Unit 4
Series, Exponential, and Logarithmic Functions

Algebra II

Unit Description:

In this unit, students study arithmetic and geometric sequences and implicit and explicit rules for defining them. Then they analyze exponential and logarithmic patterns and graphs as well as properties of logarithms. Finally, they solve exponential and logarithmic equations.

Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model with mathematics.
MP.5 Use appropriate tools strategically.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Louisiana Student Standards for Mathematics (LSSM)

The Louisiana Student Standards for Mathematics (LSSM), designates the following standards as A2: Algebra 2.
Italicized standards are designated as A1: Algebra 1 and are considered prerequisite standards for Algebra 2.
While these prerequisite standards are present in the curriculum for scaffolding purposes, teachers will focus instruction on Algebra 2 expectations.

A-SSE: Algebra-Seeing Structure in Expressions	
B. Write expressions in equivalent forms to solve problems.	
A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★ **c.** Use the properties of exponents to transform expressions for exponential functions. For example, the expression $1.15t$ can be rewritten as $\left(1.15^{\frac{1}{12}}\right)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
A-SSE.B.4	Apply the formula for the sum of a finite geometric series (when the common ratio is not 1) to solve problems. For example, calculate mortgage payments. ★
A-REI: Reasoning with Equations and Inequalities	
D. Represent and solve equations and inequalities graphically.	
A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the

	functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
F-BF: Building Functions	
A. Build a function that models a relationship between two quantities.	
F-BF.A.1	Write a function that describes a relationship between two quantities. ★ **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context. **b.** Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★
B. Build new functions from existing functions	
F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) \pm \boldsymbol{k}$, $\boldsymbol{k}\, f(x)$, $f(\boldsymbol{k}x)$, and $f(x \pm \boldsymbol{k})$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
F-BF.B.4	Find inverse functions. **a.** Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$.
F-BF.B.5 (+)	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
F-IF: Interpreting Functions	
B. Interpret functions that arise in applications in terms of the context.	
F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★
F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★
C. Analyze functions using different representations	
F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ **e.** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-LE: Linear, Quadratic, and Exponential Models	
A. Construct and compare linear, quadratic, and exponential models and solve problems.	
F-LE.A.2	Given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), construct linear and exponential functions, including arithmetic and geometric sequences, to solve multi-step problems.
F-LE.A.4	For exponential models, express as a logarithm the solution to a $b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
A-CED: Creating Equations	
A. Create equations that describe numbers or relationships	
A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.

★As defined by LSSM, the basic modeling cycle involves:
1. identifying variables in the situation and selecting those that represent essential features,
2. formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables,
3. analyzing and performing operations on these relationships to draw conclusions,
4. interpreting the results of the mathematics in terms of the original situation,
5. validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable,
6. reporting on the conclusions and the reasoning behind them.

Choices, assumptions, and approximations are present throughout this cycle.

Enduring Understandings:	**Essential Questions:**
• Some patterns in the real-world can be described using arithmetic or geometric sequences and series. • The number e frequently occurs in mathematics (especially calculus) and is an irrational constant (like π). Its value is $e = 2.718\,281\,828\ldots$ • The logarithm for base e is called a natural logarithm (ln). • Exponential and Logarithmic equations/functions model real-world situations involving growth or decay.	• How do you find the general term of an algebraic and geometric sequence? • How is Sigma notation used to represent a series? • What are some real-world applications of exponential and logarithmic functions?