Name: ____ Mr. Stanton/Mr. Lee

Calculus BC: SUMMER ASSIGNMENT 2021

This summer assignment is meant to be a refresher of the most basic and necessary fundamentals of calculus. Although you can do it whenever you want, I STRONGLY SUGGEST saving it until 2 weeks before school, so that it is fresh in your mind for when you get back to school.

This is a REVIEW for a VERY FAIR, in-class exam that will take place about a week after the first day of school.

There will be no in-class review. I will provide you with an answer key on the first day of school. I will make myself available to answer some questions. I know we'll all be a little bit rusty, but you have elected to be in the best math class that the school has to offer, and this is the first step to rising to that challenge. I will be expecting a lot from you next year.

Excellent site for help: Paul's online math notes http://tutorial.math.lamar.edu/

<u>Derivative Review:</u>

1. Determine $f'(x)$. Do ONLY the first step, and do not simplify further.
   a) $f(x) = 7(2x+3)^5$
   b) $f(x) = \frac{x+1}{x-4}$
   c) $f(x) = 4x(3x-4)^3$
   d) $f(x) = \frac{(x^2-5)^3}{2x(3-x)^8}$
   e) $f(x) = ((2x+3)^5 + (4-7x)^6)^8$

2. Determine $\frac{dy}{dx}$ for $y = 3x^2 + 5x$.
3. Determine $\frac{dy}{dt}$ (this is not a typo!) for $y = 3x^2 + 5x$.
4. Find $\frac{dy}{dx}$ by differentiating IMPLICITLY (This means DO NOT get y alone first!): $3x^2y + 2xy^5 + x = 4y$.
5. Find $\frac{dx}{dy}$ (not a typo either!) by differentiating implicitly: $5x^2 + 3y^2 + x + y = 12$

<u>Related Rates</u>

6. The opposite sides of a rectangle are decreasing at a rate of 3 ft/sec, and the other sides are being lengthened so that the rectangle has a constant area of 100 square feet. What is the rate of change of the perimeter at the exact moment that the length of the decreasing side is 10 feet? Indicate if the perimeter is increasing or decreasing at that moment.

<u>Limits</u>

7. Determine each of the following limits. You do not need to show work for it, but for each, indicate the rule or a BRIEF explanation of why the answer is what it is (ie.. "ratio of the coefficients" might be part of one of your answers.)
   a) $\lim_{x\to1}\frac{1}{x-1}$
   b) $\lim_{x\to-\infty}\frac{1}{x-1}$
   c) $\lim_{x\to-\infty}\frac{x}{x-1}$
   d) $\lim_{x\to\infty}\frac{7x}{x-1}$
   e) $\lim_{x\to3^-}\sqrt{x-3}$

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Piecewise Functions

8. Re-define each of the following functions piecewise.
   a) $f(x) = 4|x - 1|$
   b) $f(x) = |x^2 - 10|$

Curve Behavior

9. Consider the curve $f(x) = 2x^3 + 3x^2 - 12x$.
   a) Determine the critical values algebraically.
   b) Determine, **using the first derivative test (and using a little chart)** which critical value is a relative minimum and which is a relative maximum.
   c) Determine any possible points of inflection.
   d) Determine the intervals, if any, where the function is concave up.
   e) Now consider the same curve in the closed interval [-3, 2]. Find f(-3), f(2), and f(*critical points found in step b).* The ABSOLUTE MAXIMUM is the highest of these values. Determine the absolute maximum of the curve in this interval.
10. Consider the function $f(x) = \frac{x+1}{x-3}$.
    a) Determine the critical values algebraically.
    b) Determine, **using the first derivative test (and using a little chart)** which critical value is a relative minimum and which is a relative maximum. If there is none, indicate it.
    c) Determine any possible points of inflection.
    d) Determine the intervals, if any, where the function is concave down.

Tangent and normal lines to a curve

11. Find the equation of the tangent line to $y = 5x^3 + 3x$ at x=2
12. Find the equation of the normal line to the same curve at the same point.

These topics, MVT and IVT were not taught in PCH in 2021. Please learn them on your own on Khan Academy this summer.

Mean Value Theorem and Intermediate Value Theorem

13. Stating the conditions of the Mean Value Theorem, determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for $f(x) = x^3 + 2x^2 - x$ on $[-1,2]$
14. Stating the conditions of the Intermediate Value Theorem, use it to show that $f(x) = 2x^3 - 5x^2 - 10x + 5$ has a root somewhere in the interval $[-1,2]$.