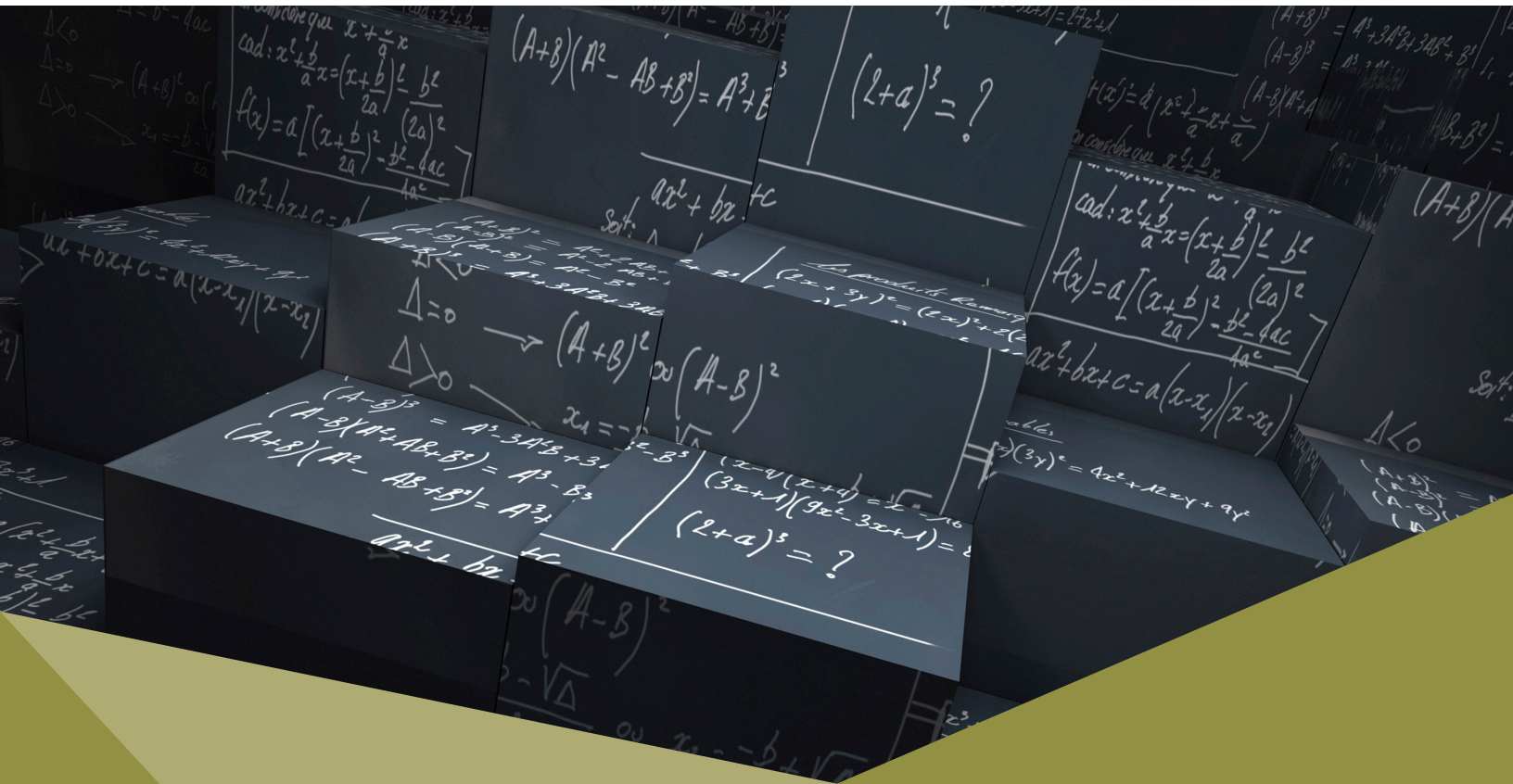


# Middle School Mathematics Parent Handbook

Common Core State Standards for Mathematics  
For California Public Schools  
Grades 6-8



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Middle School Mathematics Parent Handbook

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# Introduction

“These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that these standards are not just promises to our children, but promises we intend to keep.”

*National Governors  
Association Center for Best  
Practice and Council of Chief  
State of School Officers  
(2010)  
Common Core State  
Standards for Mathematics*

The purpose of this handbook is to provide parents and guardians with an introduction to the Common Core State Standards for Mathematics for California public middle schools.

*“If I had an hour to solve a problem I’d spend 55 minutes thinking about the problem and 5 minutes thinking about solutions.”*

Albert Einstein

The challenges our students face in the 21st century global economy are continually changing. In order to be competitive in this global economy, students need to develop the skills to be able to problem-solve creatively.

In recent years, mathematics education in California has focused more on getting an answer than understanding the problem. There are many factors that have contributed to this. The 1997 Mathematics Standards moved California in a good direction with consistent mathematical content being taught at each grade level or high school course. However, the Common Core State Standards for Mathematics are a call to take the next step.

The goal of the Common Core State Standards for Mathematics is for students to be college and career ready upon graduation from high school and to assist students in becoming competitive in a global economy. Therefore, the Common Core State Standards for Mathematics provide not only for rigorous curriculum and instruction, but also conceptual understanding, procedural skill and fluency and the ability to apply mathematics. Students will develop the skills to be able to problem-solve creatively and not be satisfied by just arriving at an answer, thus meeting the challenges of the 21st century.



# Background Information

“... more time for teachers to teach and students to master concepts.”

Jason Zimba,  
Common Core  
State Standards for  
Mathematics Lead  
Author

These new Standards for Mathematics have been developed to provide students with the knowledge, skills, and understanding in mathematics to be college and career ready when they complete high school. They are internationally benchmarked and assist students in their preparation for enrollment at a public or private university.

Common Core State Standards Initiative Mission Statement:

“The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy.”

These standards also provide focus, coherence, and rigor.

Focus	Coherence	Rigor
“Focus implies that instruction should focus deeply on only those concepts that are emphasized in the standards so that students can gain strong foundational conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems inside and outside the mathematics classroom.”	“Coherence arises from mathematical connections. Some of the connections in the standards knit topics together at a single grade level. Most connections are vertical, as the standards support a progression of increasing knowledge, skill, and sophistication across the grades.”	“Rigor requires that conceptual understanding, procedural skill and fluency, and application be approached with equal intensity.”

The Common Core State Standards for Mathematics include two types of standards:

1. Eight Standards for Mathematical Practice that are the same in each grade level and high school mathematics course.
2. Mathematical Content Standards for each grade level.

“Together these standards address both ‘habits of mind’ that students should develop to foster mathematical understanding and expertise and skills and knowledge – what students need to know and be able to do.

The mathematical content standards were built on progressions of topics across a number of grade levels, informed both by research on children’s cognitive development and by the logical structure of mathematics.”

Adapted from California Common Core State Standards – Mathematics Introduction, page 2

The California State Board of Education adopted these Standards on August 2, 2010, after determining that they were at least as rigorous as the current standards. The Common Core State Standards for Mathematics were the result of a state-led movement by the National Governors Association and the Council of Chief State School Officers. Currently, most states have adopted the Common Core State Standards for Mathematics.

## New Assessments

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For the past several years, the California Standards Test (CST) has been used to assess student understanding of the mathematics content standards. These assessments are also referred to as the [STAR Program \(Standardized Testing and Reporting\)](#). In 2014-15, the STAR will be replaced by a new assessment system that is being developed by the [Smarter Balanced Assessment Consortium](#) to test student knowledge of the Common Core State Standards for Mathematics. The new assessments will be very different from the CST's in terms of the test structure, the rigor of the mathematical content, and the delivery system. It is planned that by 2016, the entire test will be delivered on-line to each student. At the high school level, the new test will be administered at the end of eleventh grade. In addition to this test, students will still need to pass the High School Exit Exam (CAHSEE). This handbook will provide some of the [sample test items](#) that highlight characteristics of the changes and the rigor of the content that is expected at each grade level of middle school mathematics.

## Standards for Mathematical Practice

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The Standards for Mathematical Practice describe behaviors that all students will develop in the Common Core State Standards for Mathematics. These practices rest on important “processes and proficiencies” including problem solving, reasoning and proof, communication, representation, and making connections. These practices will allow students to understand and apply mathematics with confidence.

1. Make sense of problems and persevere in solving them.
  - Find meaning in problems
  - Analyze, predict and plan solution pathways
  - Verify answers
  - Ask themselves the question: “Does this make sense?”
2. Reason abstractly and quantitatively.
  - Make sense of quantities and their relationships in problems
  - Create coherent representations of problems
3. Construct viable arguments and critique the reasoning of others.
  - Understand and use information to construct arguments
  - Make and explore the truth of conjectures
  - Justify conclusions and respond to arguments of others
4. Model with mathematics.
  - Apply mathematics to problems in everyday life
  - Identify quantities in a practical situation
  - Interpret results in the context of the situation and reflect on whether the results make sense
5. Use appropriate tools strategically.
  - Consider the available tools when solving problems
  - Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)
6. Attend to precision.
  - Communicate precisely to others
  - Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes
  - Calculate accurately and efficiently
7. Look for and make use of structure.
  - Discern patterns and structures
  - Can step back for an overview and shift perspective
  - See complicated things as single objects or as being composed of several objects
8. Look for and express regularity in repeated reasoning.
  - When calculations are repeated, look for general methods, patterns and shortcuts
  - Be able to evaluate whether an answer makes sense

## Introduction

Prior to Grade 6, the focus of K-5 mathematics in the Common Core Standards will provide a student with a solid foundation. The focus is to develop a strong understanding of number and number sense. Students will be able to add, subtract, multiply, and divide fractions and decimals. Also, a student will develop a conceptual understanding of fractions. These skills and understanding will support their success as they move through middle school mathematics.

## Critical Areas of Instruction

In Grade 6 your child's mathematics experience will focus on four critical areas:

- (1) Connecting ratio and rate to whole number multiplication and division, and using concepts of ratio and rate to solve problems
- (2) Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers
- (3) Writing, interpreting, and using expressions and equations
- (4) Developing understanding of statistical thinking

## Specific Content Examples for the Critical Areas

- Understand ratio concepts and use ratio reasoning to solve problems
- Apply and extend previous understandings of multiplication and division to divide fractions by fractions
- Compute fluently with multi-digit numbers and find common factors and multiples
- Apply and extend previous understandings of numbers to the system of rational numbers
- Apply and extend previous understandings of arithmetic to algebraic expressions
- Reason about and solve one-variable equations and inequalities
- Represent and analyze quantitative relationships between dependent and independent variables
- Solve real-world and mathematical problems involving area, surface area, and volume
- Develop understanding of statistical variability
- Summarize and describe distributions

Adapted from California Common Core State Standards – Mathematics  
page 40



## Smarter Balanced Assessment Consortium Examples

Below are some sample test items (problems) from the Smarter Balanced Assessment Consortium for Grade 6. This is the consortium that is developing the new assessments (see New Assessments section in this handbook).



### Example 1

Jamal is filling bags with sand. All of the bags are the same size. Each bag must weigh less than 50 pounds. One sand bag weighs 57 pounds and another sand bag

weighs 41 pounds. Explain whether Jamal can pour sand from one bag into the other so that the weight of each bag is less than 50 pounds.

#### Sample Response:

Since the mean is less than 50,  $\frac{57 + 41}{2} = 49$ , it is possible to move sand between bags so that each bag weighs 49 pounds. Therefore,


8 pounds of sand could be moved from the 57-pound bag and placed in the 41-pound bag.

Example 1 connects student's work with operations of earlier grades to their work with statistics in Grade 6 (Critical Area #4, on the previous page). In Grade 6 students generate equivalent algebraic expressions. In Grade 7 these are expanded to include expressions with rational coefficients, and in Grade 8 students use earlier strategies to solve increasingly complex equations. A student is also asked to explain their thinking.

Smarter Balanced Assessment Consortium  
Sample Items and Performance Tasks

<http://www.smarterbalanced.org/sample-items-and-performance-tasks/>

**Smarter Balanced Assessment Consortium Examples - continued**

 **Example 2**

This problem is an interactive problem delivered on the computer. A student is asked to determine if each mathematical expression is equivalent to a given expression. The student will select yes to indicate an equivalent mathematical expression and no if it is not an equivalent mathematical expression. (Critical Area #2, see page 6).

Look at each expression. Is it equivalent to  $36x + 24y$ ?

Select Yes or No for expressions A-C.

A.  $6(6x + 4y)$        Yes       No  
 B.  $30(6x - 6y)$        Yes       No  
 C.  $12(x + 2y + 2x)$        Yes       No

**Sample Response:**


A - Yes, B - No, C - Yes

Smarter Balanced Assessment Consortium  
 Sample Items and Performance Tasks

<http://www.smarterbalanced.org/sample-items-and-performance-tasks/>

**Specific Content Examples for the Critical Areas**

Understand and solve problems using ratio concepts.

 **Example**

The students in Mr. Hill’s class played games at recess.

6 boys played soccer, 4 girls played soccer, 2 boys jumped rope, 8 girls jumped rope

Afterward, Mr. Hill asked the students to compare the boys and girls playing different games.

Mika said, “Four more girls jumped rope than played soccer.” Chaska said, “For every girl that played soccer, two girls jumped rope.”

Mr. Hill said, “Mika compared the girls by looking at the difference and Chaska compared the girls using a ratio.”

a. Compare the number of boys who played soccer and jumped rope using the difference. Write your answer as a sentence as Mika did.

b. Compare the number of boys who played soccer and jumped rope using a ratio. Write your answer as a sentence as Chaska did.

c. Compare the number of girls who played soccer to the number of boys who played soccer using a ratio. Write your answer as a sentence as Chaska did.

**Sample Response:**

a. Four more boys played soccer than jumped rope.

b. For every three boys that played soccer, one boy jumped rope. Therefore the ratio of the number of boys that played soccer to the number of boys that jumped rope is 3:1 (or “three to one”).

c. For every two girls that played soccer, three boys played soccer. Therefore the ratio of the number of girls that played soccer to the number of boys that played soccer is 2:3 (or “two to three”).

Illustrative Mathematics  
 Grade 6, Ratios and Proportional Relationships  
<http://www.illustrativemathematics.org/standards/k8>



Specific Content Examples for the Critical Areas - continued

Apply and extend student understanding of multiplication and division of fractions.



**Example**

It requires  $\frac{1}{4}$  of a credit to play a video game for one minute.

- a. Emma has  $\frac{7}{8}$  credits. Can she play for more or less than one minute? Explain how you know.

- b. How long can Emma play the video game with her  $\frac{7}{8}$  credits?

**Sample Response:**

- a. The video game requires  $\frac{1}{4}$  of a credit for one minute of playing time, which is the same as saying that the video game requires  $\frac{2}{8}$  of a credit for one minute of playing time. If Emma has  $\frac{7}{8}$  of a credit, she certainly has enough credit to play for more than one minute because  $\frac{7}{8}$  is more than  $\frac{2}{8}$ .

$$\frac{7}{8} > \frac{2}{8} \text{ and } \frac{2}{8} = \frac{1}{4} \text{ thus } \frac{7}{8} > \frac{1}{4}$$

- b. **Solution 1:**

The video game requires  $\frac{1}{4}$  of a credit for one minute of playing time, and Emma has  $\frac{7}{8}$  of a credit. If we think of  $\frac{1}{4}$  of a credit as a group, then we are really being asked, "How many groups of  $\frac{1}{4}$  are in  $\frac{7}{8}$ ?"

Using the standard fraction division algorithm, we get:

$$\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \times \frac{4}{1} = \frac{28}{8} = \frac{7}{2} = 3 \frac{1}{2}$$

Emma can play the video game for  $3 \frac{1}{2}$  minutes with her  $\frac{7}{8}$  of a credit.

- b. **Solution 2:**

We know that  $\frac{2}{8}$  of a credit gives 1 minute, so  $\frac{1}{8}$  of a credit gives  $\frac{1}{2}$  minute.

Since  $\frac{1}{8}$  of a credit would give  $\frac{1}{2}$  a minute, we know that  $\frac{7}{8}$  of a credit would give  $7 \times \frac{1}{2} = \frac{7}{2}$  minutes.

**Specific Content Examples for the Critical Areas** - continued

Apply and extend previous understandings of arithmetic to algebra. Examples of previous understanding will be built on and extended.

 **Example 1**

In Grades 3-5, students use the “x” as the symbol for multiplication. In Grade 6 and after students will transition to use a “•” as the notation for multiplication. The symbol for division “÷” in  $18 \div 2$  is sometimes replaced with  $\frac{18}{2}$ .

Grades 3-5	6th Grade through higher mathematics
$5 \times 6$	$5 \cdot 6$
$3 \times n$	$3n$
$18 \div a$	$\frac{18}{a}$

 **Example 2**

Students have been using the letters to represent an unknown quantity in word problems. In sixth grade, they begin to use variables in expressions and use repeated calculations.

Grades 3-5	6th Grade through higher mathematics
$5 + a = 8$ Five apples were on the table. Carlos put more apples on the table. There are now eight apples on the table. How many apples did Carlos add?	$10 - p$ could be used to represent the change from a \$10 bill after buying a book for various prices. (see Diagram 1)

Diagram 1

<b>Price of the book (\$)</b>	5	6.49	7.15
<b>Change from \$10</b>	$10 - 5$	$10 - 6.49$	$10 - 7.15$

Recognizing the pattern students write  $10 - p$  for a book costing  $p$  dollars. This summarizes a calculation that can be carried out repeatedly with different numbers.



## Specific Content Examples for the Critical Areas - continued

Solve one-variable equations and inequalities.



### Example

A new pet store has a \$560,000 budget for full-time employees' salary and benefits. If an employee's salary is \$35,000 and benefits are \$5,000, write an equation whose solution is the number of employees the pet store can employ if they spend their entire budget. Solve the equation.

#### Sample Response:

Let  $x$  = the number of full-time employees

$$560,000 = 35,000x + 5,000x$$

$$560,000 = 40,000x$$

$$14 = x$$

The company can hire 14 full-time people.

Solve real-world and mathematical problems involving area, surface area, and volume.



### Example

Betsy needs to paint the four exterior walls of a large rectangular shed. The length of the shed is 70 feet, the width is 40 feet, and the height is 20 feet. The paint costs \$28 per gallon, and each gallon covers 420 square feet. How much will it cost Betsy to paint the shed? Explain your work.

#### Sample Response:

Two 20 foot-by-40 foot walls and two 20 foot-by-70 foot walls need to be painted.

$$2(20 \text{ ft.})(40 \text{ ft.}) + 2(20 \text{ ft.})(70 \text{ ft.}) = 4,400 \text{ square feet}$$

Betsy will need to paint 4,400 square feet.

If each gallon covers 420 square feet, then  $4400 \text{ sq. ft.} \div 420 \text{ sq. ft per gallon} \approx 10.476$  gallons

10 gallons isn't quite enough and 11 gallons is a bit more than she needs. Since paint is usually sold in whole gallons, it makes sense for Betsy to buy 11 gallons of paint.

Finally, since paint costs \$28 per gallon, the total cost will be  $(11 \text{ gallons})(\$28 \text{ per gallon}) = \$308$ .

**Specific Content Examples for the Critical Areas** - continued

Develop an understanding of statistics.

 **Example**

Each of the 20 students in Mr. Anderson’s class timed how long it took them to solve a puzzle. Their times (in minutes) are listed below:

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Time (minutes)	3	5	4	6	4	8	5	4	9	5	3	4	7	5	8	6	3	6	5	7

Display the data using a dot plot.

Find the mean and median of the data. Does it surprise you that the values of the mean and median are not equal? Explain why or why not.

**Sample Response:**

A dot plot showing the times is shown here:



The most common times were 3, 4, 5, and 6 minutes, although there are also several times of 7, 8, and 9 minutes. There were no times of 1, 2, or 10 minutes.

To find the mean, we need to add up all of the times and divide by the number of data points, 20. This gives,

$$\frac{3 \times 3 + 4 \times 4 + 5 \times 5 + 3 \times 6 + 2 \times 7 + 2 \times 8 + 1 \times 9}{20}$$

Evaluating this expression gives 5.35 minutes.

There are 20 numbers in the list so the median will be an average of the middle two numbers:

3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 7, 7, 8, 8, 9

The middle two numbers are both 5s so the median is 5 minutes.

These two results make sense because the data are slightly skewed toward the larger numbers. Since the median is 5 we expect the mean to be larger than 5.

## Introduction

### Critical Areas of Instruction

In Grade 7 your child's mathematics experience will focus on four critical areas:

- (1) Developing understanding of and applying proportional relationships
- (2) Developing understanding of operations with rational numbers and working with expressions and linear equations
- (3) Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume
- (4) Drawing inferences about populations based on samples

### Specific Content Examples for the Critical Areas

- Analyze proportional relationships and use them to solve real-world and mathematical problems
- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers
- Use properties of operations to generate equivalent expressions
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations
- Draw, construct and describe geometrical figures and describe the relationships between them
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
- Use random sampling to draw inferences about a population
- Draw informal comparative inferences about two populations
- Investigate chance processes and develop, use, and evaluate probability models

Adapted from California Common Core State Standards – Mathematics  
page 46

**Smarter Balanced Assessment Consortium Examples**

 **Example 1**

This problem is an interactive problem delivered on the computer. A student is asked to add or subtract the mathematical expression. (A mathematical expression contains numbers like  $-3\frac{1}{2}$  and operations. An example of a mathematical expression below would be  $-3\frac{1}{2} - 3\frac{1}{2}$ . There are four listed in the sample below). The student will then drag and drop the mathematical expressions onto the appropriate place on the number line. (Critical Area #2, see above)

The point on the number line shows the location of  $-3\frac{1}{2}$ .

Move each expression into a box to show its correct location on the number line.

$-3\frac{1}{2} - 3\frac{1}{2}$        $-3\frac{1}{2} - (-5)$   
 $-3\frac{1}{2} + 3\frac{1}{2}$        $-3\frac{1}{2} + (-5)$

**Sample Response:**

$-3\frac{1}{2} + (-5)$        $-3\frac{1}{2} - (-5)$   
 $-3\frac{1}{2} - 3\frac{1}{2}$        $-3\frac{1}{2} + 3\frac{1}{2}$   
 $-3\frac{1}{2}$

Smarter Balanced Assessment Consortium  
 Sample Items and Performance Tasks  
<http://www.smarterbalanced.org/sample-items-and-performance-tasks/>

**Smarter Balanced Assessment Consortium Examples - continued**

 **Example 2**

This problem is an interactive problem delivered on the computer. A student is asked determine if each mathematical expression is equivalent to a given expression. The student will select yes to indicate and equivalent mathematical expression and no if it not an equivalent mathematical expression. (Critical Area #3, see page 13)

**Sample Response:**

A - No, B - No, C - Yes, D - No

Look at each expression. Is it equivalent to  $\frac{x+3y}{2}$ ?

Select Yes or No for expressions A-D.

A.  $\frac{4x+3y}{8}$   Yes  No

B.  $\frac{5}{4}\left(\frac{2x+6}{5}\right)$   Yes  No

C.  $\frac{1}{2}(x+3y)$   Yes  No

D.  $\frac{2}{3}\left(\frac{5x}{6} + \frac{9y}{4} - \frac{x}{12}\right)$   Yes  No

Smarter Balanced Assessment Consortium  
Sample Items and Performance Tasks  
<http://www.smarterbalanced.org/sample-items-and-performance-tasks/>

**Specific Content Examples for the Critical Areas**

Analyze proportional relationships and use them to solve real-world and mathematical problems.

 **Example**

The amount of paint needed to cover a surface is directly proportional to the area of the surface. If 3 quarts of paint are needed to paint a square with a side of 5 feet, how many quarts must be purchased to paint a square whose side is 7 feet 6 inches long?

**Sample Response:**

Let  $x$  be the number of quarts needed to paint a square with a side length of 7 feet 6 inches. 7 feet 6 inches is the same as 7.5 feet.

$$\frac{3 \text{ quarts}}{5^2 \text{ feet}^2} = \frac{x \text{ quarts}}{7.5^2 \text{ feet}^2}$$

$$\frac{3 \text{ quarts}}{25 \text{ feet}^2} = \frac{x \text{ quarts}}{56.25 \text{ feet}^2}$$

$$25x = 168.75$$

$$x = 6.75$$

It will take 6.75 quarts of paint to cover a square whose side length is 7 feet 6 inches.

**Specific Content Examples for the Critical Areas** - continued

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. Rational numbers include positive and negative whole numbers, fractions, and decimals.

 **Example 1**

Mathematics that students were taught in Grade 6 and a progression of related content in Grades 7 and 8:

6th Grade	7th Grade	8th Grade
$\frac{5}{7} \cdot 8$ $\frac{3}{4} \div \frac{2}{5}$ (started in 5th grade) What is the length of a rectangular plot of land if the width is $\frac{2}{3}$ mile and the area is $\frac{1}{4}$ square miles?	$-\frac{5}{7} \cdot -8$ $-\frac{3}{4} \div \frac{2}{5}$	Estimate the value of $\sqrt{3}$ . It is approximately 1.7.  Find the length of the diagonal of a rectangle using the Pythagorean Theorem.

 **Example 2**

Ocean water freezes at about  $-2\frac{1}{2}^{\circ}\text{C}$ . Fresh water freezes at  $0^{\circ}\text{C}$ . Antifreeze, a liquid used in the radiators of cars, freezes at  $-64^{\circ}\text{C}$ .

Imagine that the temperature has dropped to the freezing point for ocean water. How many degrees more must the temperature drop for the antifreeze to turn solid?

**Sample Response:**

The difference between the temperature that ocean water turns to a solid and antifreeze turns to a solid is  $-2.5^{\circ} - (-64^{\circ}) = 61.5^{\circ}$ .

So the temperature must drop another  $61.5^{\circ}\text{C}$  after ocean water freezes for the antifreeze to turn to ice.

**Specific Content Examples for the Critical Areas** - continued

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

 **Example**

As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.

**Sample Response:**

The salesperson would like his pay this week to be \$100 or greater.

An inequality to represent this statement is  
 $\$50 + \$3(\text{number of sales}) \geq \$100$

Solving the inequality for the number of sales  
 $\$3(\text{number of sales}) \geq \$100 - \$50$

$$\text{number of sales} \geq \frac{\$50}{\$3}$$

$$\text{number of sales} \geq \text{number of sales } 16.\overline{66}$$

Since sales are in whole number quantities, the salesperson would need 17 or more sales to have his weekly pay be at least \$100.

National Governors Association's Center for Best Practices and Council of Chief State School Officers (2010) Common Core State Standards for Mathematics

Draw, construct and describe geometrical figures and describe the relationships between them.

 **Example**

Esmeralda has a 90:1 scale drawing of her bedroom. On the drawing, her bedroom measures  $1\frac{3}{5}$  inches by  $2\frac{1}{4}$  inches. What is the area of her bedroom in square feet?

**Sample Response:**

The area of the living room in the floor plan is

$$1\frac{3}{5} \text{ in} \times 2\frac{1}{4} \text{ in} = \frac{8}{5} \text{ in} \times \frac{9}{4} \text{ in} = \frac{72}{20} \text{ in}^2.$$

Since the length scales by 90, the area scales by  $90^2$ . So the area of the real bedroom in square inches is  $\frac{72}{20} \times 90^2 = 29,160$ .

To convert square inches to square feet, you have to divide by  $12^2$ .

So the area of the living room is 202.5 square feet.

**Specific Content Examples for the Critical Areas** - continued

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.



**Example**

The 7th graders at Sunview Middle School were helping to renovate a playground for the kindergartners at a nearby elementary school. City regulations require that the sand underneath the swings be at least 15 inches deep. The sand under both swing sets was only 12 inches deep when they started.

The rectangular area under the small swing set measures 9 feet by 12 feet and required 40 bags of sand to increase the depth by 3 inches. How many bags of sand will the students need to cover the rectangular area under the large swing set if it is 1.5 times as long and 1.5 times as wide as the area under the small swing set?

**Sample Response:**

**Solution using scale factor:**

Since we have to multiply both the length and the width by 1.5, the area that needs to be covered is  $1.5^2 = 2.25$  times as large. Since the depth of sand is the same, the amount of sand needed for the large swing set is 2.25 times as much as is needed for the small swing set, and they will need 2.25 times as many bags. Since  $2.25 \times 40 = 90$ , they will need 90 bags of sand for the large swing set.

**Solution using a unit rate:**

The area they cover under the small swing set is  $9 \times 12 = 108$  square feet. Since the depth is the same everywhere, and we know that 40 bags covers 108 square feet, they can cover  $108 \div 40 = 2.7$  square feet per bag.

The area they need to cover under the large swing set is  $1.5^2 = 2.25$  times as big as the area under the small swing set, which is  $2.25 \times 108 = 243$  square feet. If we divide the number of square feet we need to cover by the area covered per bag, we will get the total number of bags we need:  $243 \div 2.7 = 90$

So they will need 90 bags of sand for the large swing set.

Illustrative Mathematics  
Grade 7, Geometry  
<http://www.illustrativemathematics.org/standards/k8>

**Specific Content Examples for the Critical Areas** - continued

Use random sampling to draw inferences about a population.

 **Example**

Below are the heights of the players on the University of Maryland women’s basketball team for the 2012-2013 season and the heights of the players on the women’s field hockey team for the 2012 season.

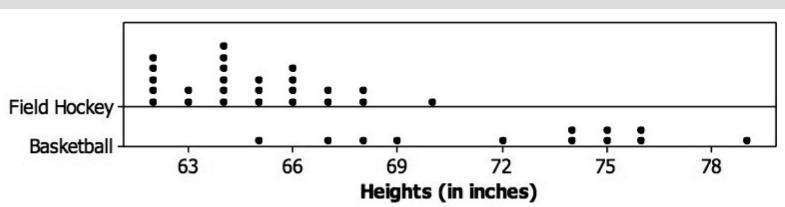
(Accessed at <http://www.umterps.com/sports/w-fieldh/mtt/md-w-fieldh-mtt.html>, <http://www.umterps.com/sports/w-baskbl/mtt/md-w-baskbl-mtt.html> on 1/13/13).

**Note:** It is typical for a women’s field hockey team to have more players than a women’s basketball team would.

Field Hockey Player Heights (inches)
66
64
66
63
67
62
62
64
64
64
64
65
66
64
63
62
62
68
68
66
70
67
65
62
64

Basketball Player Heights (inches)
75
65
76
75
76
72
67
69
74
68
74
79

- Based on visual inspection of the dot plots, which group appears to have the larger average height? Which group appears to have the greater variability in the heights?
- Compute the mean and mean absolute deviation (MAD) for each group. Do these values support your answers in part (A)?
- How many of the 12 basketball players are shorter than the tallest field hockey player?
- Imagine that an athlete from one of the two teams told you she needs to go to practice. You estimate that she is about 65 inches tall. If you had to pick, would you think that she was a field hockey player or that she was a basketball player? Explain your reasoning.
- The women on the Maryland field hockey team are not a random sample of all female college field hockey players. Similarly, the women on the Maryland basketball team are not a random sample of all female college basketball players. However, for purposes of this task, suppose that these two groups can be regarded as random samples of all female college field hockey players and all female college basketball players, respectively. If these were random samples, would you think that female college basketball players are typically taller than female college field hockey players? Explain your decision using answers to the previous questions and/or additional analysis.



**Specific Content Examples for the Critical Areas** - continued

Use random sampling to draw inferences about a population

 **Example** - continued

**Sample Response:**

- A. The center of the basketball distribution is much higher on the number line than the center of the field hockey distribution, so at first glance, it appears that the basketball group has the higher average. Similarly, the values for the basketball distribution appear to have a greater range and are less concentrated than the field hockey distribution, so it appears that the basketball group has greater variability in its observations.
- B. Field Hockey: mean = 64.76, MAD = 1.75; Basketball: mean = 72.5, MAD = 3.58. These values do support the conjectures from Part (A).
- C. The tallest field hockey player is 70 inches. Four of the basketball players are less than 70 inches (65, 67, 68, and 69).
- D. At 65 inches, she is more likely to be a field hockey player. Using the summary measures, 65 inches is approximately the mean for the field hockey players, so she would be a field hockey player of average height. A height of 65 inches is more unusual for the basketball team as that value is just over 2 MAD's below the mean. Using the raw data and a probability argument, 3 of the 25 field hockey players are 65 inches (12%) and only one out of the 12 basketball players is 65 inches (8.3%)
- E. Yes, it appears that women's college basketball players are typically taller than women's college field hockey players. In addition to any arguments/statements made earlier regarding the dot plots and summary measures, one could also mention that  $\frac{2}{3}$  of the basketball players are taller than the tallest field hockey player (and similar comparative arguments).

Illustrative Mathematics  
Grade 7, Statistics and Probability  
<http://www.illustrativemathematics.org/standards/k8>  
Direct link: <http://www.illustrativemathematics.org/illustrations/1340>

For a complete list of standards covered in Grade 7, please refer to the Common Core State Standards for Mathematics document. <http://www.cde.ca.gov/be/st/ss/documents/ccsmathstandardaug2013.pdf>

## Introduction

### Critical Areas of Instruction

In Grade 8 your child's mathematics experience will focus on three critical areas:

- (1) Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations
- (2) Grasping the concept of a function and using functions to describe quantitative relationships
- (3) Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem

### Specific Content Examples for the Critical Areas

- Know that there are numbers that are not rational, and approximate them by rational numbers
- Work with radicals and integer exponents
- Understand the connection between proportional relationships, lines, and linear equations
- Analyze and solve linear equations and pairs of simultaneous linear equations
- Define, evaluate, and compare functions
- Use functions to model relationships between quantities
- Understand congruence and similarity using physical models, transparencies, or geometry software
- Understand and apply the Pythagorean Theorem
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres
- Investigate patterns of association in bivariate data

Adapted from California Common Core State Standards – Mathematics  
page 52

**Smarter Balanced Assessment Consortium Examples**

 **Example 1**

- A. Drag into the box exactly three unique expressions whose sum is less than 10.
- B. Drag into the box exactly three unique expressions whose sum is between 10 and 20.
- C. Drag into the box exactly three unique expressions whose sum is greater than 20.

$$5\sqrt{7}$$

$$\sqrt{13}$$

$$\frac{3^8}{3^6}$$

$$20 - \sqrt{20}$$

$$(4^{-1})^{-1}$$

$$2$$

- A. Three unique expressions whose sum is less than 10.

- B. Three unique expressions whose sum is between 10 and 20.

- C. Three unique expressions whose sum is greater than 20.

**Sample Response:**

- A.  $2, (4^{-1})^{-1}, \sqrt{13}$
- B.  $2, \frac{3^8}{3^6}, 5\sqrt{7}$
- C.  $20 - \sqrt{20}, 5\sqrt{7}, \frac{3^8}{3^6}$

Modified from:  
 Smarter Balanced Assessment Consortium  
 Sample Items and Performance Tasks  
<http://www.smarterbalanced.org/sample-items38-36-and-performance-tasks/>



Smarter Balanced Assessment Consortium Examples - continued



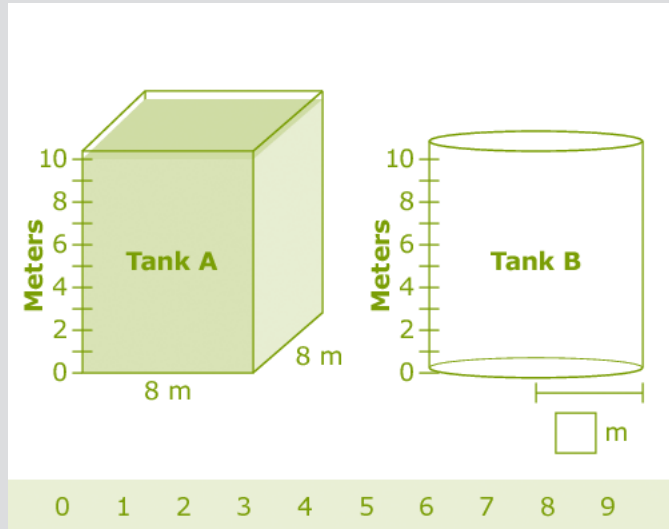
Example 2

Two water tanks are shown. Tank A is a rectangular prism and Tank B is a cylinder. The tanks are not drawn to scale.

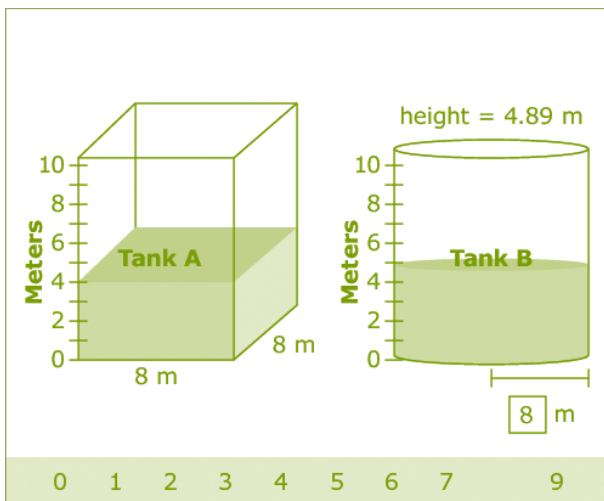
Tank A is filled with water to the 10-meter mark.

Click Tank A to change the water level. The volume of water that leaves Tank A is transferred to Tank B, and the height of the water in Tank B is shown.

Drag one number into the box to show the approximate radius of the base of Tank B.



Sample Response:



Volume of Tank A is:

$$8m \times 8m \times 4m = 256m^3$$

Volume of Tank A is also  $256m^3$ .

$$256m^3 = (4.89)(2\pi r^2)$$

$$\frac{256}{4.89 \cdot 2\pi} = r^2$$

$$8.3 \approx r$$

**Specific Content Examples for the Critical Areas**

Work with exponents and roots.

 **Example 1**

Evaluate  $(5 \times 10^{-8})(2.9 \times 10^2)$ .  
Express the result in scientific  
and standard notation.

**Sample Response:**

$$\begin{aligned} (5 \times 10^{-8})(2.9 \times 10^2) &= (5 \times 2.9)(10^{-8} \times 10^2) \\ &= 14.5 \times 10^{-6} \\ &= (1.45 \times 10^1)(10^{-6}) \\ &= 1.45 \times 10^{-5} \text{ or } 0.0000145 \end{aligned}$$

 **Example 2**

Find the product of  
 $\sqrt{6}(\sqrt{3} + 5\sqrt{2})$ .

**Sample Response:**

$$\sqrt{6}(\sqrt{3} + 5\sqrt{2}) = \sqrt{18} + 5\sqrt{12} = 3\sqrt{2} + 10\sqrt{3}$$

Solve systems of linear equations.

 **Example**

Solve the system of equations:

$$3x - 4y = -10$$

$$5x + 8y = -2$$

**Sample Response:**

Multiply the first equation by 2  $6x - 8y = -20$

Keep the second equation as is  $5x + 8y = -2$

Add the equations  $11x = -22$

Divide each side of the equation by 11  $x = -2$

Substitute  $-2$  for  $x$  in either equation and solve for  $y$ .

$$3(-2) + 4y = -10$$

$$4y = -4$$

$$y = -1$$

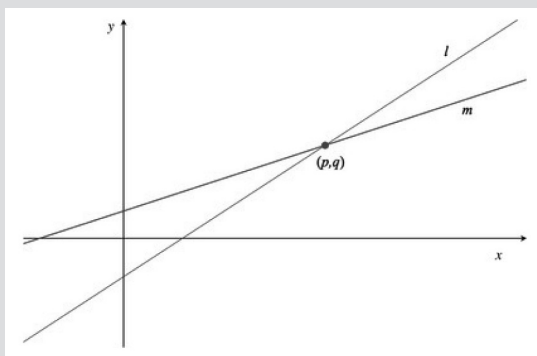
The solution for the system is  $x = -2$  and  $y = -1$

**Specific Content Examples for the Critical Areas** - continued

Understand the connection between proportional relationships, lines, and equations.

 **Example 1**

The figure below shows the lines  $l$  and  $m$  described by the equations  $4x - y = c$  and  $y = 2x + d$  respectively, for some constants  $c$  and  $d$ . They intersect at the point  $(p, q)$ .



- How can you interpret  $c$  and  $d$  in terms of the graphs of the equations above?
- Imagine you place the tip of your pencil at point  $(p, q)$  and trace line  $l$  out to the point with  $x$ -coordinate  $p + 2$ . Imagine I do the same on line  $m$ . How much greater would the  $y$ -coordinate of your ending point be than mine?

**Sample Response:**

- If we put the equation  $4x - y = c$  in the form  $y = 4x - c$ , we see that the graph has slope 4. The slope of the graph of  $y = 2x + d$  is 2. So the steeper line,  $l$ , is the one with equation  $y = 4x - c$ , and therefore  $-c$  is the  $y$ -coordinate of the point where  $l$  intersects the  $y$ -axis. The other line,  $m$ , is the one with equation  $y = 2x + d$ , so  $d$  is the  $y$ -coordinate of the point where  $m$  intersects the  $y$ -axis.
- The line  $l$  has slope 4. So on  $l$ , each increase of one unit in the  $x$ -value produces an increase of 4 units in the  $y$ -value. Thus an

increase of 2 units in the  $x$ -value produce an increase of  $2 \cdot 4 = 8$  units in the  $y$ -value. The line  $m$  has slope 2. So on  $L_2$ , each increase of 1 unit in the  $x$ -value produces an increase of 2 units in the  $y$ -value. Thus an increase of 2 units in the  $x$ -value produces an increase of  $2 \cdot 2 = 4$  units in the  $y$ -value. Thus your  $y$ -value would be  $8 - 4 = 4$  units larger than my  $y$ -value.

**Specific Content Examples for the Critical Areas** - continued

Understand functions.

8th Grade	Higher Mathematics
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .

 **Example 1**

A student has had a collection of baseball cards for several years. Suppose that  $B$ , the number of cards in the collection, can be described as a function of  $t$ , which is time in years since the collection was started. Explain what each of the following equations would tell us about the number of cards in the collection over time.

- a.  $B = 200 + 100t$
- b.  $B = 100 + 200t$
- c.  $B = 2000 - 100t$
- d.  $B = 100 - 200t$

**Sample Response:**

**Interpreting Formulas**

- a. In this equation, we can first observe what  $B$  looked like at the start of the collection, which is  $t = 0$  years. This gives

$$B = 200 + 100(0)$$

$$B = 200$$

We see that the student started out with 200 baseball cards. As  $t$  increases, the number of baseball cards increases at a rate of 100 cards per year, meaning the student added 100 cards to his collection each year.

- b. At the start of the collection,  $t = 0$ , we have

$$B = 100 + 200(0)$$

$$B = 100$$

We see that the student started with 100 baseball cards. The number of baseball cards increases at a rate of 200 cards per year, meaning the student added 200 cards to his collection each year.

- c. We observe that at the start of the collection, when  $t = 0$ , we have

$$B = 2000 - 100(0)$$

$$B = 2000$$

So at the start of the collection, the student had 2000 cards. However, in this case, the size of the collection decreases with time. As  $t$  increases,  $B$  decreases at a rate of 100 cards per year; that is, the student loses 100 cards each year. As the amount of time that has passed is only “several years,” this equation may be valid. However, after 20 years,  $B$  will become negative, in which we would either say that the equation no longer applies or possibly interpret it as meaning that he owes someone baseball cards (an unlikely scenario but it is consistent with interpretations in other contexts where negative numbers represent debt).

- d. At  $t = 0$ , the start of the collection, we have
- $$B = 100 - 200(0)$$
- $$B = 100$$

The student started his collection with 100 cards. However, after just one year,  $t = 1$ , we have

$$B = 100 - 200(1)$$

$$B = -100$$

This says that after one year, the student has -100 baseball cards. As in part (c), this could conceivably mean that he owes baseball cards to someone else, although this seems like a highly improbable situation.

Illustrative Mathematics  
Grade 8, Functions

<http://www.illustrativemathematics.org/standards/k8>

**Specific Content Examples for the Critical Areas** - continued

 **Example 2**

You work for a video streaming company that has two monthly plans to choose from:

- Plan 1: A flat rate of \$7 per month plus \$2.50 per video viewed
  - Plan 2: \$4 per video viewed
- a. What type of functions model this situation? Explain how you know.
  - b. Define variables that make sense in the context, and then write an equation with cost as a function of videos viewed, representing each monthly plan.
  - c. How much would 3 videos in a month cost for each plan? 5 videos?
  - d. Compare the two plans and explain what advice you would give to a customer trying to decide which plan is best for them, based on their viewing habits.

**Sample Response:**

- a. Each plan can be modeled by a linear function since the constant rate per video indicates a linear relationship.
- b. We let  $C_1$  be the monthly cost of Plan 1,  $C_2$  be the monthly cost of Plan 2, and  $V$  be the number of videos viewed in a given month. Then
$$C_1 = 7 + 2.5V$$
$$C_2 = 4V$$
- c. 3 videos on Plan 1:  $C_1 = 7 + 2.5(3) = \$14.50$   
5 videos on Plan 1:  $C_1 = 7 + 2.5(5) = \$19.50$   
3 videos on Plan 2:  $C_2 = 4(3) = \$12$   
5 videos on Plan 2:  $C_2 = 4(5) = \$20$
- d. Plan 1 costs less than Plan 2 for 5 or fewer videos per month. A customer who watches more than 5 videos per month should choose Plan 2. A customer who watches 5 or fewer videos per month should choose Plan 1.

Illustrative Mathematics  
Grade 8, Functions

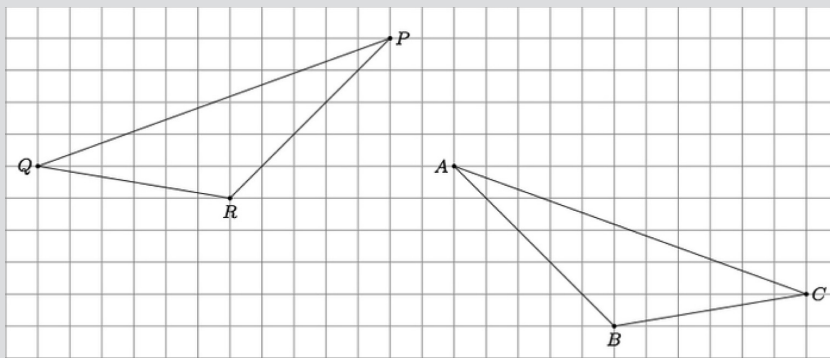
<http://www.illustrativemathematics.org/standards/k8>

**Specific Content Examples for the Critical Areas** - continued

Understand geometric relationships using physical models or software.

 **Example**

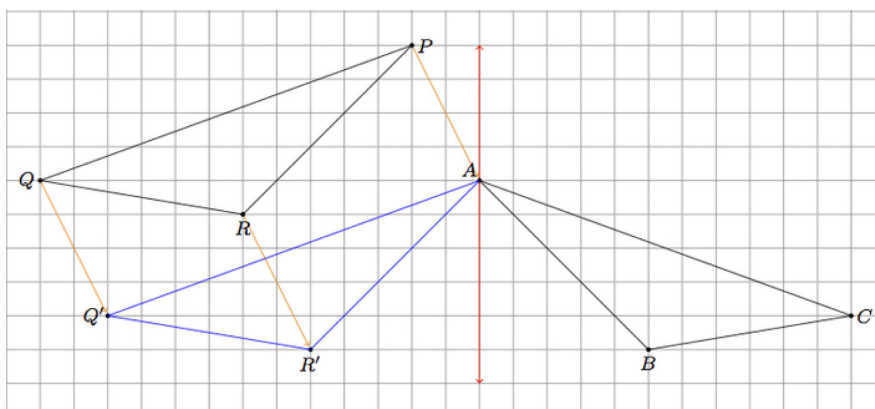
The two triangles in the picture below are congruent:



- Give a sequence of rotations, translations, and/or reflections which take  $\triangle PRQ$  to  $\triangle ABC$ .
- Is it possible to show the congruence in part (a) using only translations and rotations? Explain.

**Sample Response:**

- We may begin by translating the vertex  $P$  of  $\triangle PRQ$  to the corresponding vertex  $A$  of  $\triangle ABC$ . The image of this translation is pictured below with the image of  $R$  denoted  $R'$  and the image  $Q$  of denoted  $Q'$  (note that the image of  $P$  is  $A$ ). This translation moves each point of  $\triangle PRQ$  two boxes to the right and four boxes down.



Also pictured above, along with the translated image  $\triangle AR'Q'$  of  $\triangle PRQ$ , is a vertical red line of reflection. Reflecting about this line will send  $R'$  to  $B$  because  $R'$  is on the same horizontal line as  $B$ , 5 boxes to the left of the vertical line, while  $B$  is 5 boxes to the right of the red line. Similarly this reflection will send  $Q'$  to  $C$  as they are on the same horizontal line with  $Q'$  11 boxes to the left and  $C$  is 11 boxes to the right of the red line.

**Specific Content Examples for the Critical Areas** - continued

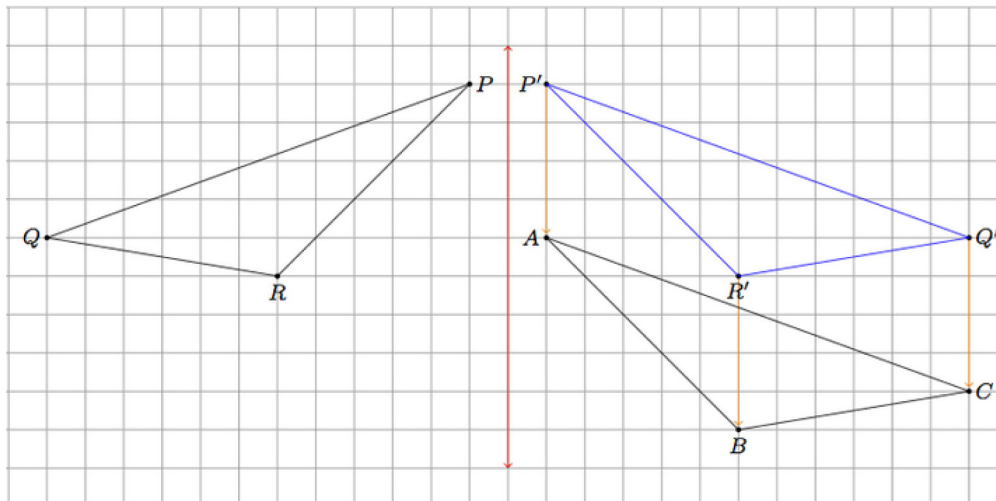
 **Example** - continued

**Sample Response:**

- b. If we move from vertex  $P$  to vertex  $R$  to vertex  $Q$  on  $\triangle PRQ$  then we are moving around  $\triangle PRQ$  in a clockwise direction. If we move from vertex  $A$  to vertex  $B$  to vertex  $C$  on  $\triangle ABC$  on the other hand, then we are moving in a counterclockwise direction. Both translations and rotations preserve the notions of clockwise and counterclockwise in the plane. Reflections, on the other hand, reverse these two directions. Since triangles  $\triangle PRQ$  and  $\triangle ABC$  have different orientations at least one reflection is needed to show this congruence.

**Alternate Approach to part (a):**

Instead of translating first and then applying a reflection, another method would be to apply a reflection first and then a translation. Below is a line of reflection, in red, and the image, denoted  $\triangle P'Q'R'$ , of  $\triangle PQR$  under that reflection:



The points  $P, P'$  lie on the same horizontal line with  $P$  one unit to the left of the red line of reflection and  $P'$  one unit to the right. Similarly,  $Q, Q'$  are on the same horizontal line with  $Q$  twelve units to the left and  $Q'$  twelve units to the right. Finally  $R$  and  $R'$  are on the same horizontal line with  $R$  six units to the left of the line of reflection and  $R'$  six units to the right. After the reflection over the red line, a translation by four boxes downward, indicated by the orange arrows, takes  $\triangle P'Q'R'$  to  $\triangle ABC$ .



## Specific Content Examples for the Critical Areas - continued

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.



### Example

A filled ice cream cone has the shape of a hemisphere atop a cone. If the cone has a height of 12 cm, and the radius of the hemisphere is 4 cm, approximately how much ice cream is there?

#### Sample Response:

Volume of the hemisphere is  $\frac{1}{2} \cdot \frac{4}{3}\pi \cdot 64 \approx 42.77\text{cm}^3$

Volume of the cone is  $\frac{1}{3} \cdot 12 \cdot 16\pi \approx 200.96\text{cm}^3$

Total volume of the ice cream is approximately  $42.77\text{cm}^3 + 200.96\text{cm}^3$  or  $243.75\text{cm}^3$ .

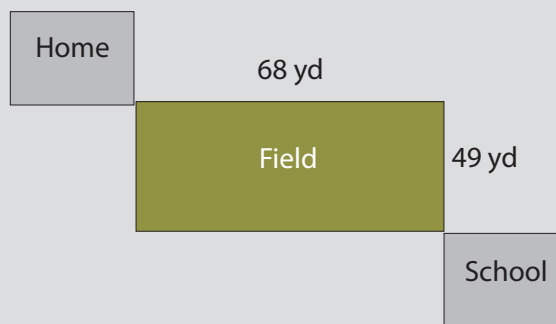
Illustrative Mathematics  
Grade 8, Expression and Equations  
<http://www.illustrativemathematics.org/standards/k8>

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.



### Example

Natalia walks around a field from school to home. Sara tells Natalia that it is a shorter distance if she walks in a straight line from school to home across the field. Is Sara correct? If so, how much shorter is Sara's path than Natalia's?



#### Sample Response:

Natalia's Path:

$$68\text{ yds} + 49\text{ yds} = 117\text{ yds}$$

Sara's Path:

$$\sqrt{(68)^2 + (49)^2}\text{ yds} \approx 83.82\text{ yds.}$$

$$117\text{ yds} - 83.82\text{ yds} = 33.18\text{ yds}$$

Sara's path is shorter by about 33.18 yds.



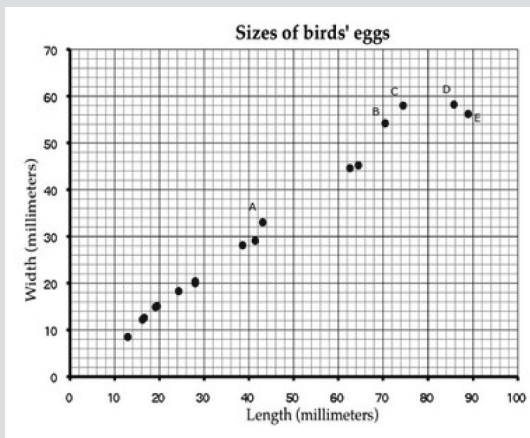
Specific Content Examples for the Critical Areas - continued

Investigate patterns of association in bivariate data.



Example

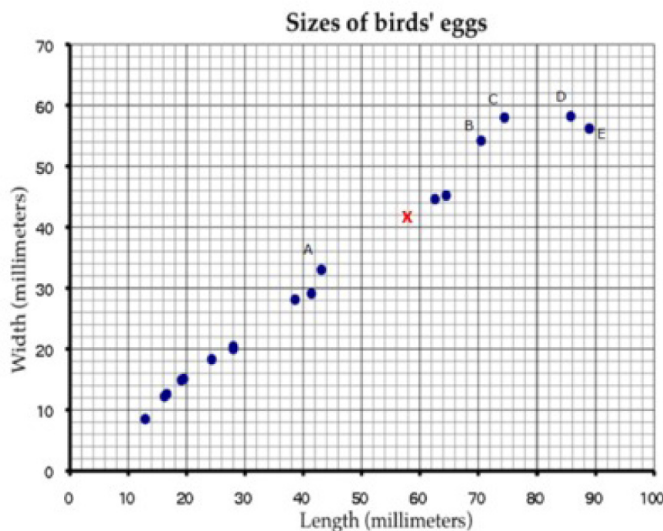
This scatter diagram shows the lengths and widths of the eggs of some American birds.



- A biologist measured a sample of one hundred Mallard duck eggs and found they had an average length of 57.8 millimeters and average width of 41.6 millimeters. Use an X to mark a point that represents this on the scatter diagram.
- What does the graph show about the relationship between the lengths of birds' eggs and their widths?
- Another sample of eggs from similar birds has an average length of 35 millimeters. If these bird eggs follow the trend in the scatter plot, about what width would you expect these eggs to have, on average?
- Describe the differences in shape of the two eggs corresponding to the data points marked C and D in the plot.
- Which of the eggs A, B, C, D, and E has the greatest ratio of length to width? Explain how you decided.

Response:

a.



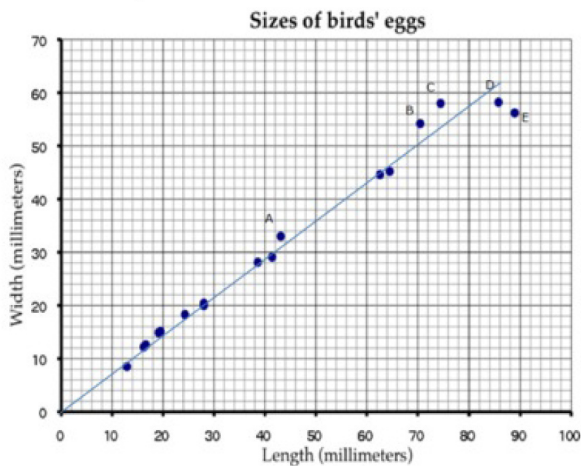
b. There seems to be a positive linear relationship between the length and width of the eggs.

Specific Content Examples for the Critical Areas - continued

 Example - continued

Response:

c. The line below appears to fit the data fairly well:



Since it passes through (0, 0) and (50, 36), its slope is  $\frac{36}{50} = 0.72$ , so the equation of the line is

$$y = 0.72x$$

If  $x = 35$ , then our line would predict that  $y = 0.72 \cdot 35 = 25.2$ . So, we would expect the width of these eggs to be, on average, about 25 mm. Answers using different lines can vary up to 1 mm in either direction.

- d. Without reading off precise numerical values from the plot, we can see that eggs C and D have very nearly the same width, but egg D is about 12 millimeters longer than egg C.
- e. First we note that egg E certainly has a higher length-to-width ratio than C or D, since it is both longer and narrower. Similarly, E has a higher ratio than B because it is significantly longer, and only a tad wider. Because it is harder to visually identify the difference between A and E, we compute their respective length-to-width ratios numerically, which turn out to be approximately 1.3 for A and 1.6 for E. So E has the greatest ratio of length to width.

Illustrative Mathematics  
Grade 8, Statistics and Probability  
<http://www.illustrativemathematics.org/standards/k8>

For a complete list of standards covered in Grade 8, please refer to the Common Core State Standards for Mathematics document. <http://www.cde.ca.gov/be/st/ss/documents/ccsmathstandardaug2013.pdf>



# Accelerated Learning for Students Who Are Ready

Success in Algebra I is critical for a student to be successful in higher mathematics.

“There are some students who are able to move through mathematics quickly. These students may choose to take high school mathematics beginning in eighth grade or earlier so they can take college-level mathematics in high school.

“Care must be taken to ensure that students master and fully understand all of the important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. In particular, the Standards for Mathematical Practice ought to continue to be emphasized in these cases.”

Common Core State Standards for Mathematics, Appendix A, page 80  
www.corestandards.org

In the Common Core State Standards for Mathematics, there are two possible mathematics courses for an eighth grade student to take:

1. Grade 8 Common Core Mathematics
2. Algebra I or Mathematics I (The course is determined by the district your student will attend. Each school district will determine which pathway their high schools will follow.)

**Note:** For additional information on pathways, see the table below.

The sequence of courses in high school is based on each eighth grade course and the traditional pathway:

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
8th Grade Common Core Standards for Mathematics	Algebra 1 or Mathematics I	Geometry or Mathematics II	Algebra II or Mathematics III	PreCalculus
Algebra 1	Geometry	Algebra II	PreCalculus	AP Calculus
Mathematics I	Mathematics II	Mathematics III	PreCalculus	AP Calculus

**Note:** For a student to reach AP Calculus in grade 12, acceleration must occur.

In recent years, a common method of acceleration was for a student to skip a mathematics class, usually sixth or seventh grade mathematics. This was possible due to the fact that there were many standards from year to year that were repeated in the former California mathematics standards. However, the Common Core State Standards for mathematics requires a new approach to acceleration for a student to reach a high school math course in eighth grade.

The recommendation with Common Core State Standards for Mathematics is to have a thoughtfully designed series of compacted courses. Compacted courses compress the standards of three years of mathematics into two years. For example, one option could be covering seventh grade, eighth grade, and Algebra I during a student’s seventh grade and eighth grade years.

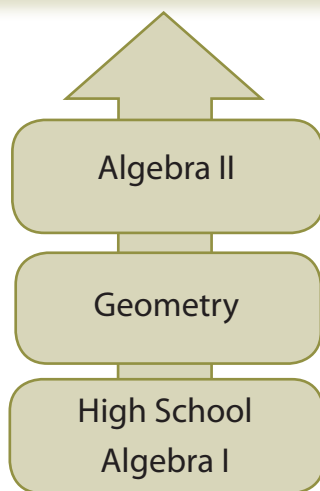
If a student does not accelerate in middle school math, acceleration can also occur in the high school level.

# Brief Overview of High School Mathematics

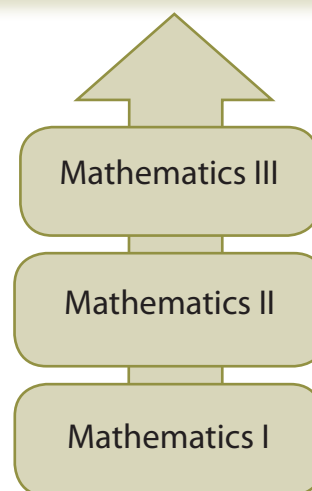
## Higher Mathematics Pathways

The Common Core State Standards for Mathematics provides two pathways to organize the standards into courses: the traditional and the integrated pathway. Both pathways require students to accomplish the same mathematical content over a three-year period and include work with making mathematical models. Each school district determines which pathway their high schools will follow. Either pathway will allow students to complete Advanced Placement Calculus with accelerated work.

Courses in higher level mathematics: Precalculus, Calculus, Advanced Statistics, Discrete Mathematics, Advanced Quantitative Reasoning, or courses designed for career technical programs of study.



**Traditional Pathway**  
Typical in U.S.



**Integrated Pathway**  
Typical Outside of U.S.

## High School Options for Acceleration

Just as care should be taken not to rush the decision to accelerate students, care should also be taken to provide more than one opportunity for acceleration. Some students may not have the preparation to enter a “Compacted Pathway” but may still develop an interest in taking advanced mathematics, such as AP Calculus or AP Statistics in their senior year. Additional opportunities for acceleration may include:

- Allowing students to take two mathematics courses simultaneously (such as Geometry and Algebra II, or PreCalculus and Statistics)
- Allowing students in schools with block scheduling to take a mathematics course in both semesters of the same academic year

- Offering summer courses that are designed to provide the equivalent experience of a full course in all regards, including attention to the Standard for Mathematical Practices
- Creating different compaction ratios, including four years of high school content into three years beginning in 9th grade
- Creating a hybrid Algebra II-PreCalculus course that allows students to go straight to Calculus

You should consult with your child’s teacher or counselor concerning the best acceleration pathway for your student.



