40 feet of fencing with which to enclose a rectangular garden. Express the garden's area in terms of its length only.

SOLUTION

Use the formula for the area of a rectangle, $A = \ell w$, and the result of Example 5.

$A = \ell w$ — **Write area formula.**

$A = \ell\left(\frac{P - 2\ell}{2}\right)$ — **Substitute $\frac{P - 2\ell}{2}$ for w.**

$A = \ell\left(\frac{40 - 2\ell}{2}\right)$ — **Substitute 40 for P.**

$A = \ell(20 - \ell)$ — **Simplify.**

GUIDED PRACTICE

Vocabulary Check ✓

1. Complete this statement: $A = \ell w$ is an example of a(n) __?__.

Concept Check ✓

2. Which of the following are equations with more than one variable?

A. $2x + 5 = 9 - 5x$ **B.** $4x + 10y = 62$ **C.** $x - 8 = 3y + 7$

3. Use the equation from Example 3. Describe how you would solve for A.

Skill Check ✓

Solve the equation for *y*.

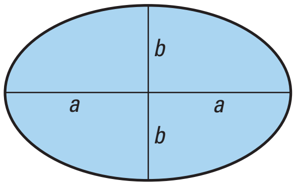
4. $4x + 8y = 17$ **5.** $5x - 3y = 9$ **6.** $5y - 3x = 15$

7. $\frac{3}{4}x + 5y = 20$ **8.** $xy + 2x = 8$ **9.** $\frac{2}{3}x - \frac{1}{2}y = 12$

In Exercises 10 and 11, use the following information.
The area A of an ellipse is given by the formula $A = \pi ab$ where a and b are half the lengths of the major and minor axes. (The longer chord is the major axis.)

10. Solve the formula for a.

11. Use the result from Exercise 10 to find the length of the major axis of an ellipse whose area is 157 square inches and whose minor axis is 10 inches long. (Use 3.14 for π.)



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 940.

EXPLORING METHODS **Find the value of *y* for the given value of *x* using two methods. First, substitute the value of *x* into the equation and then solve for *y*. Second, solve for *y* and then substitute the value of *x* into the equation.**

12. $4x + 9y = 30;\ x = 3$ **13.** $5x - 7y = 12;\ x = 1$

14. $xy + 3x = 25;\ x = 5$ **15.** $9y - 4x = -16;\ x = 8$

16. $-y - 2x = -11;\ x = -4$ **17.** $-x = 3y - 55;\ x = 20$

18. $x = 24 + xy;\ x = -12$ **19.** $-xy + 3x = 30;\ x = 15$

20. $-4x + 7y + 7 = 0;\ x = 7$ **21.** $6x - 5y - 44 = 0;\ x = 4$

22. $\frac{1}{2}x - \frac{4}{5}y = 19;\ x = 6$ **23.** $\frac{3}{4}x = -\frac{9}{11}y + 12;\ x = 10$

REWRITING FORMULAS **Solve the formula for the indicated variable.**

24. Circumference of a Circle
Solve for r: $C = 2\pi r$

25. Volume of a Cone
Solve for h: $V = \frac{1}{3}\pi r^2 h$

26. Area of a Triangle
Solve for b: $A = \frac{1}{2}bh$

27. Investment at Simple Interest
Solve for P: $I = Prt$

28. Celsius to Fahrenheit
Solve for C: $F = \frac{9}{5}C + 32$

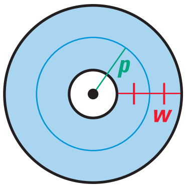
29. Area of a Trapezoid
Solve for b_2: $A = \frac{1}{2}(b_1 + b_2)h$

STUDENT HELP

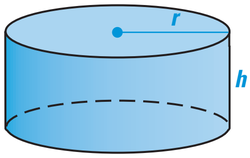
HOMEWORK HELP
Examples 1, 2: Exs. 12–23
Examples 3, 4: Exs. 33–39
Examples 5, 6: Exs. 24–32, 40–42

GEOMETRY CONNECTION **In Exercises 30–32, solve the formula for the indicated variable. Then evaluate the rewritten formula for the given values. (Include units of measure in your answer.)**

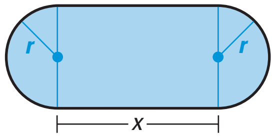
30. Area of a circular ring: $A = 2\pi pw$ Solve for p. Find p when $A = 22\text{ cm}^2$ and $w = 2$ cm.



31. Surface area of a cylinder: $S = 2\pi rh + 2\pi r^2$ Solve for h. Find h when $S = 105\text{ in.}^2$ and $r = 3$ in.



32. Perimeter of a track: $P = 2\pi r + 2x$ Solve for r. Find r when $P = 440$ yd and $x = 110$ yd.



HONEYBEES **In Exercises 33 and 34, use the following information.**
A forager honeybee spends about three weeks becoming accustomed to the immediate surroundings of its hive and spends the rest of its life collecting pollen and nectar. The total number of miles T a forager honeybee flies in its lifetime L (in days) can be modeled by $T = m(L - 21)$ where m is the number of miles it flies each day.

33. Solve the equation $T = m(L - 21)$ for L.

34. A forager honeybee's flight muscles last only about 500 miles; after that the bee dies. Some forager honeybees fly about 55 miles per day. Approximately how many days do these bees live?

BASEBALL **In Exercises 35 and 36, use the following information.**
The Pythagorean Theorem of Baseball is a formula for approximating a team's ratio of wins to games played. Let R be the number of runs the team scores during the season, A be the number of runs allowed to opponents, W be the number of wins, and T be the total number of games played. Then the formula

$$\frac{W}{T} \approx \frac{R^2}{R^2 + A^2}$$

approximates the team's ratio of wins to games played. ▶ Source: *Inside Sports*

35. Solve the formula for W.

36. The 1998 New York Yankees scored 965 runs and allowed 656. How many of its 162 games would you estimate the team won?

FOCUS ON CAREERS

SPORTS STATISTICIANS are employed by many professional sports teams, leagues, and news organizations. They collect and analyze team and individual data on items such as scoring.

CAREER LINK
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FUNDRAISER **In Exercises 37–39, use the following information.**
Your tennis team is having a fundraiser. You are going to help raise money by selling sun visors and baseball caps.

37. Write an equation that represents the total amount of money you raise.

38. How many variables are in the equation? What does each represent?

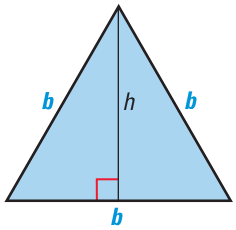
39. Your team raises a total of $4480. Give three possible combinations of sun visors and baseball caps that could have been sold if the price of a sun visor is $3.00 and the price of a baseball cap is $7.00.

40. **GEOMETRY CONNECTION** The formula for the area of a circle is $A = \pi r^2$. The formula for the circumference of a circle is $C = 2\pi r$. Write a formula for the area of a circle in terms of its circumference.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 41 and 42.

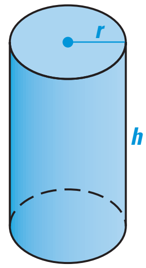
41. **GEOMETRY CONNECTION** The formula for the height h of an equilateral triangle is $h = \frac{\sqrt{3}}{2}b$ where b is the length of a side. Write a formula for the area of an equilateral triangle in terms of the following.



a. the length of a side only

b. the height only

42. **GEOMETRY CONNECTION** The surface area S of a cylinder is given by the formula $S = 2\pi rh + 2\pi r^2$. The height h of the cylinder shown at the right is 5 more than 3 times its radius r.



a. Write a formula for the surface area of the cylinder in terms of its radius.

b. Find the surface area of the cylinder for $r = 3, 4$, and 6.

Test Preparation

QUANTITATIVE COMPARISON **In Exercises 43 and 44, choose the statement that is true about the given quantities.**

Ⓐ The quantity in column A is greater.

Ⓑ The quantity in column B is greater.

Ⓒ The two quantities are equal.

Ⓓ The relationship cannot be determined from the given information.

STUDENT HELP

Skills Review
For help with the Pythagorean theorem, see p. 917.

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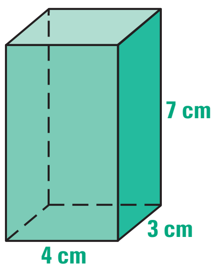
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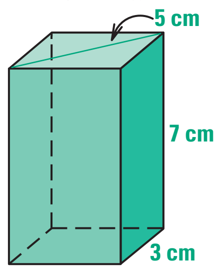
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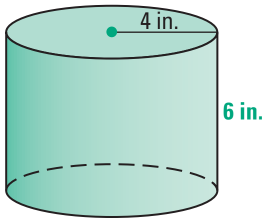
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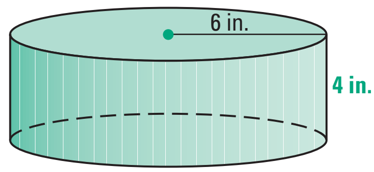
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★ **Challenge**

45. **FUEL EFFICIENCY** The more aerodynamic a vehicle is, the less fuel the vehicle's engine must use to overcome air resistance. To design vehicles that are as fuel efficient as possible, automotive engineers use the formula

$$R = 0.00256 \times D_C \times F_A \times s^2$$

where R is the air resistance (in pounds), D_C is the drag coefficient, F_A is the frontal area of the vehicle (in square feet), and s is the speed of the vehicle (in miles per hour). The formula assumes that there is no wind.

a. Rewrite the formula to find the drag coefficient in terms of the other variables.

b. Find the drag coefficient of a car when the air resistance is 50 pounds, the frontal area is 25 square feet, and the speed of the car is 45 miles per hour.

EXTRA CHALLENGE
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MIXED REVIEW

WRITING EXPRESSIONS **Write an expression to answer the question.** (Skills Review, p. 929)

46. You buy x birthday cards for \$1.85 each. How much do you spend?

47. You have \$30 and spend x dollars. How much money do you have left?

48. You drive 55 miles per hour for x hours. How many miles do you drive?

49. You have \$250 in your bank account and you deposit x dollars. How much money do you now have in your account?

50. You spend \$42 on x music cassettes. How much does each cassette cost?

51. A certain ball bearing weighs 2 ounces. A box contains x ball bearings. What is the total weight of the ball bearings?

UNIT ANALYSIS **Give the answer with the appropriate unit of measure.** (Review 1.1)

52. $\left(\frac{7 \text{ meters}}{1 \text{ minute}}\right)(60 \text{ minutes})$

53. $\left(\frac{168 \text{ hours}}{1 \text{ week}}\right)(52 \text{ weeks})$

54. $4\frac{1}{4}$ feet $+ 7\frac{3}{4}$ feet

55. $13\frac{1}{4}$ liters $- 8\frac{7}{8}$ liters

56. $\left(\frac{3 \text{ yards}}{1 \text{ second}}\right)(12 \text{ seconds}) - 10 \text{ yards}$

57. $\left(\frac{15 \text{ dollars}}{1 \text{ hour}}\right)(8 \text{ hours}) + 45 \text{ dollars}$

SOLVING EQUATIONS **Solve the equation. Check your solution.** (Review 1.3)

58. $3d + 16 = d - 4$

59. $5 - x = 23 + 2x$

60. $10(y - 1) = y + 4$

61. $p - 16 + 4 = 4(2 - p)$

62. $-10x = 5x + 5$

63. $12z = 4z - 56$

64. $\frac{2}{3}x - 7 = 1$

65. $-\frac{3}{4}x + 19 = -11$

66. $\frac{1}{4}x + \frac{3}{8} = \frac{1}{5} - \frac{1}{5}x$

67. $\frac{5}{4}x - \frac{3}{4} = \frac{5}{6}x + \frac{1}{2}$

EXPLORING DATA AND STATISTICS

1.5

Problem Solving Using Algebraic Models

What you should learn

GOAL 1 Use a general problem solving plan to solve **real-life** problems, as in **Example 2**.

GOAL 2 Use other problem solving strategies to help solve **real-life** problems, as in **Ex. 22**.

Why you should learn it

▼ To solve **real-life** problems, such as finding the average speed of the Japanese Bullet Train in **Example 1**.

REAL LIFE

GOAL 1 USING A PROBLEM SOLVING PLAN

One of your major goals in this course is to learn how to use algebra to solve real-life problems. You have solved simple problems in previous lessons, and this lesson will provide you with more experience in problem solving.

As you have seen, it is helpful when solving real-life problems to first write an equation in words *before* you write it in mathematical symbols. This word equation is called a **verbal model**. The verbal model is then used to write a mathematical statement, which is called an **algebraic model**. The key steps in this problem solving plan are shown below.

Write a verbal model. → **Assign labels.** → **Write an algebraic model.** → **Solve the algebraic model.** → **Answer the question.**

EXAMPLE 1 *Writing and Using a Formula*

The Bullet Train runs between the Japanese cities of Osaka and Fukuoka, a distance of 550 kilometers. When it makes no stops, it takes 2 hours and 15 minutes to make the trip. What is the average speed of the Bullet Train?

SOLUTION

You can use the formula $d = rt$ to write a verbal model.

VERBAL MODEL: $\boxed{\text{Distance}} = \boxed{\text{Rate}} \cdot \boxed{\text{Time}}$

LABELS:

Distance = **550** (kilometers)

Rate = r (kilometers per hour)

Time = **2.25** (hours)

ALGEBRAIC MODEL:

$550 = r(2.25)$ **Write algebraic model.**

$\frac{550}{2.25} = r$ **Divide each side by 2.25.**

$244 \approx r$ **Use a calculator.**

▶ The Bullet Train's average speed is about 244 kilometers per hour.

UNIT ANALYSIS You can use unit analysis to check your verbal model.

$$550 \text{ kilometers} \approx \frac{244 \text{ kilometers}}{\cancel{\text{hour}}} \cdot 2.25 \cancel{\text{hours}}$$

REAL LIFE

Water Conservation

EXAMPLE 2 *Writing and Using a Simple Model*

A water-saving faucet has a flow rate of at most 9.6 cubic inches per second. To test whether your faucet meets this standard, you time how long it takes the faucet to fill a 470 cubic inch pot, obtaining a time of 35 seconds. Find your faucet's flow rate. Does it meet the standard for water conservation?

SOLUTION

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\boxed{\text{Volume of pot}} = \boxed{\text{Flow rate of faucet}} \cdot \boxed{\text{Time to fill pot}}$$

LABELS

Volume of pot $= \mathbf{470}$ (cubic inches)

Flow rate of faucet $= \boxed{r}$ (cubic inches per second)

Time to fill pot $= \mathbf{35}$ (seconds)

ALGEBRAIC MODEL

$470 = r(35)$ Write algebraic model.

$13.4 \approx r$ Divide each side by 35.

▶ The flow rate is about 13.4 in.^{3}/sec, which does not meet the standard.

REAL LIFE

Gasoline Cost

EXAMPLE 3 *Writing and Using a Model*

You own a lawn care business. You want to know how much money you spend on gasoline to travel to out-of-town clients. In a typical week you drive 600 miles and use 40 gallons of gasoline. Gasoline costs $1.25 per gallon, and your truck's fuel efficiency is 21 miles per gallon on the highway and 13 miles per gallon in town.

SOLUTION

VERBAL MODEL

$$\boxed{\text{Total miles}} = \overbrace{\boxed{\text{Fuel efficiency}} \cdot \boxed{\text{Amount of gasoline}}}^{\text{highway miles}} + \overbrace{\boxed{\text{Fuel efficiency}} \cdot \boxed{\text{Amount of gasoline}}}^{\text{local miles}}$$

LABELS

Total miles $= \mathbf{600}$ (miles)

Fuel efficiency (highway) $= \mathbf{21}$ (miles per gallon)

Amount of gasoline (highway) $= \boxed{x}$ (gallons)

Fuel efficiency (local) $= \mathbf{13}$ (miles per gallon)

Amount of gasoline (local) $= \boxed{40 - x}$ (gallons)

ALGEBRAIC MODEL

$600 = 21x + 13(40 - x)$ Write algebraic model.

$600 = 8x + 520$ Simplify.

$80 = 8x$ Subtract 520 from each side.

$10 = x$ Divide each side by 8.

▶ In a typical week you use 10 gallons of gasoline to travel to out-of-town clients. The cost of the gasoline is (10 gallons)($1.25 per gallon) = $12.50.

STUDENT HELP

Study Tip

The solutions of the equations in Examples 2 and 3 are 13.4 and 10, respectively. However, these are not the answers to the questions asked. In Example 2 you must compare 13.4 to 9.6, and in Example 3 you must multiply 10 by $1.25. Be certain to answer the question asked.

FOCUS ON APPLICATIONS

RAILROADS In 1862, two companies were given the rights to build a railroad from Omaha, Nebraska to Sacramento, California. The Central Pacific Company began from Sacramento in 1863. Twenty-four months later, the Union Pacific Company began from Omaha.

INTERNET APPLICATION LINK
www.mcdougallittell.com

GOAL 2 USING OTHER PROBLEM SOLVING STRATEGIES

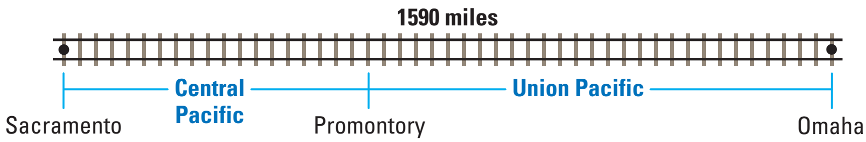
When you are writing a verbal model to represent a real-life problem, remember that you can use other problem solving strategies, such as *draw a diagram, look for a pattern*, or *guess, check, and revise,* to help create the verbal model.

EXAMPLE 4 *Drawing a Diagram*

RAILROADS Use the information under the photo at the left. The Central Pacific Company averaged 8.75 miles of track per month. The Union Pacific Company averaged 20 miles of track per month. The photo shows the two companies meeting in Promontory, Utah, as the 1590 miles of track were completed. When was the photo taken? How many miles of track did each company build?

SOLUTION

Begin by drawing and labeling a diagram, as shown below.



VERBAL MODEL

$$\boxed{\text{Total miles of track}} = \underbrace{\boxed{\text{Miles per month}} \cdot \boxed{\text{Number of months}}}_{\text{Central Pacific}} + \underbrace{\boxed{\text{Miles per month}} \cdot \boxed{\text{Number of months}}}_{\text{Union Pacific}}$$

LABELS

Total miles of track	$= \mathbf{1590}$	(miles)
Central Pacific rate	$= \mathbf{8.75}$	(miles per month)
Central Pacific time	$= \boldsymbol{t}$	(months)
Union Pacific rate	$= \mathbf{20}$	(miles per month)
Union Pacific time	$= \boldsymbol{t - 24}$	(months)

ALGEBRAIC MODEL

$\mathbf{1590 = 8.75\,t + 20\,(t - 24)}$	Write algebraic model.
$1590 = 8.75t + 20t - 480$	Distributive property
$2070 = 28.75t$	Simplify.
$72 = t$	Divide each side by 28.75.

▶ The construction took 72 months (6 years) from the time the Central Pacific Company began in 1863. So, the photo was taken in 1869. The number of miles of track built by each company is as follows.

Central Pacific: $\dfrac{8.75 \text{ miles}}{\cancel{\text{month}}} \cdot 72 \cancel{\text{ months}} = 630 \text{ miles}$

Union Pacific: $\dfrac{20 \text{ miles}}{\cancel{\text{month}}} \cdot (72 - 24) \cancel{\text{ months}} = 960 \text{ miles}$

STUDENT HELP

Skills Review
For help with additional problem solving strategies, see p. 930.

REAL LIFE Heights

EXAMPLE 5 *Looking for a Pattern*

The table gives the heights to the top of the first few stories of a tall building. Determine the height to the top of the 15th story.

Story	Lobby	1	2	3	4
Height to top of story (feet)	20	32	44	56	68

SOLUTION

Look at the differences in the heights given in the table. After the lobby, the height increases by 12 feet per story.

Heights: 20 $\xrightarrow{+12}$ 32 $\xrightarrow{+12}$ 44 $\xrightarrow{+12}$ 56 $\xrightarrow{+12}$ 68

You can use the observed pattern to write a model for the height.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Height to top of a story = Height of lobby + Height per story · Story number

LABELS

Height to top of a story = h (feet)

Height of lobby = **20** (feet)

Height per story = **12** (feet per story)

Story number = n (stories)

ALGEBRAIC MODEL

$h = \mathbf{20} + \mathbf{12}n$ **Write algebraic model.**

$= 20 + 12(15)$ **Substitute 15 for n.**

$= 200$ **Simplify.**

▶ The height to the top of the 15th story is 200 feet.

FOCUS ON APPLICATIONS

REAL LIFE **WEATHER BALLOONS** Hundreds of weather balloons are launched daily from weather stations. The balloons typically carry about 40 pounds of instruments. Balloons usually reach an altitude of about 90,000 feet.

EXAMPLE 6 *Guess, Check, and Revise*

WEATHER BALLOONS A spherical weather balloon needs to hold 175 cubic feet of helium to be buoyant enough to lift an instrument package to a desired height. To the nearest tenth of an foot, what is the radius of the balloon?

SOLUTION

Use the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$.

$175 = \frac{4}{3}\pi r^3$ **Substitute 175 for V.**

$42 \approx r^3$ **Divide each side by $\frac{4}{3}\pi$.**

You need to find a number whose cube is 42. As a first guess, try $r = 4$. This gives $4^3 = 64$. Because $64 > 42$, your guess of 4 is too high. As a second guess, try $r = 3.5$. This gives $(3.5)^3 = 42.875$, and $42.875 \approx 42$. So, the balloon's radius is about 3.5 feet.

GUIDED PRACTICE

Vocabulary Check ✓

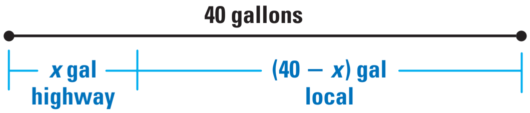
Concept Check ✓

Skill Check ✓

1. What is a verbal model? What is it used for?

2. Describe the steps of the problem solving plan.

3. How does this diagram help you set up the algebraic model in Example 3?



SCIENCE CONNECTION **In Exercises 4–7, use the following information.**
To study life in Arctic waters, scientists worked in an underwater building called a *Sub-Igloo* in Resolute Bay, Canada. The water pressure at the floor of the Sub-Igloo was 2184 pounds per square foot. Water pressure is zero at the water's surface and increases by 62.4 pounds per square foot for each foot of depth.

4. Write a verbal model for the water pressure.

5. Assign labels to the parts of the verbal model. Indicate the units of measure.

6. Use the labels to translate the verbal model into an algebraic model.

7. Solve the algebraic model to find the depth of the Sub-Igloo's floor.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 940.

BOAT TRIP **In Exercises 8–11, use the following information.**
You are on a boat on the Seine River in France. The boat's speed is 32 kilometers per hour. The Seine has a length of 764 kilometers, but only 547 kilometers can be navigated by boats. How long will your boat ride take if you travel the entire navigable portion of the Seine? Use the following verbal model.

$$\boxed{\text{Distance}} = \boxed{\text{Rate}} \cdot \boxed{\text{Time}}$$

8. Assign labels to the parts of the verbal model.

9. Use the labels to translate the verbal model into an algebraic model.

10. Solve the algebraic model.

11. Answer the question.

MUSIC **In Exercises 12–14, use the following information.**
A *metronome* is a device similar to a clock and is used to maintain the tempo of a musical piece. Suppose one particular piece has 180 measures with 3 beats per measure and a metronome marking of 80 beats per minute. Determine the length (in minutes) of the musical piece by using the following verbal model.

$$\boxed{\begin{array}{c}\text{Metronome}\\\text{marking}\end{array}} \cdot \boxed{\begin{array}{c}\text{Length of}\\\text{musical piece}\end{array}} = \boxed{\begin{array}{c}\text{Number of measures}\\\text{in musical piece}\end{array}} \cdot \boxed{\begin{array}{c}\text{Number of beats}\\\text{per measure}\end{array}}$$

12. Assign labels to the parts of the verbal model.

13. Use the labels to translate the verbal model into an algebraic model.

14. Answer the question. Use unit analysis to check your answer.

STUDENT HELP

HOMEWORK HELP
Examples 1–3: Exs. 8–17
Examples 4–6: Exs. 18–27

FOCUS ON APPLICATIONS

REAL LIFE **THE CHUNNEL** A tunnel under the English Channel was an engineering possibility for over a century before its completion. High speed trains traveling up to 300 kilometers per hour link London, England to Paris, France and Brussels, Belgium.

INTERNET APPLICATION LINK www.mcdougallittell.com

CALORIE INTAKE **In Exercises 15–17, use the following information.** To determine the total number of calories of a food, you must add the number of calories provided by the grams of fat, the grams of protein, and the grams of carbohydrates. There are 9 Calories per gram of fat. A gram of protein and a gram of carbohydrates each have about 4 Calories. ▶ Source: U.S. Department of Agriculture

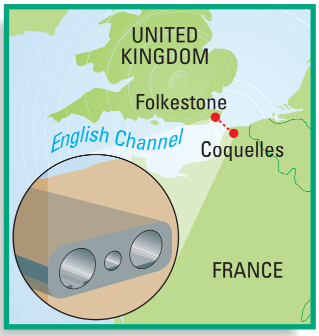
15. Write a verbal model that gives the total number of calories of a certain food.

16. Assign labels to the parts of the verbal model. Use the labels to translate the verbal model into an algebraic model.

17. One cup of raisins has 529.9 Calories and contains 0.3 gram of fat and 127.7 grams of carbohydrates. Solve the algebraic model to find the number of grams of protein in the raisins. Use unit analysis to check your answer.

18. **BORROWING MONEY** You have borrowed $529 from your parents to buy a mountain bike. Your parents are not charging you interest, but they want to be repaid as soon as possible. You can afford to repay them $20 per week. How long will it take you to repay your parents?

19. **THE CHUNNEL** The Chunnel connects the United Kingdom and France by a railway tunnel under the English Channel. The British started tunneling 2.5 months before the French and averaged 0.63 kilometer per month. The French averaged 0.47 kilometer per month. When the two sides met, they had tunneled 37.9 kilometers. How many kilometers of tunnel did each country build? If the French started tunneling on February 28, 1988, approximately when did the two sides meet?



20. **FLYING LESSONS** You are taking flying lessons to get a private pilot's license. The cost of the introductory lesson is $\frac{5}{8}$ the cost of each additional lesson, which is $80. You have a total of $375 to spend on the flying lessons. How many lessons can you afford? How much money will you have left?

21. **TYPING PAPERS** Some of your classmates ask you to type their history papers throughout a 7 week summer course. How much should you charge per page if you want to earn enough to pay for the flying lessons in Exercise 20 and have $75 left over for spending money? You estimate that you can type 40 pages per week. Assume that you have to take 9 flying lessons plus the introductory lesson and that you already have $375 to spend on the lessons.

22. **WOODSHOP** You are working on a project in woodshop. You have a wooden rod that is 72 inches long. You need to cut the rod so that one piece is 6 inches longer than the other piece. How long should each piece be?

23. **GARDENING** You have 480 feet of fencing to enclose a rectangular garden. You want the length of the garden to be 30 feet greater than the width. Find the length and width of the garden if you use all of the fencing.

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with problem solving in Ex. 24.

24. **WINDOW DISPLAYS** You are creating a window display at a toy store using wooden blocks. The display involves stacking blocks in triangular forms. You begin the display with 1 block, which is your first "triangle," and then stack 3 blocks, two on the bottom and one on the top, to get the next triangle. You create the next three triangles by stacking 6 blocks, then 10 blocks, and then 15 blocks. How many blocks will you need for the ninth triangle?

SCIENCE CONNECTION **In Exercises 25–27, use the following information.** As part of a science experiment, you drop a ball from various heights and measure how high it bounces on the first bounce. The results of six drops are given below.

Drop height (m)	0.5	1.5	2	2.5	4	5
First bounce height (m)	0.38	1.15	1.44	1.90	2.88	3.85

25. How high will the ball bounce if you drop it from a height of 6 meters?

26. To continue the experiment, you must find the number of bounces the ball will make before it bounces less than a given number of meters. Your experiment shows the ball's bounce height is always the same percent of the height from which it fell before the bounce. Find the average percent that the ball bounces each time.

27. Find the number of times the ball bounces before it bounces less than 1 meter if it is dropped from a height of 3 meters.

Test Preparation

28. **MULTIPLE CHOICE** You work at a clothing store earning \$7.50 per hour. At the end of the year, you figure out that on a weekly basis you averaged 5 hours of overtime for which you were paid time and a half. How much did you make for the entire year? (Assume that a regular workweek is 40 hours.)

(A) \$15,600 (B) \$17,550 (C) \$18,000 (D) \$18,525 (E) \$19,500

29. **MULTIPLE CHOICE** You are taking piano lessons. The cost of the first lesson is one and one half times the cost of each additional lesson. You spend \$260 for six lessons. How much did the first lesson cost?

(A) \$52 (B) \$40 (C) \$43.33 (D) \$60 (E) \$34.67

★ Challenge

30. **OWNING A BUSINESS** You have started a business making papier-mâché sculptures. The cost to make a sculpture is \$.75. Your sculptures sell for \$14.50 each at a craft store. You receive 50% of the selling price. Each sculpture takes about 2 hours to complete. If you spend 14 hours per week making sculptures, about how many weeks will you work to earn a profit of \$360?

EXTRA CHALLENGE
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MIXED REVIEW

LOGICAL REASONING **Tell whether the compound statement is *true* or *false*.** (Skills Review, p. 924)

31. $-3 < 5$ and $-3 > -5$

32. $1 > -2$ or $1 < -2$

33. $-4 > -5$ and $1 < -2$

34. $-2.7 > -2.5$ or $156 > 165$

ORDERING NUMBERS **Write the numbers in increasing order.** (Review 1.1)

35. $-1, -5, 4, -10, -55$

36. $-\frac{2}{3}, \frac{5}{8}, \frac{1}{100}, -2, 1$

37. $-1.2, 2, -2.9, 2.09, -2.1$

38. $-\sqrt{3}, 1, \sqrt{10}, \sqrt{2}, \frac{8}{5}$

SOLVING EQUATIONS **Solve the equation. Check your solution.** (Review 1.3 for 1.6)

39. $6x + 5 = 17$

40. $5x - 4 = 7x + 12$

41. $2(3x - 1) = 5 - (x + 3)$

42. $\frac{2}{3}x + \frac{1}{4} = 2x - \frac{5}{6}$

QUIZ 2

Self-Test for Lessons 1.3–1.5

Solve the equation. Check your solution. (Lesson 1.3)

1. $5x - 9 = 11$

2. $6y + 8 = 3y - 16$

3. $\frac{1}{4}z + \frac{2}{3} = \frac{1}{2}z - \frac{3}{4}$

4. $0.4(x - 50) = 0.2x + 12$

Solve the equation for y. Then find the value of y when $x = 2$. (Lesson 1.4)

5. $3x + 5y = 9$

6. $4x - 3y = 14$

7. The formula for the area of a rhombus is $A = \frac{1}{2}d_1d_2$ where d_1 and d_2 are the lengths of the diagonals. Solve the formula for d_1. (Lesson 1.4)

8. **GIRL SCOUT COOKIES** Your sister is selling Girl Scout cookies that cost \$2.80 per box. Your family bought 6 boxes. How many more boxes of cookies must your sister sell in order to collect \$154? (Lesson 1.5)

MATH & History

Problem Solving

APPLICATION LINK
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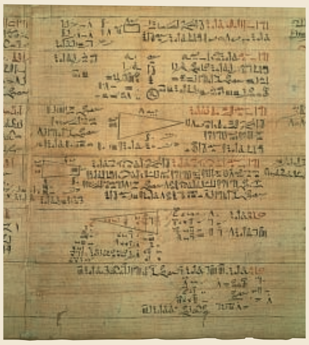
THEN

MANY CULTURES, such as the Egyptians, Greeks, Hindus, and Arabs, solved problems by using the *rule of false position*. This technique was similar to the problem solving strategy of guess, check, and revise. As an example of how to use the rule of false position, consider this problem taken from the Ahmes papyrus:

You want to divide 700 loaves of bread among four people in the ratio $\frac{2}{3}:\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. Choose a number divisible by the denominators 2, 3, and 4, such as 48. Then evaluate $\frac{2}{3}(48) + \frac{1}{2}(48) + \frac{1}{3}(48) + \frac{1}{4}(48)$, which has a value of 84.



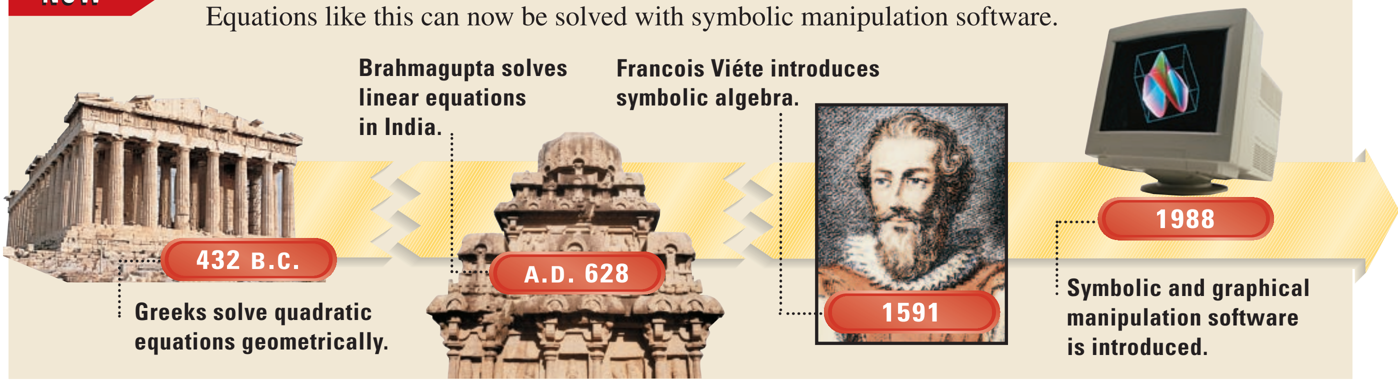
Ahmes papyrus

1. The next step is to multiply 48 by a number so that when the resulting product is substituted for 48 in the expression above, you get a new expression whose value is 700. By what number should you multiply 48? How did you use the original expression's value of 84 to get your answer?

2. Use your result from Exercise 1 to find the number of loaves for each person.

NOW

TODAY, we would model this problem using $\frac{2}{3}x + \frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x = 700$. Equations like this can now be solved with symbolic manipulation software.



1.6 Solving Linear Inequalities

GOAL 1 SOLVING SIMPLE INEQUALITIES

What you should learn

GOAL 1 Solve simple inequalities.

GOAL 2 Solve compound inequalities, as applied in **Example 6**.

Why you should learn it

▼ To model **real-life** situations, such as amusement park fees in **Ex. 50**.

REAL LIFE

Inequalities have properties that are similar to those of equations, but the properties differ in some important ways.

ACTIVITY Developing Concepts

Investigating Properties of Inequalities

1. Write two true inequalities involving integers, one using $<$ and one using $>$.
2. Add, subtract, multiply, and divide each side of your inequalities by 2 and -2. In each case, decide whether the new inequality is true or false.
3. Write a general conclusion about the operations you can perform on a true inequality to produce another true inequality.

Inequalities such as $x \le 1$ and $2n - 3 > 9$ are examples of **linear inequalities** in one variable. A **solution** of an inequality in one variable is a value of the variable that makes the inequality true. For instance, -2, 0, 0.872, and 1 are some of the many solutions of $x \le 1$.

In the activity you may have discovered some of the following properties of inequalities. You can use these properties to solve an inequality because each transformation produces a new inequality having the same solutions as the original.

TRANSFORMATIONS THAT PRODUCE EQUIVALENT INEQUALITIES

- *Add* the same number to both sides.
- *Subtract* the same number from both sides.
- *Multiply* both sides by the same *positive* number.
- *Divide* both sides by the same *positive* number.
- *Multiply* both sides by the same *negative* number and *reverse* the inequality.
- *Divide* both sides by the same *negative* number and *reverse* the inequality.

The **graph** of an inequality in one variable consists of all points on a real number line that correspond to solutions of the inequality. To graph an inequality in one variable, use an open dot for $<$ or $>$ and a solid dot for $\le$ or $\ge$. For example, the graphs of $x < 3$ and $x \ge -2$ are shown below.

Graph of $x < 3$

Graph of $x \ge -2$

EXAMPLE 1 *Solving an Inequality with a Variable on One Side*

Solve $5y - 8 < 12$.

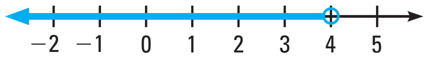
SOLUTION

$5y - 8 < 12$ **Write original inequality.**

$5y < 20$ **Add 8 to each side.**

$y < 4$ **Divide each side by 5.**

▶ The solutions are all real numbers less than 4, as shown in the graph at the right.



✓**CHECK** As a check, try several numbers that are less than 4 in the original inequality. Also, try checking some numbers that are greater than or equal to 4 to see that they are *not* solutions of the original inequality.

EXAMPLE 2 *Solving an Inequality with a Variable on Both Sides*

> **STUDENT HELP**
>
> **Study Tip**
> Don't forget that when you multiply or divide both sides of an inequality by a negative number, you must *reverse* the inequality to maintain a true statement. For instance, to reverse $\le$, replace it with $\ge$.

Solve $2x + 1 \le 6x - 1$.

SOLUTION

$2x + 1 \le 6x - 1$ **Write original inequality.**

$-4x + 1 \le -1$ **Subtract $6x$ from each side.**

$-4x \le -2$ **Subtract 1 from each side.**

$x \ge \frac{1}{2}$ **Divide each side by −4 and reverse the inequality.**

▶ The solutions are all real numbers greater than or equal to $\frac{1}{2}$. Check several numbers greater than or equal to $\frac{1}{2}$ in the original inequality.

REAL LIFE
Fish

EXAMPLE 3 *Using a Simple Inequality*

The weight w (in pounds) of an Icelandic saithe is given by

$$w = 10.4t - 2.2$$

where t is the age of the fish in years. Describe the ages of a group of Icelandic saithe that weigh up to 29 pounds. ▶ Source: Marine Research Institute

Icelandic saithe

SOLUTION

$w \le 29$ **Weights are at most 29 pounds.**

$10.4t - 2.2 \le 29$ **Substitute for w.**

$10.4t \le 31.2$ **Add 2.2 to each side.**

$t \le 3$ **Divide each side by 10.4.**

▶ The ages are less than or equal to 3 years.

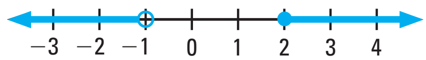
GOAL 2 SOLVING COMPOUND INEQUALITIES

A **compound inequality** is two simple inequalities joined by "and" or "or." Here are two examples.

$-2 \le x < 1$

All real numbers that are greater than or equal to -2 *and* less than 1.

$x < -1$ or $x \ge 2$



All real numbers that are less than -1 *or* greater than or equal to 2.

> **STUDENT HELP**
> **Study Tip**
> The inequality $a < x < b$ is read as "x is between a and b." The inequality $a \le x \le b$ is read as "x is between a and b, inclusive."

EXAMPLE 4 *Solving an "And" Compound Inequality*

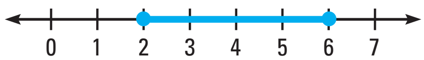
Solve $-2 \le 3t - 8 \le 10$.

SOLUTION

To solve, you must isolate the variable between the two inequality signs.

$-2 \le 3t - 8 \le 10$	**Write original inequality.**
$6 \le 3t \le 18$	**Add 8 to each expression.**
$2 \le t \le 6$	**Divide each expression by 3.**

▶ Because t is between 2 and 6, inclusive, the solutions are all real numbers greater than or equal to 2 *and* less than or equal to 6. Check several of these numbers in the original inequality. The graph is shown below.



EXAMPLE 5 *Solving an "Or" Compound Inequality*

Solve $2x + 3 < 5$ or $4x - 7 > 9$.

SOLUTION

A solution of this compound inequality is a solution of *either* of its simple parts, so you should solve each part separately.

> **STUDENT HELP**
> **HOMEWORK HELP**
> Visit our Web site www.mcdougallittell.com for extra examples.

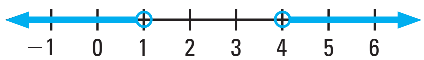
SOLUTION OF FIRST INEQUALITY

$2x + 3 < 5$	**Write first inequality.**
$2x < 2$	**Subtract 3 from each side.**
$x < 1$	**Divide each side by 2.**

SOLUTION OF SECOND INEQUALITY

$4x - 7 > 9$	**Write second inequality.**
$4x > 16$	**Add 7 to each side.**
$x > 4$	**Divide each side by 4.**

▶ The solutions are all real numbers less than 1 *or* greater than 4. Check several of these numbers to see that they satisfy one of the simple parts of the original inequality. The graph is shown below.



REAL LIFE
Automotive Maintenance

EXAMPLE 6 *Using an "And" Compound Inequality*

You have added enough antifreeze to your car's cooling system to lower the freezing point to $-35°$C and raise the boiling point to $125°$C. The coolant will remain a liquid as long as the temperature C (in degrees Celsius) satisfies the inequality $-35 < C < 125$. Write the inequality in degrees Fahrenheit.

SOLUTION

Let F represent the temperature in degrees Fahrenheit, and use the formula $C = \frac{5}{9}(F - 32)$.

$-35 < C < 125$	Write original inequality.
$-35 < \frac{5}{9}(F - 32) < 125$	Substitute $\frac{5}{9}(F - 32)$ for C.
$-63 < F - 32 < 225$	Multiply each expression by $\frac{9}{5}$, the reciprocal of $\frac{5}{9}$.
$-31 < F < 257$	Add 32 to each expression.

▶ The coolant will remain a liquid as long as the temperature stays between $-31°$F and $257°$F.

EXAMPLE 7 *Using an "Or" Compound Inequality*

TRAFFIC ENFORCEMENT You are a state patrol officer who is assigned to work traffic enforcement on a highway. The posted minimum speed on the highway is 45 miles per hour and the posted maximum speed is 65 miles per hour. You need to detect vehicles that are traveling outside the posted speed limits.

a. Write these conditions as a compound inequality.

b. Rewrite the conditions in kilometers per hour.

SOLUTION

a. Let m represent the vehicle speeds in miles per hour. The speeds that you need to detect are given by:

$$m < 45 \text{ or } m > 65$$

b. Let k be the vehicle speeds in kilometers per hour. The relationship between miles per hour and kilometers per hour is given by the formula $m \approx 0.621k$. You can rewrite the conditions in kilometers per hour by substituting $0.621k$ for m in each inequality and then solving for k.

$$m < 45 \quad \text{or} \quad m > 65$$

$$0.621k < 45 \quad \text{or} \quad 0.621k > 65$$

$$k < 72.5 \quad \text{or} \quad k > 105$$

▶ You need to detect vehicles whose speeds are less than 72.5 kilometers per hour or greater than 105 kilometers per hour.

FOCUS ON APPLICATIONS

REAL LIFE **POLICE RADAR** Police radar guns emit a continuous radio wave of known frequency. The radar gun compares the frequency of the wave reflected from a vehicle to the frequency of the transmitted wave and then displays the vehicle's speed.

GUIDED PRACTICE

Vocabulary Check ✓

1. Explain the difference between a simple linear inequality and a compound linear inequality.

Concept Check ✓

2. Tell whether this statement is *true* or *false:* Multiplying both sides of an inequality by the same number always produces an equivalent inequality. Explain.

3. Explain the difference between solving $2x < 7$ and solving $-2x < 7$.

Skill Check ✓

Solve the inequality. Then graph your solution.

4. $x - 5 < 8$ **5.** $3x \geq 15$ **6.** $-x + 4 > 3$

7. $\frac{1}{2}x \leq 6$ **8.** $x + 8 > -2$ **9.** $-x - 3 < -5$

Graph the inequality.

10. $-2 \leq x < 5$ **11.** $x \geq 3$ or $x < -3$

12. **WINTER DRIVING** You are moving to Montana and need to lower the freezing point of the cooling system in the car from Example 6 to $-50°C$. This will also raise the boiling point to $140°C$. Write a compound inequality that models this situation. Then write the inequality in degrees Fahrenheit.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 941.

MATCHING INEQUALITIES **Match the inequality with its graph.**

13. $x \geq 4$ **14.** $x < 4$ **15.** $-4 < x \leq 4$

16. $x \geq 4$ or $x < -4$ **17.** $-4 \leq x \leq 4$ **18.** $x > 4$ or $x \leq -4$

A.


B.


C.


D.


E.


F.


CHECKING SOLUTIONS **Decide whether the given number is a solution of the inequality.**

19. $2x + 9 < 16; 4$ **20.** $10 - x \geq 3; 7$ **21.** $7x - 12 < 8; 3$

22. $-\frac{1}{3}x - 2 \leq -4; 9$ **23.** $-3 < 2x \leq 6; 3$ **24.** $-8 < x - 11 < -6; 5$

STUDENT HELP

HOMEWORK HELP
Examples 1, 2: Exs. 13, 14, 19–22, 25–36
Example 3: Exs. 49–51
Examples 4, 5: Exs. 15–18, 23, 24, 37–48
Example 6: Exs. 52–54
Example 7: Exs. 55, 56

SIMPLE INEQUALITIES **Solve the inequality. Then graph your solution.**

25. $4x + 5 > 25$ **26.** $7 - n \leq 19$ **27.** $5 - 2x \geq 27$

28. $\frac{1}{2}x - 4 > -6$ **29.** $\frac{3}{2}x - 7 < 2$ **30.** $5 + \frac{1}{3}n \leq 6$

31. $4x - 1 > 14 - x$ **32.** $-n + 6 < 7n + 4$ **33.** $4.7 - 2.1x > -7.9$

34. $2(n - 4) \leq 6$ **35.** $2(4 - x) > 8$ **36.** $5 - 5x > 4(3 - x)$

COMPOUND INEQUALITIES **Solve the inequality. Then graph your solution.**

37. $-2 \leq x - 7 \leq 11$ **38.** $-16 \leq 3x - 4 \leq 2$ **39.** $-5 \leq -n - 6 \leq 0$

40. $-2 < -2n + 1 \leq 7$ **41.** $-7 < 6x - 1 < 5$ **42.** $-8 < \frac{2}{3}x - 4 < 10$

43. $x + 2 \leq 5$ or $x - 4 \geq 2$ **44.** $3x + 2 < -10$ or $2x - 4 > -4$

45. $-5x - 4 < -1.4$ or $-2x + 1 > 11$ **46.** $x - 1 \leq 5$ or $x + 3 \geq 10$

47. $-0.1 \leq 3.4x - 1.8 < 6.7$ **48.** $0.4x + 0.6 < 2.2$ or $0.6x > 3.6$

49. **COMMISSION** Your salary is \$1250 per week and you receive a 5% commission on your sales each week. What are the possible amounts (in dollars) that you can sell each week to earn at least \$1500 per week?

50. **PARK FEES** You have \$50 and are going to an amusement park. You spend \$25 for the entrance fee and \$15 for food. You want to play a game that costs \$.75. Write and solve an inequality to find the possible numbers of times you can play the game. If you play the game the maximum number of times, will you have spent the entire \$50? Explain.

51. **GRADES** A professor announces that course grades will be computed by taking 40% of a student's project score (0–100 points) and adding 60% of the student's final exam score (0–100 points). If a student gets an 86 on the project, what scores can she get on the final exam to get a course grade of at least 90?

SCIENCE CONNECTION **In Exercises 52–54, use the following information.**
The international standard for scientific temperature measurement is the Kelvin scale. A Kelvin temperature can be obtained by adding 273.15 to a Celsius temperature. The daytime temperature on Mars ranges from −89.15°C to −31.15°C. ▶ Source: NASA

52. Write the daytime temperature range on Mars as a compound inequality in degrees Celsius.

53. Rewrite the compound inequality in degrees Kelvin.

54. **RESEARCH** Find the high and low temperatures in your area for any particular day. Write three compound inequalities representing the temperature range in degrees Fahrenheit, in degrees Celsius, and in degrees Kelvin.

FOCUS ON APPLICATIONS

REAL LIFE **MARS** is the fourth planet from the sun. A Martian year is 687 Earth days long, but a Martian day is only 40 minutes longer than an Earth day. Mars is also much colder than Earth, as discussed in Exs. 52–54.

INTERNET **APPLICATION LINK**
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WINTER **In Exercises 55 and 56, use the following information.**
The Ontario Winter Severity Index (OWSI) is a weekly calculation used to determine the severity of winter conditions. The OWSI for deer is given by

$$I = \frac{p}{30} + \frac{d}{30} + c$$

where p represents the average Snow Penetration Gauge reading (in centimeters), d represents the average snow depth (in centimeters), and c represents the *chillometer reading*, which is a measure of the cold (in kilowatt-hours) based on temperature and wind chill. An extremely mild winter occurs when $I < 5$ on average, and an extremely severe winter occurs when $I > 6.5$ on average. A deer can tolerate a maximum snow penetration of 50 centimeters. Assume the average snow depth is 60 centimeters. ▶ Source: Snow Network for Ontario Wildlife

55. What weekly chillometer readings will produce extremely severe winter readings?

56. What weekly chillometer readings will produce extremely mild winter readings?

57. *Writing* The first transformation listed in the box on page 41 can be written symbolically as follows: If a, b, and c are real numbers and $a > b$, then $a + c > b + c$. Write similar statements for the other transformations.

Test Preparation

58. **MULTI-STEP PROBLEM** You are vacationing at Lake Tahoe, California. You decide to spend a day sightseeing in other places. You want to go from Lake Tahoe to Sacramento, from Sacramento to Sonora, and then from Sonora back to Lake Tahoe. You know that it is about 85 miles from Lake Tahoe to Sacramento and about 75 miles from Sacramento to Sonora.

a. The triangle inequality theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. Write a compound inequality that represents the distance from Sonora to Lake Tahoe.

b. **CRITICAL THINKING** You are reading a brochure which states that the distance between Sonora and Lake Tahoe is 170 miles. You know that the distance is a misprint. How can you be so sure? Explain.

c. You keep a journal of the distances you have traveled. Many of your distances represent triangular circuits. Your friend is reading your journal and states that you must have recorded a wrong distance for one of these circuits. To which one of the following is your friend referring? Explain.

A. 35 miles, 65 miles, 45 miles　　**B.** 15 miles, 50 miles, 64 miles

C. 49 miles, 78 miles, 28 miles　　**D.** 55 miles, 72 miles, 41 miles

★ **Challenge**

59. Write an inequality that has no solution. Show why it has no solution.

60. Write an inequality whose solutions are all real numbers. Show why the solutions are all real numbers.

MIXED REVIEW

IDENTIFYING PROPERTIES **Identify the property shown.** (Review 1.1)

61. $(7 \cdot 3) \cdot 11 = 7 \cdot (3 \cdot 11)$　　62. $34 + (-34) = 0$

63. $37 + 29 = 29 + 37$　　64. $3(9 + 4) = 3(9) + 3(4)$

SOLVING EQUATIONS **Solve the equation. Check your solution.** (Review 1.3 for 1.7)

65. $5x + 4 = -2(x + 3)$　　66. $2(3 - x) = 16(x + 1)$

67. $-(x - 1) + 10 = -3(x - 3)$　　68. $\frac{1}{8}x + \frac{3}{2} = \frac{3}{4}x - 1$

69. **CONCERT TRIP** You are going to a concert in another town 48 miles away. You can average 40 miles per hour on the road you plan to take to the concert. What is the minimum number of hours before the concert starts that you should leave to get to the concert on time? (Review 1.5)

1.7 Solving Absolute Value Equations and Inequalities

What *you should learn*

GOAL 1 Solve absolute value equations and inequalities.

GOAL 2 Use absolute value equations and inequalities to solve **real-life** problems, such as finding acceptable weights in **Example 4**.

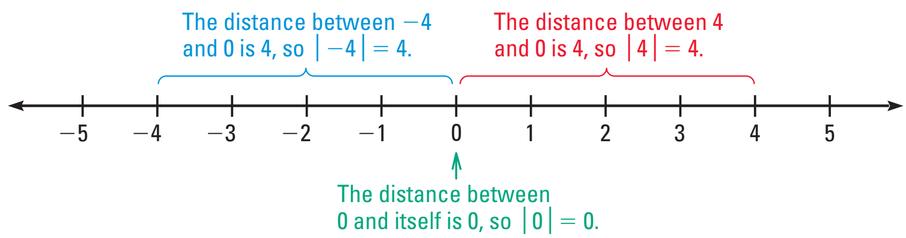
Why *you should learn it*

▼ To solve **real-life** problems, such as finding recommended weight ranges for sports equipment in **Ex. 72**.

REAL LIFE

GOAL 1 SOLVING EQUATIONS AND INEQUALITIES

The **absolute value** of a number x, written $|x|$, is the distance the number is from 0 on a number line. Notice that the absolute value of a number is always nonnegative.



The absolute value of x can be defined algebraically as follows.

$$|x| = \begin{cases} x, & \text{if } x \text{ is positive} \\ 0, & \text{if } x = 0 \\ -x, & \text{if } x \text{ is negative} \end{cases}$$

To solve an absolute value equation of the form $|x| = c$ where $c > 0$, use the fact that x can have two possible values: a positive value c or a negative value $-c$. For instance, if $|x| = 5$, then $x = 5$ or $x = -5$.

SOLVING AN ABSOLUTE VALUE EQUATION

The absolute value equation $|ax + b| = c$, where $c > 0$, is equivalent to the compound statement $ax + b = c$ or $ax + b = -c$.

EXAMPLE 1 *Solving an Absolute Value Equation*

Solve $|2x - 5| = 9$.

SOLUTION

Rewrite the absolute value equation as two linear equations and then solve each linear equation.

$|2x - 5| = 9$			**Write original equation.**
$2x - 5 = 9$	or	$2x - 5 = -9$	**Expression can be 9 or −9.**
$2x = 14$	or	$2x = -4$	**Add 5 to each side.**
$x = 7$	or	$x = -2$	**Divide each side by 2.**

▶ The solutions are 7 and −2. Check these by substituting each solution into the original equation.

An absolute value inequality such as $|x - 2| < 4$ can be solved by rewriting it as a compound inequality, in this case as $-4 < x - 2 < 4$.

TRANSFORMATIONS OF ABSOLUTE VALUE INEQUALITIES

- The inequality $|ax + b| < c$, where $c > 0$, means that $ax + b$ is *between* $-c$ and c. This is equivalent to $-c < ax + b < c$.
- The inequality $|ax + b| > c$, where $c > 0$, means that $ax + b$ is *beyond* $-c$ and c. This is equivalent to $ax + b < -c$ *or* $ax + b > c$.

In the first transformation, $<$ can be replaced by $\leq$. In the second transformation, $>$ can be replaced by $\geq$.

EXAMPLE 2 Solving an Inequality of the Form $|ax + b| < c$

STUDENT HELP
INTERNET
HOMEWORK HELP
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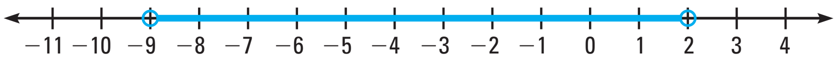
Solve $|2x + 7| < 11$.

SOLUTION

$\|2x + 7\| < 11$	Write original inequality.
$-11 < 2x + 7 < 11$	Write equivalent compound inequality.
$-18 < 2x < 4$	Subtract 7 from each expression.
$-9 < x < 2$	Divide each expression by 2.

▶ The solutions are all real numbers greater than -9 and less than 2. Check several solutions in the original inequality. The graph is shown below.



EXAMPLE 3 Solving an Inequality of the Form $|ax + b| \geq c$

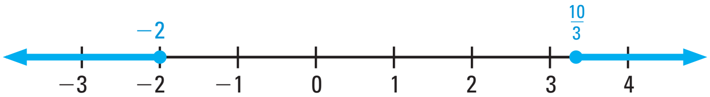
Solve $|3x - 2| \geq 8$.

SOLUTION

This absolute value inequality is equivalent to $3x - 2 \leq -8$ or $3x - 2 \geq 8$.

SOLVE FIRST INEQUALITY		SOLVE SECOND INEQUALITY
$3x - 2 \leq -8$	Write inequality.	$3x - 2 \geq 8$
$3x \leq -6$	Add 2 to each side.	$3x \geq 10$
$x \leq -2$	Divide each side by 3.	$x \geq \frac{10}{3}$

▶ The solutions are all real numbers less than or equal to -2 or greater than or equal to $\frac{10}{3}$. Check several solutions in the original inequality. The graph is shown below.



GOAL 2 USING ABSOLUTE VALUE IN REAL LIFE

In manufacturing applications, the maximum acceptable deviation of a product from some ideal or average measurement is called the *tolerance*.

REAL LIFE
Manufacturing

EXAMPLE 4 *Writing a Model for Tolerance*

A cereal manufacturer has a tolerance of 0.75 ounce for a box of cereal that is supposed to weigh 20 ounces. Write and solve an absolute value inequality that describes the acceptable weights for “20 ounce” boxes.

PROBLEM SOLVING STRATEGY

SOLUTION

VERBAL MODEL

$$\left| \boxed{\text{Actual weight}} - \boxed{\text{Ideal weight}} \right| \le \boxed{\text{Tolerance}}$$

LABELS

Actual weight $= x$ (ounces)

Ideal weight $= 20$ (ounces)

Tolerance $= 0.75$ (ounces)

ALGEBRAIC MODEL

$|x - 20| \le 0.75$ **Write algebraic model.**

$-0.75 \le x - 20 \le 0.75$ **Write equivalent compound inequality.**

$19.25 \le x \le 20.75$ **Add 20 to each expression.**

▶ The weights can range between 19.25 ounces and 20.75 ounces, inclusive.

EXAMPLE 5 *Writing an Absolute Value Model*

QUALITY CONTROL You are a quality control inspector at a bowling pin company. A regulation pin must weigh between 50 ounces and 58 ounces, inclusive. Write an absolute value inequality describing the weights you should reject.

SOLUTION

VERBAL MODEL

$$\left| \boxed{\text{Weight of pin}} - \boxed{\text{Average of extreme weights}} \right| > \boxed{\text{Tolerance}}$$

LABELS

Weight of pin $= w$ (ounces)

Average of extreme weights $= \dfrac{50 + 58}{2} = 54$ (ounces)

Tolerance $= 58 - 54 = 4$ (ounces)

ALGEBRAIC MODEL

$|w - 54| > 4$

▶ You should reject a bowling pin if its weight w satisfies $|w - 54| > 4$.

FOCUS ON APPLICATIONS

REAL LIFE **BOWLING** Bowling pins are made from maple wood, either solid or laminated. They are given a tough plastic coating to resist cracking. The lighter the pin, the easier it is to knock down.

GUIDED PRACTICE

Vocabulary Check ✓

1. What is the absolute value of a number?

Concept Check ✓

2. The absolute value of a number cannot be negative. How, then, can the absolute value of a be $-a$?

3. Give an example of the absolute value of a number. How many other numbers have this absolute value? State the number or numbers.

Skill Check ✓

Decide whether the given number is a solution of the equation.

4. $|3x + 8| = 20; -4$
5. $|11 - 4x| = 7; 1$
6. $|2x - 9| = 11; -1$
7. $|-x + 9| = 4; -5$
8. $|6 + 3x| = 0; -2$
9. $|-5x - 3| = 8; -1$

Rewrite the absolute value inequality as a compound inequality.

10. $|x + 8| < 5$
11. $|11 - 2x| \geq 13$
12. $|9 - x| > 21$
13. $|x + 5| \leq 9$
14. $|10 - 3x| \geq 17$
15. $\left|\frac{1}{4}x + 10\right| < 18$
16. **TOLERANCE** Suppose the tolerance for the "20 ounce" cereal boxes in Example 4 is now 0.45 ounce. Write and solve an absolute value inequality that describes the new acceptable weights of the boxes.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 941.

REWRITING EQUATIONS **Rewrite the absolute value equation as two linear equations.**

17. $|x - 8| = 11$
18. $|5 - 2x| = 13$
19. $|6n + 1| = \frac{1}{2}$
20. $|5n - 4| = 16$
21. $|2x + 1| = 5$
22. $|2 - x| = 3$
23. $|15 - 2x| = 8$
24. $\left|\frac{1}{2}x + 4\right| = 6$
25. $\left|\frac{2}{3}x - 9\right| = 18$

CHECKING A SOLUTION **Decide whether the given number is a solution of the equation.**

26. $|4x + 1| = 11; 3$
27. $|8 - 2n| = 2; -5$
28. $\left|6 + \frac{1}{2}x\right| = 14; -40$
29. $\left|\frac{1}{5}x - 2\right| = 4; 10$
30. $|4n + 7| = 1; 2$
31. $|-3x + 5| = 7; 4$

SOLVING EQUATIONS **Solve the equation.**

32. $|11 + 2x| = 5$
33. $|10 - 4x| = 2$
34. $|22 - 3n| = 5$
35. $|2n - 5| = 7$
36. $|8x + 1| = 23$
37. $|30 - 7x| = 4$
38. $\left|\frac{1}{4}x - 5\right| = 8$
39. $\left|\frac{2}{3}x + 2\right| = 10$
40. $\left|\frac{1}{2}x - 3\right| = 2$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 17–40
Examples 2, 3: Exs. 41–58
Examples 4, 5: Exs. 65–76

REWRITING INEQUALITIES **Rewrite the absolute value inequality as a compound inequality.**

41. $|3 + 4x| \leq 15$
42. $|4n - 12| > 16$
43. $|3x + 2| < 7$
44. $|2x - 1| \geq 12$
45. $|8 - 3n| \leq 18$
46. $|11 + 4x| < 23$

SOLVING AND GRAPHING Solve the inequality. Then graph your solution.

47. $|x + 1| < 8$ **48.** $|12 - x| \le 19$ **49.** $|16 - x| \ge 10$

50. $|x + 5| > 12$ **51.** $|x - 8| \le 5$ **52.** $|x - 16| > 24$

53. $|14 - 3x| > 18$ **54.** $|4x + 10| < 20$ **55.** $|8x + 28| \ge 32$

56. $\left|20 + \frac{1}{2}x\right| > 6$ **57.** $|7x + 5| < 23$ **58.** $|11 + 6x| \le 47$

STUDENT HELP

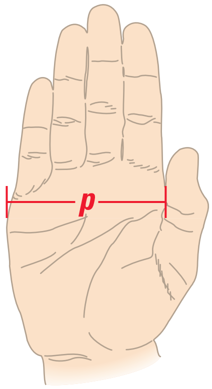
INTERNET **KEYSTROKE HELP** Visit our Web site www.mcdougallittell.com to see keystrokes for several models of calculators.

SOLVING INEQUALITIES Use the *Test* feature of a graphing calculator to solve the inequality. Most calculators use *abs* for absolute value. For example, you enter $|x + 1|$ as abs(x + 1).

59. $|x + 1| < 3$ **60.** $\left|\frac{2}{3}x - \frac{1}{3}\right| \le \frac{1}{3}$ **61.** $|2x - 4| > 10$

62. $\left|\frac{1}{2}x - 1\right| \le 3$ **63.** $|4x - 10| > 6$ **64.** $|1 - 2x| \ge 13$

PALM WIDTHS In Exercises 65 and 66, use the following information.
In a sampling conducted by the United States Air Force, the right-hand dimensions of 4000 Air Force men were measured. The gathering of such information is useful when designing control panels, keyboards, gloves, and so on.



65. Ninety-five percent of the palm widths p were within 0.26 inch of 3.49 inches. Write an absolute value inequality that describes these values of p. Graph the inequality.

66. Ninety-nine percent of the palm widths p were within 0.37 inch of 3.49 inches. Write an absolute value inequality that describes these values of p. Graph the inequality.

67. **ACCURACY OF MEASUREMENTS** Your woodshop instructor requires that you cut several pieces of wood within $\frac{3}{16}$ inch of his specifications. Let p represent the specification and let x represent the length of a cut piece of wood. Write an absolute value inequality that describes the acceptable values of x. One piece of wood is specified to be $p = 9\frac{1}{8}$ inches. Describe the acceptable lengths for the piece of wood.

68. **BASKETBALL** The length of a standard basketball court can vary from 84 feet to 94 feet, inclusive. Write an absolute value inequality that describes the possible lengths of a standard basketball court.

69. **BODY TEMPERATURE** Physicians consider an adult's normal body temperature to be within 1°F of 98.6°F, inclusive. Write an absolute value inequality that describes the range of normal body temperatures.

FOCUS ON APPLICATIONS

REAL LIFE **BODY TEMPERATURE** Doctors routinely use ear thermometers to measure body temperature. The first ear thermometers were used in 1990. The thermometers use an infrared sensor and microprocessors.

WEIGHING FLOUR In Exercises 70 and 71, use the following information.
A 16 ounce bag of flour probably does not weigh exactly 16 ounces. Suppose the actual weight can be between 15.6 ounces and 16.4 ounces, inclusive.

70. Write an absolute value inequality that describes the acceptable weights for a "16 ounce" bag of flour.

71. A case of flour contains 24 of these "16 ounce" bags. What is the greatest possible weight of the flour in a case? What is the least possible weight? Write an absolute value inequality that describes the acceptable weights of a case.

SPORTS EQUIPMENT In Exercises 72 and 73, use the table giving the recommended weight ranges for the balls from five different sports.

Sport	Weight range of ball used
Volleyball	260–280 grams
Basketball	600–650 grams
Water polo	400–450 grams
Lacrosse	142–149 grams
Football	14–15 ounces

72. Write an absolute value inequality for the weight range of each ball.

73. For each ball, write an absolute value inequality describing the weights of balls that are *outside* the recommended range.

74. SCIENCE CONNECTION Green plants can live in the ocean only at depths of 0 feet to 100 feet. Write an absolute value inequality describing the range of possible depths for green plants in an ocean.

75. **BOTTLING** A juice bottler has a tolerance of 9 milliliters in a two liter bottle, of 5 milliliters in a one liter bottle, and of 2 milliliters in a 500 milliliter bottle. For each size of bottle, write an absolute value inequality describing the capacities that are outside the acceptable range.

76. SCIENCE CONNECTION To determine height from skeletal remains, scientists use the equation $H = 2.26f + 66.4$ where H is the person's height (in centimeters) and f is the skeleton's femur length (in centimeters). The equation has a margin of error of ± 3.42 centimeters. Suppose a skeleton's femur length is 51.6 centimeters. Write an absolute value inequality that describes the person's height. Then solve the inequality to find the range of possible heights.

Test Preparation

77. **MULTIPLE CHOICE** Which of the following are solutions of $|3x - 7| = 14$?

Ⓐ $x = \frac{7}{3}$ or $x = 7$ Ⓑ $x = -\frac{7}{3}$ or $x = 7$

Ⓒ $x = \frac{7}{3}$ or $x = -7$ Ⓓ $x = -\frac{7}{3}$ or $x = -7$

78. **MULTIPLE CHOICE** Which of the following is equivalent to $|2x - 9| < 3$?

Ⓐ $-3 \le x \le 6$ Ⓑ $3 < x < 6$

Ⓒ $3 \le x \le 6$ Ⓓ $-3 < x < -6$

79. **MULTIPLE CHOICE** Which of the following is equivalent to $|3x + 5| \ge 19$?

Ⓐ $x \le -\frac{14}{3}$ or $x \ge 8$ Ⓑ $x < -8$ or $x > \frac{14}{3}$

Ⓒ $x \le -8$ or $x \ge \frac{14}{3}$ Ⓓ $x < -\frac{14}{3}$ or $x > 8$

★ Challenge

SOLVING INEQUALITIES Solve the inequality. If there is no solution, write *no solution*.

80. $|2x + 3| \ge -13$ **81.** $|5x + 2| \le -2$

82. $|3x - 8| < -10$ **83.** $|4x - 2| > -6$

84. $|6 - 2x| > -8$ **85.** $|7 - 3x| \le -14$

SOLVING INEQUALITIES Solve for *x*. Assume *a* and *b* are positive numbers.

86. $|x + a| < b$ **87.** $|x - a| > b$

88. $|x + a| \ge a$ **89.** $|x - a| \le a$

EXTRA CHALLENGE
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Mixed Review

Logical Reasoning **Tell whether the statement is *true* or *false*. If the statement is false, explain why.** (Skills Review, p. 926)

90. A triangle is a right triangle if and only if it has a right angle.

91. $2x = 14$ if and only if $x = -7$.

92. All rectangles are squares.

Evaluating Expressions **Evaluate the expression for the given value(s) of the variable(s).** (Review 1.2 for 2.1)

93. $5x - 9$ when $x = 6$

94. $-2y + 4$ when $y = 14$

95. $11c + 6$ when $c = -3$

96. $-8a - 3$ when $a = -4$

97. $a - 11b + 2$ when $a = 61$ and $b = 7$

98. $15x + 8y$ when $x = \frac{1}{2}$ and $y = \frac{1}{3}$

99. $\frac{1}{5}\left(8g + \frac{1}{3}h\right)$ when $g = 6$ and $h = 6$

100. $\frac{1}{5}(p + q) - 7$ when $p = 5$ and $q = 3$

Solving Inequalities **Solve the inequality.** (Review 1.6)

101. $6x + 9 > 11$

102. $15 - 2x \geq 45$

103. $-3x - 5 \leq 10$

104. $13 + 4x < 9$

105. $-18 < 2x + 10 < 6$

106. $x + 2 \leq -1$ or $4x \geq 8$

Quiz 3

Self-Test for Lessons 1.6 and 1.7

Solve the inequality. Then graph your solution. (Lesson 1.6)

1. $4x - 3 \leq 17$

2. $2y - 9 > 5y + 12$

3. $-8 < 3x + 4 < 22$

4. $3x - 5 \leq -11$ or $2x - 3 > 3$

Solve the equation. (Lesson 1.7)

5. $|x + 5| = 4$

6. $|x - 3| = 2$

7. $|6 - x| = 9$

8. $|4x - 7| = 13$

9. $|3x + 4| = 20$

10. $|15 - 3x| = 12$

Solve the inequality. Then graph your solution. (Lesson 1.7)

11. $|y + 2| \geq 3$

12. $|x + 6| < 4$

13. $|x - 3| > 7$

14. $|2y - 5| \leq 3$

15. $|2x - 3| > 1$

16. $|4x + 5| \geq 13$

17. **Fuel Efficiency** Your car gets between 20 miles per gallon and 28 miles per gallon of gasoline and has a 16 gallon gasoline tank. Write a compound inequality that represents your fuel efficiency. How many miles can you travel on one tank of gasoline? (Lesson 1.6)

18. **Manufacturing Tolerance** The ideal diameter of a certain type of ball bearing is 30 millimeters. The manufacturer has a tolerance of 0.045 millimeter. Write an absolute value inequality that describes the acceptable diameters for these ball bearings. Then solve the inequality to find the range of acceptable diameters. (Lesson 1.7)

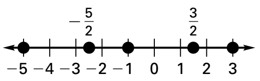
Selected Answers

CHAPTER 1

SKILL REVIEW (p. 2) **1.** 11 **2.** –70 **3.** 8 **4.** 9 **5.** 24 **6.** –7 **7.** –10 **8.** –8 **9.** 60 units^2 **10.** 121 units^2 **11.** 165 units^2 **12.** 20.25π units^2, or about 63.6 units^2

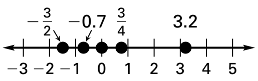
1.1 PRACTICE (pp. 7–10)

5.



3

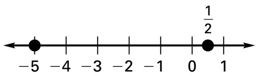
7.



3.2

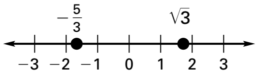
9. inverse property of addition **11.** commutative property of multiplication **13.** inverse property of multiplication

15.



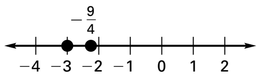
; $\frac{1}{2} > -5$ **17.** $2.3 > -0.6$

19.



$-\frac{5}{3} < \sqrt{3}$

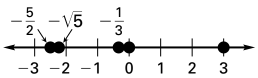
21.



$-\frac{9}{4} > -3$

23. $\sqrt{5} > 2$ **25.** $\sqrt{8} > 2.5$ **27.** $-6, -3, -\frac{1}{2}, 2, \frac{13}{4}$

29.



; $-\frac{5}{2}, -\sqrt{5}, -\frac{1}{3}, 0, 3$

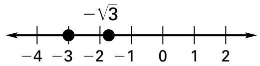
31. $-\sqrt{12}, -\frac{12}{5}, -1.5, 0, 0.3$ **33.** inverse property of addition **35.** commutative property of multiplication **37.** identity property of multiplication **39.** Yes; the associative property of addition is true for all real numbers a, b, and c. **41.** Yes; the associative property of multiplication is true for all real numbers a, b, and c. **43.** $32 + (-7) = 25$ **45.** $-5 - 8 = -13$ **47.** $9 \cdot (-4) = -36$ **49.** $-5 \div \left(-\frac{1}{2}\right) = 10$ **51.** 13 ft **53.** \$612.50 **55.** Honolulu, HI; New Orleans, LA; Jackson, MS; Seattle-Tacoma, WA; Norfolk, VA; Atlanta, GA; Detroit, MI; Milwaukee, WI; Albany, NY; Helena, MT; three **57.** Yes; the result of performing the given operations is 9, the check digit. **59.** Sky Central Plaza: 352 yd, 12,672 in., 0.2 mi; Petronas Tower I: about 494.3 yd, 17,796 in., about 0.2809 mi **61.** yes **63.** \$214 **65.** –15°F

1.1 MIXED REVIEW (p. 10) **69.** 63 **71.** –30 **73.** 19 **75.** –34 **77.** $x - 3$ **79.** $\frac{1}{4}x$ **81.** 10.5 in.^{2} **83.** 750 in.^{2}

1.2 PRACTICE (pp. 14–16) **7.** 5 **9.** 27 **11.** $9x + 9y$ **13.** $8x^2 - 8x$ **15.** 8^3 **17.** 5^n **19.** 256 **21.** –32 **23.** 125 **25.** 256 **27.** 24 **29.** 19 **31.** 0 **33.** –5 **35.** 125 **37.** –8 **39.** 76 **41.** $\frac{9}{5}$ **43.** $-\frac{5}{13}$ **45.** 16 **47.** $6x^2 - 28x$ **49.** $16n - 88$ **51.** $-5x - y$ **53.** $\frac{1}{2}n(n + 10)$; 1000 **55.** $(x + y)^2$; 289 **57.** about 1,200,000; about 238,000 **59.** $149 + 3.85(12)n$, where n is the number of movies rented each month; \$426.20 **61.** $[4n + 8(3 - n)]15$, or $360 - 60n$, where n is the number of hours spent walking; \$240

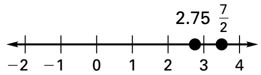
1.2 MIXED REVIEW (p. 17) **69.** 20 **71.** 15 **73.** 105

75.



$-\sqrt{3} > -3$

77.

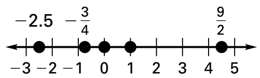


$2.75 < \frac{7}{2}$

79. inverse property of addition **81.** identity property of multiplication **83.** $\frac{8}{7}$ **85.** $-\frac{4}{5}$ **87.** –9 **89.** $-\frac{1}{14}$

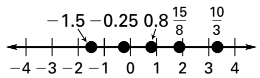
QUIZ 1 (p. 17)

1.



$-2.5, -\frac{3}{4}, 0, 1, \frac{9}{2}$

2.



$-1.5, -0.25, 0.8, \frac{15}{8}, \frac{10}{3}$

3. distributive property **4.** associative property of addition **5.** 15 **6.** $-\frac{17}{3}$ **7.** –14 **8.** 76 **9.** –124 **10.** $8x - 11y + 4$ **11.** $2x - 10$ **12.** $-2x^2 + 5x - 6$ **13.** $-2x^2 + 14x$ **14.** $0.35n + 13.95(15 - n)$, or $209.25 - 13.60n$, where n is the number of regular floppy disks bought

TECHNOLOGY ACTIVITY 1.2 (p. 18) **1.** $(-4)^2 - 5$; 11 **3.** $(1 + 4)^6$; 15,625 **5.** 4.32 **7.** 160.989 **9.** 7.833 **11.** 5912.099 **13.** 0.81

1.3 PRACTICE (pp. 22–24) **7.** 5 **9.** 5 **11.** $\frac{5}{4}$ **13.** –3 **15.** 28 **17.** Subtract 5 from each side. **19.** Multiply each side by $-\frac{7}{4}$. **21.** Subtract 2 from each side; then multiply each side by 3. **23.** 5 **25.** $\frac{7}{2}$ **27.** $\frac{4}{5}$ **29.** –1 **31.** 0 **33.** 4 **35.** $\frac{85}{12}$ **37.** 3.2 **39.** 7.5 **41.** length: 36, width: 14 **43.** –78.5°C **45.** 5 h **47.** \$635,000 **49.** 16.25 ft

1.3 MIXED REVIEW (p. 24) **57.** 25π in.^{2}, or about 78.5 in.^{2} **59.** 49π in.^{2}, or about 154 in.^{2} **61.** 8 **63.** 21 **65.** 11 **67.** –28 **69.** $21 - 5x$ **71.** $7x - 6$ **73.** $x + 35$ **75.** $3x^2 - x + 11$ **77.** $4x^2 + 16x$

TECHNOLOGY ACTIVITY 1.3 (p. 25) **1.** False; $y_1 = y_2$ when $x = -2$, not when $x = 2$. **3.** –2 **5.** 1 **7.** 1

1.4 PRACTICE (pp. 29–32) **5.** $y = \frac{5}{3}x - 3$ **7.** $y = -\frac{3}{20}x + 4$ **9.** $y = \frac{4}{3}x - 24$ **11.** 20 in. **13.** –1 **15.** $\frac{16}{9}$ **17.** $\frac{35}{3}$ **19.** 1 **21.** –4 **23.** $\frac{11}{2}$ **25.** $h = \frac{3V}{\pi r^2}$ **27.** $P = \frac{I}{rt}$ **29.** $b_2 = \frac{2A}{h} - b_1$ **31.** $h = \frac{S - 2\pi r^2}{2\pi r}$; $\frac{35 - 6\pi}{2\pi}$, or about 2.57 in. **33.** $L = \frac{T}{m} + 21$

SELECTED ANSWERS

SELECTED ANSWERS

35. $W \approx \frac{TR^2}{R^2 + A^2}$ **37.** $R = p_1V + p_2C$ **39.** *Sample answer:* 210 sun visors, 550 baseball caps; 490 sun visors, 430 baseball caps; 700 sun visors, 340 baseball caps

41. a. $A = \frac{\sqrt{3}}{4}b^2$ **b.** $A = \frac{\sqrt{3}}{3}h^2$

1.4 MIXED REVIEW (p. 32) **47.** $30 - x$ **49.** $250 + x$ **51.** $2x$ **53.** 8736 h **55.** $4\frac{3}{8}$ L **57.** \$165 **59.** -6 **61.** 4 **63.** -7 **65.** 40 **67.** 3

1.5 PRACTICE (pp. 37–39) **3.** The diagram helps you see how to express the numbers of gallons used in town in terms of x, the label given to the number of gallons used on the highway. **5.** water pressure = 2184 (lb/ft^2); pressure per ft of depth = 62.4 (lb/ft^2 per ft); depth = d (ft) **7.** 35 ft **9.** $547 = 32t$ **11.** about 17 h **13.** $80t = (180)(3)$ **15.** total calories = (calories/gram of fat)(number of grams of fat) + (calories/gram of protein)(number of grams of protein) + (calories/gram of carbohydrate)(number of grams of carbohydrate) **17.** 4.1 g **19.** Great Britain: 22.4 km, France: 15.5 km; Dec. 1, 1990 **21.** \$1.68 per page **23.** length: 135 ft, width: 105 ft **25.** 4.5 m **27.** 4 bounces

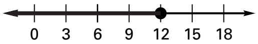
1.5 MIXED REVIEW (p. 39) **31.** true **33.** false **35.** $-55, -10, -5, -1, 4$ **37.** $-2.9, -2.1, -1.2, 2, 2.09$ **39.** 2 **41.** $\frac{4}{7}$

QUIZ 2 (p. 40) **1.** 4 **2.** -8 **3.** $\frac{17}{3}$ **4.** 160 **5.** $y = -\frac{3}{5}x + \frac{9}{5}; \frac{3}{5}$ **6.** $y = \frac{4}{3}x - \frac{14}{3}; -2$ **7.** $d_1 = \frac{2A}{d_2}$ **8.** 49 boxes

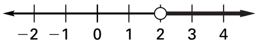
1.6 PRACTICE (pp. 45–47)

5. $x \geq 5$;

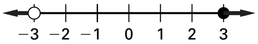
7. $x \leq 12$;



9. $x > 2$;

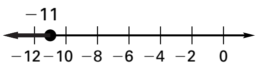


11.

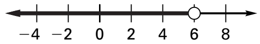


13. C **15.** D **17.** F **19.** no **21.** no **23.** yes **25.** $x > 5$

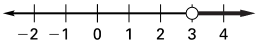
27. $x \leq -11$;



29. $x < 6$;

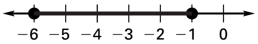


31. $x > 3$;

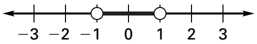


33. $x < 6$ **35.** $x < 0$

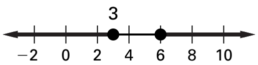
37. $5 \leq x \leq 18$ **39.** $-6 \leq n \leq -1$;



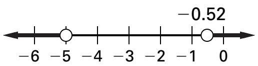
41. $-1 < x < 1$;



43. $x \leq 3$ or $x \geq 6$;



45. $x < -5$ or $x > -0.52$;



47. $0.5 \leq x < 2.5$

49. Your sales must be greater than or equal to \$5000.
51. Her score must be between 93 and 100, inclusive.
53. $184 \leq K \leq 242$ **55.** $c > 2.83$

1.6 MIXED REVIEW (p. 47) **61.** associative property of multiplication **63.** commutative property of addition **65.** $-\frac{10}{7}$ **67.** -1 **69.** $1\frac{1}{5}$ h, or 1 h 12 min

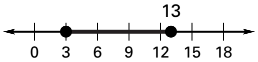
TECHNOLOGY ACTIVITY 1.6 (p. 48) **1.** $x \leq 4$ **3.** $x > 3$ **5.** $x \leq -6$ **7.** $x < 2$ **9.** $x < 6$ **11.** $x \leq 9$ **13.** $x < -7$

1.7 PRACTICE (pp. 53–55) **5.** yes **7.** no **9.** no **11.** $11 - 2x \leq -13$ or $11 - 2x \geq 13$ **13.** $-9 \leq x + 5 \leq 9$ **15.** $-18 < \frac{1}{4}x + 10 < 18$ **17.** $x - 8 = 11$ or $x - 8 = -11$ **19.** $6n + 1 = \frac{1}{2}$ or $6n + 1 = -\frac{1}{2}$ **21.** $2x + 1 = 5$ or $2x + 1 = -5$ **23.** $15 - 2x = 8$ or $15 - 2x = -8$ **25.** $\frac{2}{3}x - 9 = 18$ or $\frac{2}{3}x - 9 = -18$ **27.** no **29.** no **31.** yes **33.** 2, 3 **35.** $6, -1$ **37.** $\frac{26}{7}, \frac{34}{7}$ **39.** $12, -18$ **41.** $-15 \leq 3 + 4x \leq 15$ **43.** $-7 < 3x + 2 < 7$ **45.** $-18 \leq 8 - 3n \leq 18$

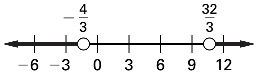
47. $-9 < x < 7$;

49. $x \leq 6$ or $x \geq 26$

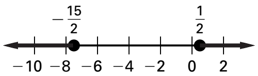
51. $3 \leq x \leq 13$;



53. $x < -\frac{4}{3}$ or $x > \frac{32}{3}$;

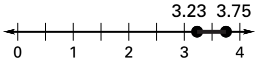


55. $x \leq -\frac{15}{2}$ or $x \geq \frac{1}{2}$;



57. $-4 < x < \frac{18}{7}$ **59.** $-4 < x < 2$ **61.** $x < -3$ or $x > 7$ **63.** $x < 1$ or $x > 4$

65. $|p - 3.49| \leq 0.26$;

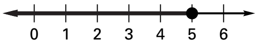


67. $|x - p| \leq \frac{3}{16}$; between $8\frac{15}{16}$ in. and $9\frac{5}{16}$ in., inclusive. **69.** $|t - 98.6| \leq 1$ **71.** 393.6 oz; 374.4 oz; $|c - 384| \leq 9.6$ **73.** volleyball: $|v - 270| > 10$, basketball: $|b - 625| > 25$, water polo: $|w - 425| > 25$, lacrosse: $|l - 145.5| > 3.5$, football: $|f - 14.5| > 0.5$ **75.** 2 L: $|c - 2000| > 9$, 1 L: $|c - 1000| > 5$, 500 mL: $|c - 500| > 2$

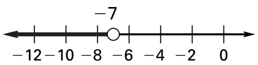
1.7 MIXED REVIEW (p. 56) **91.** False; if $x = -7$, then $2x = 2(-7) = -14$, not 14. **93.** 21 **95.** -27 **97.** -14 **99.** 10 **101.** $x > \frac{1}{3}$ **103.** $x \geq -5$ **105.** $-14 < x < -2$

QUIZ 3 (p. 56)

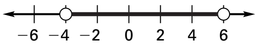
1. $x \leq 5$;



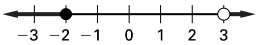
2. $x < -7$;



3. $-4 < x < 6$;

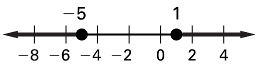


4. $x \leq -2$ or $x > 3$;

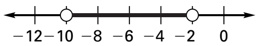


5. $-1, -9$ **6.** 5, 1 **7.** $-3, 15$ **8.** $5, -\frac{3}{2}$ **9.** $\frac{16}{3}, -8$ **10.** 1, 9

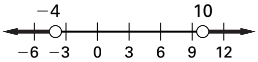
11. $y \leq -5$ or $y \geq 1$;



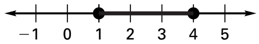
12. $-10 < x < -2$;



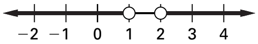
13. $x < -4$ or $x > 10$;



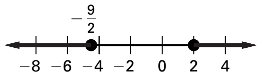
14. $1 \le y \le 4$;



15. $x < 1$ or $x > 2$;



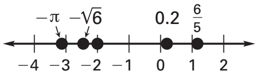
16. $x \le -\frac{9}{2}$ or $x \ge 2$;



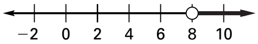
17. $20 \le e \le 28$; between 320 mi and 448 mi, inclusive

18. $|d - 30| \le 0.045$; between 29.955 mm and 30.045 mm, inclusive

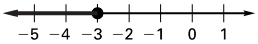
Chapter 1 Review (pp. 58–60)

1.  ; $-\pi, -\sqrt{6}, -2, 0.2, \frac{6}{5}$

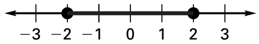
3. distributive property **5.** -18 **7.** 4 **9.** $5x + 4y$
11. $11x^2 - x$ **13.** -3 **15.** -32 **17.** 4 **19.** $y = 5x - 10$
21. $y = -0.2x + 7$ **23.** $y = \frac{5}{6}x + 2$ **25.** $l = \frac{P - 2w}{2}$
27. about 5 h 55 min **29.** $x > 8$;



31. $x \le -3$;



33. $-2 \le y \le 2$;



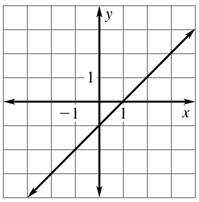
35. $-5, 3$ **37.** $-\frac{8}{3}, 6$ **39.** $-2 < x < 7$

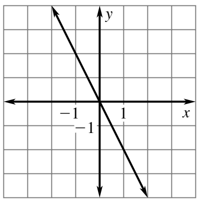
Chapter 2

Skill Review (p. 66)

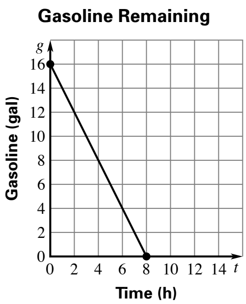
1. 2 **2.** 2 **3.** 3 **4.** $y = -3x + 4$
5. $y = \frac{1}{2}x - 5$ **6.** $y = -\frac{5}{6}x - 10$ **7.** $x < \frac{9}{2}$ **8.** $y \ge -26$ **9.** $x < \frac{5}{2}$

2.1 Practice (pp. 71–74)

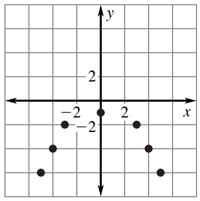
5. 

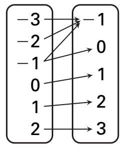
9. 

11. 3 **13.** 9 **15.** 1
17. domain: $0 \le t \le 8$; range: $0 \le g \le 16$;

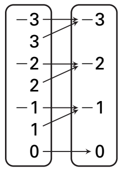


19. domain: $-1, 2, 5, 6$; range: $-2, 3$
21. domain: $1, 2, 3, 4$; range: $1, 2, 3, 4$

23.  yes

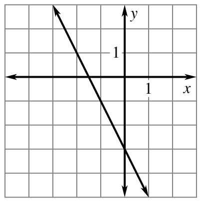
25. 

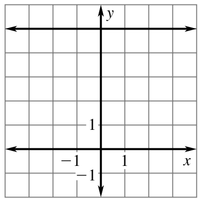
no

27. 

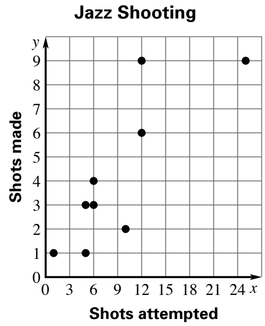
yes

29. If a relation is a function, then no vertical line intersects the graph of the relation at more than one point. If no vertical line intersects the graph of a relation at more than one point, then the relation is a function. **31.** yes

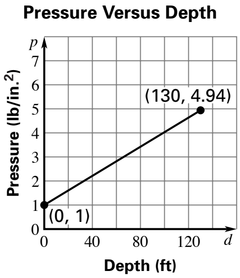
39. 

41. 

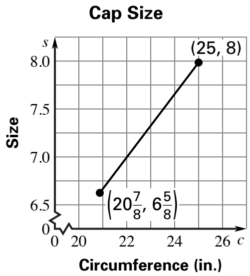
43. linear; -7 **45.** not linear; 1 **47.** not linear; -25
49. 125; the volume of a cube with sides of length 5 units
51. No. *Sample answer:* Not every age corresponds to exactly one place. For example, there were 24-year-olds with finishes of first and third.
53. domain: 1, 5, 6, 10, 12, 25; range: 1, 2, 3, 4, 6, 9;



55. domain: $0 \le d \le 130$; range: $1 \le p \le 4\frac{31}{33}$;



57. domain: $20\frac{7}{8} \le c \le 25$; range: $6\frac{5}{8} \le s \le 8$;



2.1 Mixed Review (p. 74)

65. 1 **67.** $\frac{1}{2}$ **69.** $\frac{1}{4}$ **71.** -7.5
73. $-4\frac{11}{16}$ **75.** $-\frac{12}{11}$ **77.** yes **79.** yes **81.** yes

2.2 Practice (pp. 79–81)

5. undefined; vertical **7.** -1; falls
9. 2; rises **11.** line 2 **13.** neither **15.** parallel **17.** 1
19. undefined **21.** 10; rises **23.** $\frac{1}{2}$; rises **25.** -1; falls
27. undefined; vertical **29.** $-\frac{1}{2}$; falls **31.** undefined; vertical **33.** C **35.** A **37.** line 1 **39.** line 2 **41.** parallel

SELECTED ANSWERS