3rd Grade Mathematics – Scope and Sequence – MSD of Pike Township

Purpose Statement

The purpose of this scope and sequence document is to ensure that MSD of Pike Township has a viable and guaranteed Mathematics curriculum. It is meant to provide the foundational skills, strategies, and concepts necessary for our students to leave Pike Township college and career ready. Please remember that this scope and sequence is based on the Indiana Academic Standards and the typical progress of students. Use your professional judgment when addressing the individual needs of your students. If you need to shorten or lengthen a unit, then do so based on mastery of standards, evidence from your classroom assessments and professional observations. Always consider the students' need and interest as well as social studies and science content area topics to guide your units of study. Collaborate with your instructional coach and school librarian to plan and implement the units of study, mini-lesson ideas, and instructional resources.

Components Included in the Scope and Sequence

Mathematics Content

As you work with your students, please remember the following:

- *Students are expected to **apply** the math they previously learned as they progress through the year and to the next level.*
- *Math content builds on previous lessons and years, but students learn in different ways and take different paths to learn concepts. Provide them with many opportunities every day to tackle and master math content in their world.*
- *The workshop model for Reading and Writing is applicable to Mathematics as well. Use your knowledge of creating mini-lessons, allowing for independent and collaborative work time (problem solving), small group instruction, conferring, etc. to guide your math instruction.*
- *All students are mathematicians. Find ways to allow our students to make true connections with math content.*
- *Math instruction should incorporate reading and writing daily.*
- ***Fluency Expectation:***
  - ***Fluently add and subtract within 1000.***
  - ***Fluently multiply using factors 0 – 10 and use the corresponding division facts.***

Process Standards for Mathematics

As you work with your students, please remember the following:

- *The Process Standards for Mathematics are the "how" when delivering mathematics instruction. They rely on students communicating with each other about mathematics in order to learn mathematics.*
- *The **Process Standards for Mathematics** are expected student behaviors.*

1. ***Make sense of problems and persevere in solving them.***
2. ***Reason abstractly and quantitatively.***
3. ***Construct viable arguments and critique the reasoning of others.***
4. ***Model with mathematics.***
5. ***Use appropriate tools strategically.***
6. ***Attend to precision.***
7. ***Look for and make use of structure.***
8. ***Look for and express regularity in repeated reasoning.***

Each Component Includes:

Essential Goals	Priority Standards	Strategies/Skills	Academic Vocabulary	Possible Instructional Resources	Scaffolding Support
These goals define the necessary habits, skills, and dispositions we want students to know and be able to do when the unit is completed.	The Indiana Academic Standards listed represent the priority standards for the unit. Other standards may be taught explicitly or implicitly.	The strategy and skill focus gives guidance for mini-lesson topics and ideas. It also gives ideas for teaching strategies you might rely upon for instruction. The goal is for students to gain understanding of these skills/strategies by the end of the unit.	Academic vocabulary includes the words that are needed to understand the content. You will also teach other vocabulary throughout the rest of your day. Assessment vocabulary should be integrated throughout the year. They should be explicitly taught and used regularly.	Professional and mentor text ideas are suggested in this section. Additionally, this is where you will find your connection to the Go Math resources. Ask your school librarian and/or instructional coach for assistance in this area also!	Ideas for scaffolding support for striving mathematicians, English learners and Exceptional Learners are provided. Please use your available resources to differentiate for students. Ask your building resource teachers for assistance if needed.

Testing Vocabulary to be Taught Throughout the Year

Best: "Which statement **BEST** describes the two shapes?" Although more than one option might make sense, students will need to choose one that is better than all the others.
Choose: "**Choose** TWO fractions that are greater than 0.50." Students will be asked to choose one or more items that fit the criteria.
Complete: "**Complete** the table by filling in the missing numbers." Students may be asked to complete tables, graphs, and/or statements.
Define the Variable: Students should be able to provide a precise description of a variable used in an equation.
Enter: "**Enter** the product. 214 x 12". Students will be asked to enter items. This means they type the answer in the appropriate place.
Greatest: "What is the **GREATEST** number of hats Sarah can buy?"
Identify: "**Identify** all errors in Jenna's work." Students will need to choose one or more items that fit the criteria.
In ALL: "How much money does Amy spend **IN ALL**?" Students will need to give a total. This does not mean to simply apply an operation, though. Students will need to read critically to determine the operation(s) that make(s) sense.
More/Fewer: "How many **MORE** stickers does Jimmy need to complete his collection?" Students will need to compare two quantities and determine how much more or less one quantity is than another.
Plot: "**Plot** an X on the line plot to represent Eric's data." Students will need to place data points on a coordinate grid, data display, or number line.
Represent: "**Represent** 0.20 as a fraction." Students will need to be able to translate between different forms of the same concept (i.e. fractions and decimals, equivalent fractions, equivalent expressions).
Select: "Select the shape(s) that match the given attributes**."** Students will need to choose one or more items that fit the criteria.
Shade: "Shade squares in the grid that represents the given fraction." Students will need to fill in the appropriate amount
Show All Work: Students will need to show all work needed to solve problems in order to receive full credit.
Solve/Evaluate: "Solve. 145 + 82" or **"Evaluate.** 145 + 82" Students will need to give an answer for the test item.
Use Word, Numbers and/or Symbols: "Use words, numbers, and/or symbols to support your answer." Students will need to explain their ideas clearly using math words, numbers or symbols. It should include evidence from the problem and student work.

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Quarter 1 Weeks 1 – 5	Essential Goals	Priority Standards	Strategies/Skills	Academic Vocabulary	Possible Instructional Resources	Differentiation:
Unit 1: **Introducing your Math Block and** **Place Value/Number Sense – Whole Numbers**	• Mathematicians write about and share their thinking. • Mathematicians show their work. • Mathematicians ask questions of themselves, others, and the world around them. • Mathematicians find math ideas in their world. • Mathematicians use words, models, and expanded form to represent numbers up to 10,000. • Mathematicians understand place value in order to write any number up to 10,000 in various combinations. • Mathematicians understand how to round whole numbers to the nearest 10 and 100. • Mathematicians use estimation to decide whether answers are reasonable in their problems. ***text does not emphasize larger numbers (1,000 – 10,000) be sure to include large numbers in your daily work with students.**	3.NS.1 3.NS.2 3.NS.9 3.AT.1	• Introduce procedures and routines. • Help students to see they are mathematicians. • Write about and discuss math concepts daily. • Reinforce there are multiple ways to find solutions to problems. • Compare two whole numbers up to 10,000 using relational symbols (less than, greater than, equal to). • Round whole numbers to the nearest 10 or 100. • Distinguish between place value and place. • Distinguish between digit, number, and numeral. • Compose and decompose whole numbers up to 10,000. • Estimate sums and differences to determine reasonableness of a solution. • Use mental math strategies, daily, to improve number sense and understanding. • Solve real-world addition and subtraction problems within 1,000 (using equations and expressions).	add compatible numbers digit equal to equivalent forms estimate even expanded form explain greater than hundreds less than mathematicians procedures routines model notation number line numeral odd ones pattern place value round standard form strategies subtract sum ten thousands tens thousands	**"First 20 Days"** resources/examples/ideas on P: Drive **Go Math:** IN Success Lessons: 1.0a, 1.0b, 1.0c, 1.0d **Optional Lesson: 1.1** **Weeks of Inspirational Math** https://www.youcubed.org/week-inspirational-math/ **Illustrative Mathematics Tasks:** (www.illustrativemathematics.org) Rounding to the Nearest 10 and 100 (3.NS.9) Classroom Supplies (3.C.1) **Learning Math in the Intermediate Grades (Madison MSD)** https://mathweb.madison.k12.wi.us/files/math/LMIGcomplete.pdf **Good Questions for Math Teaching** Place Value (p. 34-36) Counting & Ordering (p. 37-39) **Inside Mathematics:** A Question of Numbers (3.C.1/PS1, PS6) **http://www.insidemathematics.org/assets/common-core-math-tasks/a%20question%20of%20numbers.pdf** **Mentor Text(s)** A Place for Zero A Million Dots Even Steven and Odd Todd A Million Fish…More or Less How Much is a Million? If You Made a Million	**Spiral review:** **-2.NS.1, 2.NS.2, 2.CA.1** **Prerequisite Skills:** • Knowledge of basic addition and subtraction facts. **Scaffolding Support (ELL; SPED; Striving Learners)** • Go Math: differentiation materials • Open ended problems with multiple entry points • Written instructions • Small group instruction • Graphic Organizers • Manipulatives • Encourage multiple ways to solve problems • Provide models of finished products • Provide picture support • Ask all students to show their strategies while explaining • Send home games to practice specific skills
Process Standards for Mathematics	• Mathematicians progress from concrete to pictorial to more abstract reasoning. • Mathematicians explain the meaning of the symbols they choose. • Mathematicians strategically choose tools to solve problems. • Mathematicians look closely to discern a pattern or structure for the base-ten number system.	PS. 2 PS. 5 PS. 7	• Compose and decompose numbers using equations and expressions. • Make connections and find patterns between different forms of numbers (standard form, expanded form, word form, decomposed, etc.). • Use number lines, fact families, place value charts and other tools to make sense of the structure of the number system. Explain how the tool helped to make connections.			

3.NS.1: Read and write whole numbers up to 10,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 10,000.
3.NS.2: Compare two whole numbers up to 10,000 using >, =, and < symbols.
3.NS.9: Use place value understanding to round 2- and 3-digit whole numbers to the nearest 10 or 100.
3.AT.1: Solve real-world problems involving addition and subtraction of whole numbers within 1000 (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
2.NS.1: Count by ones, twos, fives, tens and hundreds up to at least 1,000.
2.NS.2: Read and write whole numbers up to 1,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 1,000.
2.CA.1: Add and subtract fluently within 100.

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Quarter 1 Weeks 6 – 9	Essential Goals	Priority Standards	Strategies/Skills	Academic Vocabulary	Possible Instructional Resources	Differentiation:
Unit 2: **Addition and Subtraction Strategies with Data Displays**	• Mathematicians interpret in picture graphs, bar graphs, and frequency tables. • Mathematicians collect and record data from observations, surveys, and scientific experiments. • Mathematicians use addition and subtraction to solve multi-step real-world problems. • Mathematicians use estimation to decide whether answers are reasonable in their problems.	3.DA.1 3.AT.1 3.AT.3 3.C.1	• Create scaled picture graphs, bar graphs, and frequency tables. • Collect data through observations, surveys, and experiments (with several categories). • Solve one- and two- step "how many more" and "how many less" problems regarding the data. • Make predictions based on the data. • Create and solve equations using addition and subtraction of whole numbers. • Create and solve addition and subtraction equations within 1,000 for real-world problems. • Solve addition and subtraction equations, with unknowns in all places, for whole numbers up to 1,000. • Compose and decompose whole numbers up to 10,000. • Estimate sums and differences to determine reasonableness of a solution. • Use mental math strategies, daily, to improve number sense and understanding. • Add and subtract within 1,000 fluently.	add bar graph compare conclusions data defend describe difference experiment explain fewer frequency table horizontal bar graph investigate key more observation observations picture graph predictions scale subtract sum survey table tally units vertical bar graph	**Go Math:** Chapter 1 Chapter 2 IN Success Lessons: 2.1a, 2.7a **Illustrative Mathematics Tasks:** (www.illustrativemathematics.org) Classroom Supplies (3.DA.1) **Learning Math in the Intermediate Grades (Madison MSD)** https://mathweb.madison.k12.wi.us/files/math/LMIGcomplete.pdf **Good Questions for Math Teaching** Chance (p. 94-95) **Mentor Text(s)** Probably Pistachio The Best Vacation Ever Do YaWanna Bet? Pigs at Odds A Very Improbably Story It's Probably Penny Bad Luck Brad **Everyday Mathematics Teacher's Guide to Games** **Inside Mathematics:** Adding Numbers (3.C.1, 3.C.5/PS1, PS6) **http://www.insidemathematics.org/assets/common-core-math-tasks/adding%20numbers.pdf**	**Spiral review:** **3.NS.1, 3.NS.9, 2.CA.1** **Prerequisite Skills:** • Knowledge of basic addition and subtraction facts • Skip counting • Identifying (subitizing) whole numbers to 20 **Scaffolding Support (ELL; SPED; Striving Learners)** • Go Math: differentiation materials • Open ended problems with multiple entry points • Written instructions • Small group instruction • Graphic Organizers • Manipulatives • Encourage multiple ways to solve problems • Provide models of finished products • Provide picture support • Ask all students to show their strategies while explaining • Send home games to practice specific skills
Process Standards for Mathematics Problem Solving	• Mathematicians create and use representations to organize, record, and communicate mathematical ideas. • Mathematicians display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication. • Mathematicians apply mathematics to problems arising in everyday life, society, and the workplace.	PS. 2 PS. 3 PS. 4	• Create and use data displays to organize and record numeric and categorical data in meaningful ways. • Read, interpret, and make predictions from picture graphs, bar graphs, and frequency tables. • Model real-world situations with graphs, equations, data, and tables. Use these mathematical models to help answer questions about the situations.			

3.DA.1: Create scaled picture graphs, scaled bar graphs, and frequency tables to represent a data set—including data collected through observations, surveys, and experiments—with several categories. Solve one- and two-step "how many more" and "how many less" problems regarding the data and make predictions based on the data.
3.AT.1: Solve real-world problems involving addition and subtraction of whole numbers within 1000 (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.AT.3: Solve two-step real-world problems using the four operations of addition, subtraction, multiplication and division (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.C.1: Fluently add and subtract whole numbers within 1000 using strategies based on place value, properties of operations, and relationships between addition and subtraction.

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Quarters 2 Weeks 10 -14	Essential Goals	Priority Standards	Strategies/Skills	Academic Vocabulary	Possible Instructional Resources	Differentiation:
Unit 3: **Multiplication & Division**	• Mathematicians understand and apply the inverse relationship between multiplication and division. • Mathematicians represent the concepts of multiplication and division in many ways, in order to develop a deep understanding. • Mathematicians create, describe, and extend number patterns using multiplication and addition. • Mathematicians use algebraic expressions and equations to solve real-world problems.	3.C.2 3.C.3 3.C.4 3.C.5 3.C.6 3.AT.2 3.AT.3 3.AT.4 3.AT.5 3.AT.6	• Use multiple strategies for performing multiplication, including: equal sized groups, arrays, area models, equal jumps on a number line, repeated addition, and multiplication equations. • Use multiple strategies for performing division, including: equal groups, partitioning, sharing, inverse of multiplication, bar models, arrays, repeated subtraction, and division equations. • Understand the inverse relationship between multiplication and division. • Understand and use the distributive, commutative, and associative properties for multiplication. • Understand the properties of 0 and 1 in division and multiplication. • Explain the meaning of quotients and products. • Create and solve equations or expressions using all four operations to represent real-world problems. • Determine the unknown whole number in a multiplication or division equation relating 3 whole numbers (unknown can be in any position). • Create, extend, and give appropriate rule for number patterns within 100. • Represent the concept of multiplication and division in multiple ways. • Interpret whole number quotients of whole numbers. • Know and fluently use multiplication and corresponding division facts from 0 to 10. • Solve real-world multiplication problems within 100 using multiple strategies. • Interpret rules for number patterns using multiplication. • Solve two step, real-world problems using all four operations. • Interpret multiplication equations as equal groups.	addend addition arrays associative property commutative property distributive property divide dividend division divisor equal groups equation even factors hundreds identity property of multiplication inverse operations model multiple multiplication multiply number sentence odd ones patterns place value product quotient related facts repeated addition strategy structure subtraction sum tens unknown	**Go Math:** Chapters 3, 4, 5, 6, 7 IN Success Lessons: 5.5a, 7.9a Optional Lesson: 5.3, 5.4, 5.5 **Skip: Lesson 7.11 – 4th grade standard** **Illustrative Mathematics Tasks:** (www.illustrativemathematics.org) Fish Tanks (3.C.4) Markers in Boxes (3.C.4) Analyzing Word Problems Involving Multiplication (3.AT.2) Gifts from Grandma (3.AT.2) Interpretations of Division (3.AT.2) Valid Equalities? (3.C.5) Class Trip (3.AT.3) Stamp Collection (3.AT.3) **Learning Math in the Intermediate Grades (Madison MSD)** https://mathweb.madison.k12.wi.us/files/math/LMIGcomplete.pdf **Inside Mathematics:** Boxing the Pots (3.C.5, 3.AT.3/PS2, PS7) Houses in a Row (3.AT.3, 3.AT.6/ PS.4, PS.8) http://www.insidemathematics.org/performance-assessment-tasks **Good Questions for Math Teaching** Operations (p. 41-43) **Mentor Text(s)** Two of Everything Divide & Ride Too Many Kangaroo Things to Do! A Remainder of One One Hundred Hungry Ants Sea Squares Anno's Magic Seeds	**Spiral review:** **3.AT.1*, 3.AT.3*, 3.C.1*** **Prerequisite Skills:** • Skip counting with 2's and 5's • Modeling repeated addition with arrays • Use "counting on" strategy and number line to solve addition problems. • Doubles and doubles plus one facts • Equal groups • Renaming groups of tens as hundreds • Multiplication facts through 9 • Counting back to subtract • Inverse operations • Identifying missing factors **Scaffolding Support (ELL; SPED; Striving Learners)** • Go Math: differentiation materials • Open ended problems with multiple entry points • Written instructions • Small group instruction • Graphic Organizers • Manipulatives • Encourage multiple ways to solve problems • Provide models of finished products • Provide picture support • Ask all students to show their strategies while explaining • Send home games to practice specific skills
Process Standards for Mathematics Problem Solving	• Mathematicians frequently check their work to make sure they are on the correct pathway. • Mathematicians check their estimation with the calculation to determine if they are correct. • Mathematicians use appropriate tools strategically. • Mathematicians use place value and other strategies to create their own strategy for multiplying and dividing. • Mathematicians use symbols, expressions, and equations to show their thinking.	PS. 1 PS. 2 PS. 5 PS. 8	• Provide contextual problems and time for students to solve real problems that involve multiplying and dividing. • Explore multiple strategies for multiplication and division to help make sense of different strategies and contexts. • Use number lines, fact families, place value charts, multiplication charts, bar models, related facts and other tools to find patterns and make generalizations about multiplication. • Write and evaluate algebraic and numerical expressions and equations for contextual problems, relating the solution(s) back to the original context. • Number work (rename any number within 1000 in various ways)			

3.C.2: Represent the concept of multiplication of whole numbers with the following models: equal-sized groups, arrays, area models, and equal "jumps" on a number line. Understand the properties of 0 and 1 in multiplication.
3.C.3: Represent the concept of division of whole numbers with the following models: partitioning, sharing, and an inverse of multiplication. Understand the properties of 0 and 1 in division.
3.C.4: Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each).
3.C.5: Multiply and divide within 100 using strategies, such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8), or properties of operations.
3.C.6: Demonstrate fluency with mastery of multiplication facts and corresponding division facts of 0 to 10.
3.AT.2: Solve real-world problems involving whole number multiplication and division within 100 in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.AT.3: Solve two-step real-world problems using the four operations of addition, subtraction, multiplication and division (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.AT.4: Interpret a multiplication equation as equal groups (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). Represent verbal statements of equal groups as multiplication equations.
3.AT.5: Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
3.AT.6: Create, extend, and give an appropriate rule for number patterns within 100 (including patterns in the addition table or multiplication table).

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Quarter 2 Weeks 15 – 18	Essential Goals	Priority Standards	Strategies/Skills	Academic Vocabulary	Possible Instructional Resources	Differentiation:
Unit 4: **Fractions**	• Mathematicians understand why and how to use fractions • Mathematicians show equivalent fractions • Mathematicians compare fractions using objects or pictures • Mathematicians understand a fraction as a number on the number line ***Denominators limited to 2, 3, 4, 6 and 8***	3.NS.3 3.NS.4 3.NS.5 3.NS.6 3.NS.7 3.NS.8 3.G.4	• Understand the meaning of the numerator, the denominator, and their relationship (in relation to one whole). • Compare fractions with the same numerator and/or the same denominator • Understand and generate equivalent fractions using pictures and models. • Represent and explain fractions using pictures, words, diagrams, etc. • Represent fractions on a number line between 0 and 1 • Relate fractions on a ruler to fractions on a number line and pictures of fractions. • Use rulers to measure to the nearest quarter of an inch. • Use shapes and their areas to represent fractions. • Partition shapes into parts with equal areas. Express the area of each part as a unit fraction	denominator eighths equal parts equal shares equal to equivalent equivalent fractions fourths fractions halves inch (in.) numerator sixths thirds unit fraction whole	**Go Math:** Chapters 8: Lessons 1 – 5 Chapter 9 IN Success Lessons: 10.5a, 10.5b **Illustrative Mathematics Tasks:** (www.illustrativemathematics.org) Geometric Pictures of One-Half (3.G.4) Representing Half a Circle (3.G.4) Halves, Thirds and Sixths (3.NS.3, 3.G.4) Naming the Whole for a Fraction (3.NS.3) Closest to ½ (3.NS.4) Find 1 (3.NS.4) Which is Closer to 1? (3.NS.4) **Learning Math in the Intermediate Grades (Madison MSD)** https://mathweb.madison.k12.wi.us/files/math/LMIGcomplete.pdf **Good Questions for Math Teaching** Fractions (p. 26-27) **Inside Mathematics:** Leapfrog Fractions (3.NS.7/PS.2, PS.7) http://www.insidemathematics.org/assets/common-core-math-tasks/time%20to%20get%20clean.pdf **Mentor Text(s)** Fraction Fun Give Me Half!	**Spiral review:** **3.AT.1*, 3.C.1*** **Prerequisite Skills:** • Equal parts • Combining 2-dimensional shapes • Counting equal groups • Identifying parts of a whole **Scaffolding Support (ELL; SPED; Striving Learners)** • Go Math: differentiation materials • Open ended problems with multiple entry points • Written instructions • Small group instruction • Graphic Organizers • Manipulatives • Encourage multiple ways to solve problems • Provide models of finished products • Provide picture support • Ask all students to show their strategies while explaining • Send home games to practice specific skills
Process Standards for Mathematics	• Mathematicians solve real-world problems. • Mathematicians choose appropriate tools, and can explain why they chose the tool, to solve problems. • Mathematicians look for patterns, make generalizations, and apply similar reasoning to other problems..	PS. 5 PS. 6 PS. 8	• Provide contextual problems and time to tackle real-world problems that involve fractions. • Use tools such as rulers, base-ten blocks, number lines, visual models, etc. to solve problems with fractions. • Relate strategies for adding and subtracting whole numbers to fractions.			

3.NS.3: Understand a fraction, 1/b, as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction, a/b, as the quantity formed by a parts of size 1/b. [In grade 3, limit denominators of fractions to 2, 3, 4, 6, 8.]
3.NS.4: Represent a fraction, 1/b, on a number line by defining the interval from 0 to 1 as the whole, and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
3.NS.5: Represent a fraction, a/b, on a number line by marking off lengths 1/b from 0. Recognize that the resulting interval has size a/b, and that its endpoint locates the number a/b on the number line.
3.NS.6: Understand two fractions as equivalent (equal) if they are the same size, based on the same whole or the same point on a number line.
3.NS.7: Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model).
3.NS.8: Compare two fractions with the same numerator or the same denominator by reasoning about their size based on the same whole. Record the results of comparisons with the symbols >, =, or <
3.G.4: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole (1/2, 1/3, 1/4, 1/6, 1/8).

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Quarter 3 Weeks 19 – 24	Essential Goals	Priority Standards	Strategies/Skills	Academic Vocabulary	Possible Instructional Resources	Differentiation:
Unit 5: **Measurement**	• Mathematicians measure line segments to the nearest fourth of an inch • Mathematicians estimate, measure and record the length of the sides of geometric shapes. • Mathematicians use correct labels. • Mathematicians find and use the area and perimeter of geometric shapes. • Mathematicians find the perimeter of polygons given the side lengths. • Mathematicians find unknown side lengths of polygons given the perimeter. • Mathematicians recognize area as an attribute of plane figures and understand concepts of area measurement.	3.NS.3 3.NS.4 3.NS.5 3.NS.6 3.NS.7 3.NS.8 3.G.4 3.M.1 3.M.2 3.M.5 3.M.6 3.M.7 3.DA.2	• Use rulers to measure to the nearest quarter of an inch. • Use shapes and their areas to represent fractions. • Partition shapes into parts with equal areas. Express the area of each part as a unit fraction. • Estimate and measure length (quarter inch), mass (grams and kilograms), weight (pounds), temperature (Celsius and Fahrenheit), and volume (quarts, gallons and liters), using appropriate tools. • Display measurement data on a line plot. • Find and/or measure the perimeter of polygons. • Find area in multiple ways of rectangles, including counting unit squares and arrays. • Relate finding area using arrays to multiplying side lengths of rectangles. • Draw rectangles with the same areas and different perimeters or same perimeters and different areas. • Find unknown side lengths of polygons, given the perimeter. • Estimate and measure mass and volume. • Solve one-step real-world problems involving mass and volume. • Choose appropriate units and tools to measure objects. • Generate measurement data by measuring lengths with rulers.	area compare customary units degrees Celsius degrees Fahrenheit distributive property gallons (gal) gram (g) greater than kilogram (kg) less than liquid volume liter (l) mass multiplication order pattern perimeter pound (lb) quart (qt) repeated addition square unit	**Go Math:** Chapters 10 (lessons 6 – 9), 11 IN Success Lessons 10.9a, 10.9b, 10.9c, 10.9d Optional Lesson: 11.8 **Illustrative Mathematics Tasks:** (www.illustrativemathematics.org) Finding the Area of Polygons (3.M.5) India's Bathroom Tiles (3.M.5) How Heavy? (3.M.1) **Learning Math in the Intermediate Grades (Madison MSD)** https://mathweb.madison.k12.wi.us/files/math/LMIGcomplete.pdf **Good Questions for Math Teaching** Two-Dimensional Shapes (p. 79-81) **Inside Mathematics:** Garden Design (3.M.5/PS.2, PS.3) http://www.insidemathematics.org/performance-assessment-tasks **Mentor Text(s)** Clean Sweep Campers Jump, Kangaroo, Jump! Ed Emberley's Picture Pie A Cloak for the Dreamer	**Spiral review:** **3.C.5*, 3.C.6*, 3.AT.2*, 3.AT.3*** **Prerequisite Skills:** • Combining 2-dimensional shapes • Basic understandings of an inch • Use non-standard units to measure length • Addition with three addends • Modeling with arrays **Scaffolding Support (ELL; SPED; Striving Learners)** • Go Math: differentiation materials • Open ended problems with multiple entry points • Written instructions • Small group instruction • Graphic Organizers • Manipulatives • Encourage multiple ways to solve problems • Provide models of finished products • Provide picture support • Ask all students to show their strategies while explaining • Send home games to practice specific skills
Process Standards for Mathematics	• Mathematicians solve real-world problems. • Mathematicians choose appropriate tools, and can explain why they chose the tool, to solve problems. • Mathematicians look for patterns, make generalizations, and apply similar reasoning to other problems.	PS.1 PS.5 PS.8	• Provide contextual problems and time to tackle real-world problems that involve area and perimeter. • Write and evaluate algebraic and numerical expressions and equations for contextual problems, relating the solution(s) back to the original context. • Use tools such as rulers, base-ten blocks, number lines, visual models, etc. to solve problems with perimeter and area.			

3.M.1: Estimate and measure the mass of objects in grams (g) and kilograms (kg) and the volume of objects in quarts (qt), gallons (gal), and liters (l). Add, subtract, multiply, or divide to solve one-step real-world problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem).
3.M.2: Choose and use appropriate units and tools to estimate and measure length, weight, and temperature. Estimate and measure length to a quarter-inch, weight in pounds, and temperature in degrees Celsius and Fahrenheit.
3.M.5: Find the area of a rectangle with whole-number side lengths by modeling with unit squares, and show that the area is the same as would be found by multiplying the side lengths. Identify and draw rectangles with the same perimeter and different areas or with the same area and different perimeters.
3.M.6: Multiply side lengths to find areas of rectangles with whole-number side lengths to solve real-world problems and other mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
3.M.7: Find perimeters of polygons given the side lengths or by finding an unknown side length.
3.DA.2: Generate measurement data by measuring lengths with rulers to the nearest quarter of an inch. Display the data by making a line plot, where the horizontal scale is marked off in appropriate units, such as whole numbers, halves, or quarters.

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Quarter 3 Weeks 25 – 27	Essential Goals	Priority Standards	Strategies/Skills	Academic Vocabulary	Possible Instructional Resources	Differentiation:
Unit 6: **Time and Money**	• Mathematicians tell time to the nearest minute on an analog clock. • Mathematicians find how much time has elapsed to the nearest minute. • Mathematicians understand and use a.m. and p.m. appropriately. • Mathematicians find the value of any collection of coins and bills. • Mathematicians use play or real money to decide whether there is enough money to make a purchase.	3.M.3 3.M.4	• Read, write, and tell time on analog clocks to the nearest minute. • Use number lines and analog clocks to help determine elapsed time, starting times, or ending times. • Skip count on an analog clock. • Use a.m. and p.m. appropriately. • Match activities with the appropriate time of day. • Add and subtract time intervals in minutes. • Introduce the decimal point as notation for money. • Count collections of bills and coins. • Use the $ and the ¢ symbols appropriately. • Solve real-world problems with money to determine whether there is enough money to make a purchase. • Solve real-world problems involving time in minutes and money in dollars and cents.	a.m. analog clock cents clock decimal point digital clock dime dollars elapsed time equivalent half hour half-dollar hour (hr.) midnight minute (min.) nickel noon notation p.m. penny quarter quarter hour time interval	**Go Math:** Chapter 10: Lessons 1 – 5 IN Success Lessons: 10.5a, 10.5b **Illustrative Mathematics Tasks:** (www.illustrativemathematics.org) Dajuana's Homework (3.M.3) **Learning Math in the Intermediate Grades (Madison MSD)** https://mathweb.madison.k12.wi.us/files/math/LMIGcomplete.pdf **Good Questions for Math Teaching** Time (p. 63-65) Money (p. 22-23) **Inside Mathematics:** Time to Get Clean (3.M.3/PS1, PS5) http://www.insidemathematics.org/assets/common-core-math-tasks/time%20to%20get%20clean.pdf **Mentor Text(s)** Game Time Rodeo Time The Clock Struck One It's About Time	**Spiral review:** **3.AT.3*, 3.C.1*** **Prerequisite Skills:** • Telling time to the half hour • Skip counting by fives **Scaffolding Support (ELL; SPED; Striving Learners)** • Go Math: differentiation materials • Open ended problems with multiple entry points • Written instructions • Small group instruction • Graphic Organizers • Manipulatives • Encourage multiple ways to solve problems • Provide models of finished products • Provide picture support • Ask all students to show their strategies while explaining • Send home games to practice specific skills
Process Standards for Mathematics	• Mathematicians use clocks (analog and digital) to assist in solving problems. • Mathematicians use structure of a clock to assist in skip counting. • Mathematicians find shortcuts to reading time from an analog clock. • Mathematicians use their knowledge of time to determine elapsed time. • Mathematicians precisely label money.	PS. 5 PS. 6 PS. 8	• Allow students to choose (and explain the reasoning behind the choice) tools, such as paper clocks, number lines, play money, real money, drawings, etc. to help them solve problems involving money and time. • Use precise language when discussing and explaining concepts in time and money. • Look for patterns and make generalizations about relationships on analog clocks. • Look for patterns and make generalizations about relationships between different forms of money.			

3.M.3: Tell and write time to the nearest minute from analog clocks, using a.m. and p.m., and measure time intervals in minutes. Solve real world problems involving addition and subtraction of time intervals in minutes.
3.M.4: Find the value of any collection of coins and bills. Write amounts less than a dollar using the ¢ symbol and write larger amounts using the $ symbol in the form of dollars and cents (e.g., $4.59). Solve real-world problems to determine whether there is enough money to make a purchase

3rd Grade Mathematics – Scope and Sequence – MSD of Pike Township

Quarter 4 Weeks 28 – 32	Essential Goals	Priority Standards	Strategies/Skills	Academic Vocabulary	Possible Instructional Resources	Differentiation:
Unit 7: **Geometry**	• Mathematicians identify, classify, and describe geometric figures. • Mathematicians draw lines, points, and line segments and use those words when describing other shapes **Geometric Figures: triangles, circles, quadrilaterals (squares, rectangles, rhombuses), cubes, spheres, prisms, pyramids, cones, cylinders**	3.G.1 3.G.2 3.G.3	• Identify, classify, and describe geometric figures. • Classify 2D and 3D shapes by edges, vertices, and faces. • Understand that squares and rectangles have right angles. • Identify similar and different attributes of shapes that make up larger categories (i.e. squares, rectangles, and others are four sided, and fit into quadrilaterals). • Identify, describe, and draw points, lines, and line segments. • Partition shapes into parts with equal areas. Express the area of each part as a unit fraction.	angle closed shape cone cube cylinder decagon edge endpoint face height hexagon intersecting lines length line line segment octagon open shape parallel lines pentagon perpendicular lines plane shape point polygon quadrilateral ray rectangle rectangular prism rhombus right angle side sphere square square pyramid three-dimensional shape trapezoid triangle two-dimensional shape unit fraction Venn diagram vertex width	**Go Math:** Chapter 12 IN Success Lessons: 12.1a, 12.9a **Illustrative Mathematics Tasks:** (www.illustrativemathematics.org) **Learning Math in the Intermediate Grades (Madison MSD)** **https://mathweb.madison.k12.wi.us/files/math/LMIGcomplete.pdf** **Inside Mathematics:** Which Shape? (3.G.2, 3.G.3/PS.1, PS.6) http://www.insidemathematics.org/assets/common-core-math-tasks/which%20shape.pdf **Good Questions for Math Teaching** Two-Dimensional Shapes (p. 79-81) Three-Dimensional Shapes (p. 85-88) **Mentor Text(s)** Cubes, Cones, Cylinders, Spheres The Greedy Triangle	**Spiral review:** **3.C.1*, 3.AT.1*, 3.M.3*** **Prerequisite Skills:** • Identify plane shapes and number of sides **Scaffolding Support (ELL; SPED; Striving Learners)** • Go Math: differentiation materials • Open ended problems with multiple entry points • Written instructions • Small group instruction • Graphic Organizers • Manipulatives • Encourage multiple ways to solve problems • Provide models of finished products • Provide picture support • Ask all students to show their strategies while explaining • Send home games to practice specific skills
Process Standards for Mathematics Problem Solving	• Mathematicians construct viable arguments and critique the reasoning of others. • Mathematicians use appropriate tools strategically. • Mathematicians use clear definitions when discussing and reasoning about geometric figures.	PS. 1 PS. 3 PS. 6	• Choose multiples strategies, manipulatives, and concrete models to solve problems. Explain why a tools was chosen. • Use precise vocabulary to describe and classify geometric figures. • Justify claims using mathematical arguments. • Politely defend or reject others' claims, using mathematical arguments.			

3.G.1: Identify and describe the following: cube, sphere, prism, pyramid, cone, and cylinder.
3.G.2: Understand that shapes (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize and draw rhombuses, rectangles, and squares as examples of quadrilaterals. Recognize and draw examples of quadrilaterals that do not belong to any of these subcategories.
3.G.3: Identify, describe and draw points, lines and line segments using appropriate tools (e.g., ruler, straightedge, and technology), and use these terms when describing two-dimensional shapes.
3.G.4: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole (1/2, 1/3, 1/4, 1/6, 1/8).

3rd Grade Mathematics – Scope and Sequence – MSD of Pike Township

Quarter 4 Weeks 33 – 36	Essential Goals	Priority Standards	Strategies/Skills	Academic Vocabulary	Possible Instructional Resources	Differentiation:
Unit 8: **Reinforce and Extend Mathematical Understandings of Addition, Subtraction, Multiplication and Division**	• Mathematicians practice critical grade level skills, in a variety of formats, to prepare them for more complex mathematical understandings. • Mathematicians make sense of a variety of problems and persevere in solving them. • Mathematicians reason abstractly and quantitatively to solve problems. • Mathematicians construct viable arguments and critique the reasoning of others. • Mathematicians apply math skills and understandings to solve real-world problems. • Mathematicians use a variety of tools to aid them in solving mathematical and real-world problems. • Mathematicians use structures and patterns to solve problems efficiently.	3.NS.7 3.C.1 3.C.2 3.C.5 3.C.6 3.AT.1 3.AT.2 3.AT.3 3.M.3	• Use pre and post assessments to determine reinforcement or extension of skills and understandings. • Use multiple representations to solve problems. • Connect mathematics to students' daily lives. • Use correct mathematical vocabulary. • Use accurate labels, symbols, and calculations. • Work in teams to solve problems and justify solutions. • Select an appropriate tool to solve a problem and explain why the tool makes sense to use. • Explain whether or not a solution or process is reasonable for the problem situation	abstract reasoning argument construct critique efficient mathematical models persevere precision proficient quantitative reasoning reasonable reasoning repeated reasoning strategically structure	**Go Math:** Feel free to pull ideas from: IN Success Supplement – STEM Activities End of Year Review Projects Unit Projects Getting Ready for 4th Grade **Illustrative Mathematics Tasks:** http://www.illustrativemathematics.org **Learning Math in the Intermediate Grades (Madison MSD)** https://mathweb.madison.k12.wi.us/files/math/LMIGcomplete.pdf **Inside Mathematics:** The Answer is 36 (3.C.5, 3.AT.5/PS.2, PS.7) http://www.insidemathematics.org/assets/common-core-math-tasks/the%20answer%20is%2036.pdf **Making Sense: Teaching and Learning Mathematics with Understanding** **Good Questions for Math Teaching** **Mentor Text** *Mystery Math: A first book of algebra* *Math for all seasons* by Greg Tang *Math Fables: Lessons that count* by Greg Tang	**Scaffolding Support (ELL; SPED; Striving Learners)** • Go Math: differentiation materials • Open ended problems with multiple entry points • Written instructions • Small group instruction • Graphic Organizers • Manipulatives • Encourage multiple ways to solve problems • Provide models of finished products • Provide picture support • Ask all students to show their strategies while explaining • Send home games to practice specific skills

3.NS.7: Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model).
3.C.1: Fluently add and subtract whole numbers within 1000 using strategies based on place value, properties of operations, and relationships between addition and subtraction.
3.M.1: Estimate and measure the mass of objects in grams (g) and kilograms (kg) and the volume of objects in quarts (qt), gallons (gal), and liters (l). Add, subtract, multiply, or divide to solve one-step real-world problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem).
3.C.2: Represent the concept of multiplication of whole numbers with the following models: equal-sized groups, arrays, area models, and equal "jumps" on a number line. Understand the properties of 0 and 1 in multiplication.
3.C.5: Multiply and divide within 100 using strategies, such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8), or properties of operations.
3.C.6: Demonstrate fluency with mastery of multiplication facts and corresponding division facts of 0 to 10.
3.AT.1: Solve real-world problems involving addition and subtraction of whole numbers within 1000 (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.AT.2: Solve real-world problems involving whole number multiplication and division within 100 in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.AT.3: Solve two-step real-world problems using the four operations of addition, subtraction, multiplication and division (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.M.3: Tell and write time to the nearest minute from analog clocks, using a.m. and p.m., and measure time intervals in minutes. Solve real world problems involving addition and subtraction of time intervals in minutes.

Process Standards for Mathematics

The Process Standards demonstrate the ways in which students should develop conceptual understanding of mathematical content, and the ways in which students should synthesize and apply mathematical skills.	
PS.1: Make sense of problems and persevere in solving them.	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway, rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” and "Is my answer reasonable?" They understand the approaches of others to solving complex problems and identify correspondences between different approaches. Mathematically proficient students understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
PS.2: Reason abstractly and quantitatively.	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
PS.3: Construct viable arguments and critique the reasoning of others.	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They analyze situations by breaking them into cases and recognize and use counterexamples. They organize their mathematical thinking, justify their conclusions and communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. They justify whether a given statement is true always, sometimes, or never. Mathematically proficient students participate and collaborate in a mathematics community. They listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
PS.4: Model with mathematics.	Mathematically proficient students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace using a variety of appropriate strategies. They create and use a variety of representations to solve problems and to organize and communicate mathematical ideas. Mathematically proficient students apply what they know and are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
PS.5: Use appropriate tools strategically.	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Mathematically proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Mathematically proficient students identify relevant external mathematical resources, such as digital content, and use them to pose or solve problems. They use technological tools to explore and deepen their understanding of concepts and to support the development of learning mathematics. They use technology to contribute to concept development, simulation, representation, reasoning, communication and problem solving.
PS.6: Attend to precision.	Mathematically proficient students communicate precisely to others. They use clear definitions, including correct mathematical language, in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They express solutions clearly and logically by using the appropriate mathematical terms and notation. They specify units of measure and label axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and check the validity of their results in the context of the problem. They express numerical answers with a degree of precision appropriate for the problem context.
PS.7: Look for and make use of structure.	Mathematically proficient students look closely to discern a pattern or structure. They step back for an overview and shift perspective. They recognize and use properties of operations and equality. They organize and classify geometric shapes based on their attributes. They see expressions, equations, and geometric figures as single objects or as being composed of several objects.
PS.8: Look for and express regularity in repeated reasoning.	Mathematically proficient students notice if calculations are repeated and look for general methods and shortcuts. They notice regularity in mathematical problems and their work to create a rule or formula. Mathematically proficient students maintain oversight of the process, while attending to the details as they solve a problem. They continually evaluate the reasonableness of their intermediate results.

NUMBER SENSE	
3.NS.1	Read and write whole numbers up to 10,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 10,000.
3.NS.2	Compare two whole numbers up to 10,000 using >, =, and < symbols.
3.NS.3	Understand a fraction, 1/b, as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction, a/b, as the quantity formed by *a* parts of size 1/b. [*In grade 3, limit denominators of fractions to 2, 3, 4, 6, 8.*]
3.NS.4	Represent a fraction, 1/b, on a number line by defining the interval from 0 to 1 as the whole, and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
3.NS.5	Represent a fraction, a/b, on a number line by marking off lengths 1/b from 0. Recognize that the resulting interval has size a/b, and that its endpoint locates the number a/b on the number line.
3.NS.6	Understand two fractions as equivalent (equal) if they are the same size, based on the same whole or the same point on a number line.
3.NS.7	Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model).
3.NS.8	Compare two fractions with the same numerator or the same denominator by reasoning about their size based on the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).
3.NS.9	Use place value understanding to round 2- and 3-digit whole numbers to the nearest 10 or 100.

COMPUTATION	
3.C.1	Fluently add and subtract whole numbers within 1000 using strategies and algorithms based on place value, properties of operations, and relationships between addition and subtraction.
3.C.2	Represent the concept of multiplication of whole numbers with the following models: equal-sized groups, arrays, area models, and equal "jumps" on a number line. Understand the properties of 0 and 1 in multiplication.
3.C.3	Represent the concept of division of whole numbers with the following models: partitioning, sharing, and an inverse of multiplication. Understand the properties of 0 and 1 in division.
3.C.4	Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each).

3.C.5	Multiply and divide within 100 using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8), or properties of operations.
3.C.6	Demonstrate fluency with mastery of multiplication facts and corresponding division facts of 0 to 10.

ALGEBRAIC THINKING	
3.AT.1	Solve real-world problems involving addition and subtraction of whole numbers within 1000 (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.AT.2	Solve real-world problems involving whole number multiplication and division within 100 in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.AT.3	Solve two-step real-world problems using the four operations of addition, subtraction, multiplication and division (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).
3.AT.4	Interpret a multiplication equation as equal groups (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). Represent verbal statements of equal groups as multiplication equations.
3.AT.5	Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
3.AT.6	Create, extend, and give an appropriate rule for number patterns within 100 (including patterns in the addition table or multiplication table).

GEOMETRY	
3.G.1	Identify and describe the following: cube, sphere, prism, pyramid, cone, and cylinder.
3.G.2	Understand that shapes (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize and draw rhombuses, rectangles, and squares as examples of quadrilaterals. Recognize and draw examples of quadrilaterals that do not belong to any of these subcategories.
3.G.3	Identify, describe and draw points, lines and line segments using appropriate tools (e.g., ruler, straightedge, and technology), and use these terms when describing two-dimensional shapes.
3.G.4	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole (1/2, 1/3, 1/4, 1/6, 1/8).

MEASUREMENT	
3.M.1	Estimate and measure the mass of objects in grams (g) and kilograms (kg) and the volume of objects in quarts (qt), gallons (gal), and liters (l). Add, subtract, multiply, or divide to solve one-step real-world problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem).
3.M.2	Choose and use appropriate units and tools to estimate and measure length, weight, and temperature. Estimate and measure length to a quarter-inch, weight in pounds, and temperature in degrees Celsius and Fahrenheit.
3.M.3	Tell and write time to the nearest minute from analog clocks, using a.m. and p.m., and measure time intervals in minutes. Solve real-world problems involving addition and subtraction of time intervals in minutes.
3.M.4	Find the value of any collection of coins and bills. Write amounts less than a dollar using the ¢ symbol and write larger amounts using the $ symbol in the form of dollars and cents (e.g., $4.59). Solve real-world problems to determine whether there is enough money to make a purchase.
3.M.5	Find the area of a rectangle with whole-number side lengths by modeling with unit squares, and show that the area is the same as would be found by multiplying the side lengths. Identify and draw rectangles with the same perimeter and different areas or with the same area and different perimeters.
3.M.6	Multiply side lengths to find areas of rectangles with whole-number side lengths to solve real-world problems and other mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
3.M.7	Find perimeters of polygons given the side lengths or given an unknown side length.

DATA ANALYSIS	
3.DA.1	Create scaled picture graphs, scaled bar graphs, and frequency tables to represent a data set—including data collected through observations, surveys, and experiments—with several categories. Solve one- and two-step “how many more” and “how many less” problems regarding the data and make predictions based on the data.
3.DA.2	Generate measurement data by measuring lengths with rulers to the nearest quarter of an inch. Display the data by making a line plot, where the horizontal scale is marked off in appropriate units, such as whole numbers, halves, or quarters.

<table>
<tr><th colspan="3">Number Sense</th></tr>
<tr><th>Third Grade</th><th>Fourth Grade</th><th>Fifth Grade</th></tr>
<tr><td rowspan="2">3.NS.1: Read and write whole numbers up to 10,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 10,000.</td><td rowspan="2">4.NS.1: Read and write whole numbers up to 1,000,000. Use words, models, standard form and expanded form to represent and show equivalent forms of whole numbers up to 1,000,000.</td><td>5.NS.3: Recognize the relationship that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right, and inversely, a digit in one place represents 1/10 of what it represents in the place to its left.</td></tr>
<tr><td>5.NS.4: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</td></tr>
<tr><td>3.NS.2: Compare two whole numbers up to 10,000 using >, =, and < symbols.</td><td>4.NS.2: Compare two whole numbers up to 1,000,000 using >, =, and < symbols.</td><td></td></tr>
<tr><td>3.NS.3: Understand a fraction, 1/b, as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction, a/b, as the quantity formed by a parts of size 1/b. [In grade 3, limit denominators of fractions to 2, 3, 4, 6, 8.]</td><td rowspan="3">4.NS.3: Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. Name and write mixed numbers using objects or pictures. Name and write mixed numbers as improper fractions using objects or pictures.</td><td rowspan="5">5.NS.2: Explain different interpretations of fractions, including: as parts of a whole, parts of a set, and division of whole numbers by whole numbers.</td></tr>
<tr><td>3.NS.4: Represent a fraction, 1/b, on a number line by defining the interval from 0 to 1 as the whole, and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.</td></tr>
<tr><td>3.NS.5: Represent a fraction, a/b, on a number line by marking off lengths 1/b from 0. Recognize that the resulting interval has size a/b, and that its endpoint locates the number a/b on the number line.</td></tr>
<tr><td>3.NS.6: Understand two fractions as equivalent (equal) if they are the same size, based on the same whole or the same point on a number line.</td><td rowspan="2">4.NS.4: Explain why a fraction, a/b, is equivalent to a fraction, (n × a)/(n × b), by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. [In grade 4, limit denominators of fractions to 2, 3, 4, 5, 6, 8, 10, 25, 100.]</td></tr>
<tr><td>3.NS.7: Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model).</td></tr>
<tr><td rowspan="2">3.NS.8: Compare two fractions with the same numerator or the same denominator by reasoning about their size based on the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).</td><td>4.NS.5: Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, or by comparing to a benchmark, such as 0, 1/2, and 1). Recognize comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions (e.g., by using a visual fraction model).</td><td rowspan="2">5.NS.1: Use a number line to compare and order fractions, mixed numbers, and decimals to thousandths. Write the results using >, =, and < symbols.</td></tr>
<tr><td>4.NS.7: Compare two decimals to hundredths by reasoning about their size based on the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions (e.g., by using a visual model).</td></tr>
<tr><td>3.NS.9: Use place value understanding to round 2- and 3-digit whole numbers to the nearest 10 or 100.</td><td>4.NS.9: Use place value understanding to round multi-digit whole numbers to any given place value.</td><td>5.NS.5: Use place value understanding to round decimal numbers up to thousandths to any given place value.</td></tr>
<tr><td rowspan="2"></td><td>4.NS.6: Write tenths and hundredths in decimal and fraction notations. Use words, models, standard form and expanded form to represent decimal numbers to hundredths. Know the fraction and decimal equivalents for halves and fourths (e.g., 1/2 = 0.5 = 0.50, 7/4 = 1 3/4 = 1.75).</td><td>5.NS.6: Understand, interpret, and model percents as part of a hundred (e.g. by using pictures, diagrams, and other visual models).</td></tr>
<tr><td>4.NS.8: Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number.</td><td></td></tr>
</table>

<table>
<tr><th colspan="3">Computation</th></tr>
<tr><th>Third Grade</th><th>Fourth Grade</th><th>Fifth Grade</th></tr>
<tr><td>3.C.1: Add and subtract whole numbers fluently within 1000.</td><td>4.C.1: Add and subtract multi-digit whole numbers fluently using a standard algorithmic approach.</td><td></td></tr>
<tr><td>3.C.2: Represent the concept of multiplication of whole numbers with the following models: equal-sized groups, arrays, area models, and equal "jumps" on a number line. Understand the properties of 0 and 1 in multiplication.</td><td>4.C.7: Show how the order in which two numbers are multiplied (commutative property) and how numbers are grouped in multiplication (associative property) will not change the product. Use these properties to show that numbers can by multiplied in any order. Understand and use the distributive property.</td><td>5.C.3: Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.</td></tr>
<tr><td>3.C.3: Represent the concept of division of whole numbers with the following models: partitioning, sharing, and an inverse of multiplication. Understand the properties of 0 and 1 in division.</td><td rowspan="2">4.C.3: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Describe the strategy and explain the reasoning.</td><td rowspan="2">5.C.2: Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Describe the strategy and explain the reasoning used.</td></tr>
<tr><td>3.C.4: Interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each).</td></tr>
<tr><td>3.C.5: Multiply and divide within 100 using strategies, such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8), or properties of operations.</td><td>4.C.2: Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Describe the strategy and explain the reasoning.</td><td></td></tr>
<tr><td>3.C.6: Demonstrate fluency with multiplication facts and corresponding division facts of 0 to 10.</td><td>4.C.4: Multiply fluently within 100.</td><td>5.C.1: Multiply multi-digit whole numbers fluently using a standard algorithmic approach.</td></tr>
<tr><td rowspan="2"></td><td>4.C.5: Add and subtract fractions with common denominators. Decompose a fraction into a sum of fractions with common denominators. Understand addition and subtraction of fractions as combining and separating parts referring to the same whole.</td><td rowspan="2">5.C.4: Add and subtract fractions with unlike denominators, including mixed numbers.</td></tr>
<tr><td>4.C.6: Add and subtract mixed numbers with common denominators (e.g. by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction).</td></tr>
<tr><td rowspan="5" colspan="2"></td><td>5.C.5: Use visual fraction models and numbers to multiply a fraction by a fraction or a whole number.</td></tr>
<tr><td>5.C.6: Explain why multiplying a positive number by a fraction greater than 1 results in a product greater than the given number. Explain why multiplying a positive number by a fraction less than 1 results in a product smaller than the given number. Relate the principle of fraction equivalence, a/b = (n × a)/(n × b), to the effect of multiplying a/b by 1.</td></tr>
<tr><td>5.C.7: Use visual fraction models and numbers to divide a unit fraction by a non-zero whole number and to divide a whole number by a unit fraction.</td></tr>
<tr><td>5.C.8: Add, subtract, multiply, and divide decimals to hundredths, using models or drawings and strategies based on place value or the properties of operations. Describe the strategy and explain the reasoning.</td></tr>
<tr><td>5.C.9: Evaluate expressions with parentheses or brackets involving whole numbers using the commutative properties of addition and multiplication, associative properties of addition and multiplication, and distributive property.</td></tr>
</table>

<table>
<tr><th colspan="3">Algebraic Thinking</th></tr>
<tr><th>Third Grade</th><th>Fourth Grade</th><th>Fifth Grade</th></tr>
<tr><td>3.AT.1: Solve real-world problems involving addition and subtraction of whole numbers within 1000 (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).</td><td>4.AT.1: Solve real-world problems involving addition and subtraction of multi-digit whole numbers (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).</td><td></td></tr>
<tr><td rowspan="2">3.AT.2: Solve real-world problems involving whole number multiplication and division within 100 in situations involving equal groups, arrays, and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).</td><td>4.AT.2: Recognize and apply the relationships between addition and multiplication, between subtraction and division, and the inverse relationship between multiplication and division to solve real-world and other mathematical problems.</td><td rowspan="3">5.AT.1: Solve real-world problems involving multiplication and division of whole numbers (e.g. by using equations to represent the problem). In division problems that involve a remainder, explain how the remainder affects the solution to the problem.</td></tr>
<tr><td rowspan="2">4.AT.4: Solve real-world problems with whole numbers involving multiplicative comparison (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem), distinguishing multiplicative comparison from additive comparison. [In grade 4, division problems should not include a remainder.]</td></tr>
<tr><td>3.AT.3: Solve two-step real-world problems using the four operations of addition, subtraction, multiplication and division (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).</td></tr>
<tr><td>3.AT.4: Interpret a multiplication equation as equal groups (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). Represent verbal statements of equal groups as multiplication equations.</td><td>4.AT.3: Interpret a multiplication equation as a comparison (e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7, and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations.</td><td></td></tr>
<tr><td>3.AT.5: Determine the unknown whole number in a multiplication or division equation relating three whole numbers.</td><td colspan="2"></td></tr>
<tr><td>3.AT.6: Create, extend, and give an appropriate rule for number patterns using multiplication within 1000.</td><td>4.AT.6: Understand that an equation, such as y = 3x + 5, is a rule to describe a relationship between two variables and can be used to find a second number when a first number is given. Generate a number pattern that follows a given rule.</td><td>5.AT.8: Define and use up to two variables to write linear expressions that arise from real-world problems, and evaluate them for given values.</td></tr>
<tr><td rowspan="8"></td><td rowspan="2">4.AT.5: Solve real-world problems involving addition and subtraction of fractions referring to the same whole and having common denominators (e.g., by using visual fraction models and equations to represent the problem).</td><td>5.AT.2: Solve real-world problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models and equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and assess whether the answer is reasonable.</td></tr>
<tr><td rowspan="7"></td></tr>
<tr><td>5.AT.3: Solve real-world problems involving multiplication of fractions, including mixed numbers (e.g., by using visual fraction models and equations to represent the problem).</td></tr>
<tr><td>5.AT.4: Solve real-world problems involving division of unit fractions by non-zero whole numbers, and division of whole numbers by unit fractions (e.g., by using visual fraction models and equations to represent the problem).</td></tr>
<tr><td>5.AT.5: Solve real-world problems involving addition, subtraction, mutliplication, and division with decimals to hundredths, including problems that involve money in decimal notation (e.g. by using equations to represent the problem).</td></tr>
<tr><td>5.AT.6: Graph points with whole number coordinates on a coordinate plane. Explain how the coordinates relate the point as the distance from the origin on each axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).</td></tr>
<tr><td>5.AT.7: Represent real-world problems and equations by graphing ordered pairs in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.</td></tr>
</table>

Geometry		
Third Grade	**Fourth Grade**	**Fifth Grade**
3.G.1: Identify and describe the following: cube, sphere, prism, pyramid, cone, and cylinder.		
3.G.2: Understand that shapes (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize and draw rhombuses, rectangles, and squares as examples of quadrilaterals. Recognize and draw examples of quadrilaterals that do not belong to any of these subcategories.	**4.G.5:** Classify triangles and quadrilaterals based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles (right, acute, obtuse).	**5.G.2:** Identify and classify polygons including quadrilaterals, pentagons, hexagons, and triangles (equilateral, isosceles, scalene, right, acute and obtuse) based on angle measures and sides. Classify polygons in a hierarchy based on properties.
3.G.3: Identify, describe and draw points, lines and line segments using appropriate tools (e.g., ruler, straightedge, and technology), and use these terms when describing two-dimensional shapes.	**4.G.1:** Identify, describe, and draw parallelograms, rhombuses, and trapezoids using appropriate tools (e.g., ruler, straightedge and technology).	**5.G.1:** Identify, describe, and draw triangles (right, acute, obtuse) and circles using appropriate tools (e.g., ruler or straightedge, compass and technology). Understand the relationship between radius and diameter.
	4.G.3: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint.	
	4.G.4: Identify, describe, and draw rays, angles (right, acute, obtuse), and perpendicular and parallel lines using appropriate tools (e.g., ruler, straightedge and technology). Identify these in two-dimensional figures.	
3.G.4: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole (1/2, 1/3, 1/4, 1/6, 1/8).	**4.G.2:** Recognize and draw lines of symmetry in two-dimensional figures. Identify figures that have lines of symmetry.	

Data Analysis (and Statistics in Gr.5)		
Third Grade	**Fourth Grade**	**Fifth Grade**
3.DA.1: Create scaled picture graphs, scaled bar graphs, and frequency tables to represent a data set—including data collected through observations, surveys, and experiments—with several categories. Solve one- and two-step "how many more" and "how many less" problems regarding the data and make predictions based on the data.	**4.DA.1:** Formulate questions that can be addressed with data. Use observations, surveys, and experiments to collect, represent, and interpret the data using tables (including frequency tables), line plots, and bar graphs.	**5.DS.1:** Formulate questions that can be addressed with data and make predictions about the data. Use observations, surveys, and experiments to collect, represent, and interpret the data using tables (including frequency tables), line plots, bar graphs, and line graphs. Recognize the differences in representing categorical and numerical data.
3.DA.2: Generate measurement data by measuring lengths with rulers to the nearest quarter of an inch. Display the data by making a line plot, where the horizontal scale is marked off in appropriate units, such as whole numbers, halves, or quarters.	**4.DA.2:** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using data displayed in line plots.	
	4.DA.3: Interpret data displayed in a circle graph.	
		5.DS.2: Understand and use measures of center (mean and median) and frequency (mode) to describe a data set.

Measurement		
Third Grade	**Fourth Grade**	**Fifth Grade**
3.M.1: Estimate and measure the mass of objects in grams (g) and kilograms (kg) and the volume of objects in quarts (qt), gallons (gal), and liters (l). Add, subtract, multiply, or divide to solve one-step real-world problems involving masses or volumes that are given in the same units (e.g., by using drawings, such as a beaker with a measurement scale, to represent the problem).	**4.M.2**: Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; hr, min, sec. Express measurements in a larger unit in terms of a smaller unit within a single system of measurement. Record measurement equivalents in a two-column table.	**5.M.1:** Convert among different-sized standard measurement units within a given measurement system, and use these conversions in solving multi-step real-world problems.
	4.M.3: Use the four operations (addition, subtraction, multiplication and division) to solve real-world problems involving distances, intervals of time, volumes, masses of objects, and money. Include addition and subtraction problems involving simple fractions and problems that require expressing measurements given in a larger unit in terms of a smaller unit.	
3.M.2: Choose and use appropriate units and tools to estimate and measure length, weight, and temperature. Estimate and measure length to a quarter-inch, weight in pounds, and temperature in degrees Celsius and Fahrenheit.	**4.M.1:** Measure length to the nearest quarter-inch, eighth-inch, and millimeter.	
3.M.3: Tell and write time to the nearest minute from analog clocks, using a.m. and p.m., and measure time intervals in minutes. Solve real-world problems involving addition and subtraction of time intervals in minutes.		
3.M.4: Find the value of any collection of coins and bills. Write amounts less than a dollar using the ¢ symbol and write larger amounts using the $ symbol in the form of dollars and cents (e.g., $4.59). Solve real-world problems to determine whether there is enough money to make a purchase.		
3.M.5: Find the area of a rectangle with whole-number side lengths by modeling with unit squares, and show that the area is the same as would be found by multiplying the side lengths. Identify and draw rectangles with the same perimeter and different areas or with the same area and different perimeters.	**4.M.4:** Apply the area and perimeter formulas for rectangles to solve real-world problems and other mathematical problems. Recognize area as additive and find the area of complex shapes composed of rectangles by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts; apply this technique to solve real-world problems and other mathematical problems involving shapes.	**5.M.2:** Find the area of a rectangle with fractional side lengths by modeling with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
3.M.6: Multiply side lengths to find areas of rectangles with whole-number side lengths to solve real-world problems and other mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.		**5.M.3:** Develop and use formulas for the area of triangles, parallelograms and trapezoids. Solve real-world and other mathematical problems that involve perimeter and area of triangles, parallelograms and trapezoids, using appropriate units for measures.
3.M.7: Find perimeters of polygons given the side lengths or by finding an unknown side length.		
	4.M.5: Understand that an angle is measured with reference to a circle, with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. Understand an angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure other angles. Understand an angle that turns through n one-degree angles is said to have an angle measure of n degrees.	
	4.M.6: Measure angles in whole-number degrees using appropriate tools. Sketch angles of specified measure.	
		5.M.4: Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths or multiplying the height by the area of the base.
		5.M.5: Apply the formulas V = l × w × h and V = B × h for right rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths to solve real-world problems and other mathematical problems.
		5.M.6: Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems and other mathematical problems.