Dear Calculus II/III student,

I hope that you're all enjoying your first few days of summer! Here's something that will make it a little more fun! Enclosed you will find a packet of review questions that you should complete before the first day of classes. This packet covers mainly limit, differentiation and integration techniques. These are skills that we will work on having down pat so that you won't even have to think about, "how do I solve this problem?" The packet will be graded upon your return to school and will be worth 3 homework assignments (a week's worth of assignments). Solutions are also attached, but I expect to see full work for each problem.

There will be a quiz covering this material in the second week of classes.

Many of you may want to complete this as soon as possible and have it over and done with. If this is the case, please be sure to review your solutions in the days before school starts. The aim of this summer work is to keep your mathematical mind from rusting in the months that you'll spend away from school. We'll be hitting the ground running in August and you don't want to be out of breath. Enjoy!
Email any questions: jbowman@keyschool.org

See you all in August!
Mrs. Bowman

If you come across a topic with which you are uncomfortable, you may find the following websites useful:

http://www.khanacademy.org **Companion Site to our Textbook**

Help is also available in your Calculus notebook or textbook. Did you save your notes?

The following problems involve the skills that you will be using throughout the year in Calculus II/III. Let's see what you remember! We will answer some questions on the first day of class, but your goal is to answer your own questions before class starts. You can phone a friend, google, use khanacademy.org, use your Calculus notes/textbook etc. Be honest with yourself and how comfortable you are with these skills. We'll discuss in August!

Provide your work for these problems on separate paper
Skills

1. Evaluate the following limits:

a.) $\lim_{x\to 0}\frac{\sqrt{4+x}-3}{x+2}$

d.) $\lim_{x\to 1^+}\frac{x^2-5x}{x-1}$

b.) $\lim_{x\to -7}\frac{x^2+5x-14}{x+7}$

e.) $\frac{2n^2+5n-6}{3n^2+10n}$

c.) $\lim_{x\to 0}\frac{\frac{2}{x+2}-1}{x}$

f.) $\frac{sin(3x)}{x}$

2. Differentiate the following functions:

a.) $G(x)=(3x-2)^{10}(5x^2-x+1)^{12}$

b.) $g(\theta)=\tan\left(5\theta^2+3\theta-4\right)$

c.) $y=\frac{\sqrt{2x+3}}{\left(6x^2+7\right)^8}$ (Do not simplify at all—I want to see the rules you've used!)

d.) Find $\frac{dy}{dx}$ implicitly if $\sin^2 y=6xy$

e.) $y=e^{\sqrt{x}}$

f.) $y = x^{\sin x}$

g.) $g(x) = \int_{-1}^{\ln x} \sin t dt$

h.) $f(x) = \ln|\cos(x)|$

i.) $y = \arctan(e^x)$

3.) Evaluate the following integrals:

a.) $\int\left(\sqrt{x} + \sin x - 2x^5\right)dx$

b.) $\int \frac{x^2 + 2x - 3}{x^4} dx$

c.) $\int 2\sqrt{\tan x} \sec^2 x dx$

d.) $\int \frac{x^2}{x^3 + 1} dx$

e.) $\int \frac{3}{\sqrt{4 - (x+1)^2}} dx$

f.) $\int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x} dx$

g.) $\int_0^{\pi} x^2 sinx\, dx$

Applications

4.) Let $f(x) = x^4 - 4x^3 + 4x^2 + 1$

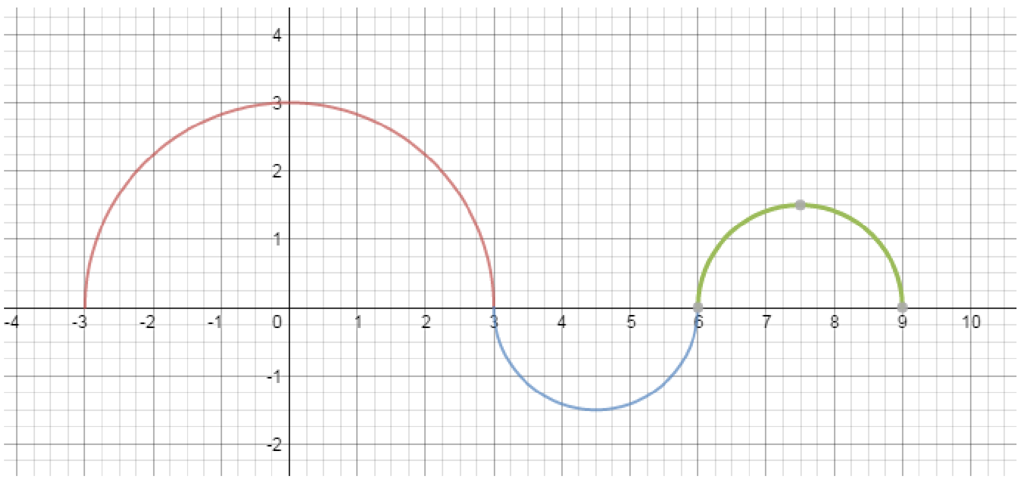
a. Find all critical numbers of *f*.

b. State the intervals over which *f* is increasing or decreasing. Justify your response.

5.) Determine the tangent line approximation for $f(x) = (x+1)^2 + \tan(x)$ at $x = 0$. Use the tangent line approximation to approximate $f(.2)$.

6.) A balloon rises at the rate of 8 feet per second from a point on the ground 60 feet from an observer. Find the rate of change of the angle of elevation when the balloon is 25 feet above the ground.

7.)



The figure above shows the graph of $g(x)$, where g is the derivative of f, for $-3 \le x \le 9$. The graph consists of three semicircular regions and has horizontal tangents at $x = 0$, $x = 4.5$, and $x = 7.5$. [10 pts]

a.) Find all values of x, for $-3 < x < 9$, at which f attains a relative minimum. Justify your answer.

b.) Find all values of x, for $-3 < x < 9$, at which f attains a relative maximum. Justify your answer.

c.) If $f(x)=\int_{-3}^{x} g(t)dt$, find $f(6)$.

d.) Determine the *x*-coordinates where $f''(x)=0$.

8.) Find the area bound by the curves f(x) = 2sin(x) and g(x) = tan(x), $\frac{-\pi}{3}\le x\le\frac{\pi}{3}$

9.) Set up and evaluate the integral that gives the volume of the solid formed by revolving the region bounded by the graphs of f(x) = x^2 and g(x) = x^3 about the line y = 3.
[You are permitted a calculator for your final answer here)

Answer Key

1.) a.) $-\frac{1}{2}$ b.) -9 c.) $-\frac{1}{2}$ d.) $-\infty$ e.) $\frac{2}{3}$ f.) 3

2.) a.) $G'(x) = 6(3x - 2)^9(5x^2 - x + 1)^{11}(85x^2 - 51x + 9)$

b.) $g'(\theta) = sec^2(5\theta^2 + 3\theta - 4)(10\theta + 3)$

2c.) $y' = \dfrac{(6x^2+7)^8\left(\frac{1}{2}\right)(2x+3)^{-\frac{1}{2}}(2) - (2x+3)^{\frac{1}{2}}(8)(6x^2+7)^7(12x)}{(6x^2+7)^{16}}$

2d.) $y' = \dfrac{6y}{\sin(2y) - 6x}$ 2e.) $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$ 2f.) $y' = \left(\dfrac{\sin x}{x} + \ln x \cos x\right) x^{\sin x}$

2g.) $g'(x) = \dfrac{\sin(\ln x)}{x}$ 2h.) $f'(x) = -\tan(x)$ 2i.) $y' = \dfrac{e^x}{1 + e^{2x}}$

3.) a.) $\frac{2}{3}x^{\frac{3}{2}} - \cos x - \frac{1}{3}x^6 + C$ b.) $-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$ c.) $\frac{4}{3}(\tan x)^{\frac{3}{2}} + C$

d.) $\frac{1}{3}\ln|x^3 + 1| + C$ e.) $3\arcsin\left(\frac{x+1}{2}\right) + C$ f.) $\frac{\pi}{4}$ g.) $\pi^2 - 4$

4.) a.) $x = 0, 1, 2$ b.) inc: $(0, 1)\cup(2, \infty)$; dec: $(-\infty, 0)\cup(1, 2)$

5.) $y = 3x + 1$; $f(0.2)\approx 1.6$

6.) $0.1136\ \frac{rad}{sec}$

7.) a.) $f'(x)$ changes from negative to positive at $x = 6$
b.) $f'(x)$ changes from positive to negative at $x = 3$
c.) $\frac{27\pi}{8}$ d.) $x = 0,\ 4.5,\ 7.5$

8.) 0.614