Unit 5
Radical and Rational Functions

Algebra II

Unit Description:

In this unit, students will study radical and rational functions. They will graph these functions and explore transformations. Students will examine domain and range, find the roots of these functions, and identify asymptotes. Students will also explore inverse variation and solve rational inequalities.

Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model with mathematics.
MP.5 Use appropriate tools strategically.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Louisiana Student Standards for Mathematics (LSSM)

The Louisiana Student Standards for Mathematics (LSSM), designates the following standards as A2: Algebra 2.
Italicized standards are designated as A1: Algebra 1 and are considered prerequisite standards for Algebra 2. While these prerequisite standards are present in the curriculum for scaffolding purposes, teachers will focus instruction on Algebra 2 expectations.

A-SSE: Algebra-Seeing Structure in Expressions	
A. Interpret the structure of expressions	
A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context. ★ **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.
A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
A-REI: Reasoning with Equations and Inequalities	
A. Understand solving equations as a process of reasoning and explain the reasoning	
A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
N-RN: The Real Number System	
A. Extend the properties of exponents to rational exponents.	
N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.
N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
F-BF: Building Functions	
A. Build a function that models a relationship between two quantities.	
F-BF.A.1	Write a function that describes a relationship between two quantities. ★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context. **b.** Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
B. Build new functions from existing functions	
F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + \boldsymbol{k}$, $\boldsymbol{k}\, f(x)$, $f(\boldsymbol{k}x)$, and $f(x + \boldsymbol{k})$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
F-BF.B.4	Find inverse functions. **a.** Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$.
F-IF: Interpreting Functions	
B. Interpret functions that arise in applications in terms of the context.	
F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★

F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★
C. Analyze functions using different representations	
F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
A-APR: Arithmetic with Polynomials and Rational Expressions	
D. Rewrite rational expressions	
A-APR.D.6	Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x)+\frac{r(x)}{b(x)}$, where $a(x), b(x), q(x),$ and $r(x)$ are polynomials with the degree of $r(x)$less than the degree of $b(x),$ using inspection, long division, or, for the more complicated examples, a computer algebra system.
A-APR.D.7 (+)	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
A-CED: Creating Equations	
A. Create equations that describe numbers or relationships	
A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★

***As defined by LSSM, the basic modeling cycle involves:**

1. identifying variables in the situation and selecting those that represent essential features,
2. formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables,
3. analyzing and performing operations on these relationships to draw conclusions,
4. interpreting the results of the mathematics in terms of the original situation,
5. validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable,
6. reporting on the conclusions and the reasoning behind them.

Choices, assumptions, and approximations are present throughout this cycle.

Enduring Understandings:	**Essential Questions:**
• Radical expressions can be rewritten with rational exponents. • Square root and cube root functions can be used to model real-world data. • Rational functions have asymptotes for which the function is undefined.	• How do the properties of exponents apply to rational exponents? • How are the asymptotes of a rational function found? • Why is it important to consider the domain and range of a function?

• Transformations can be performed with square root, cube root, and rational functions.	