

MATHEMATICS GRADE 10

Unit 2 Sequences and Series

CRITERIA A and B - Summative Assessment

ASSESSMENT TASK:

Your **goal** is to demonstrate your understanding of the unit so far. You will complete a written test (**product**). This is a formative assessment of criteria A. You will have 60 minutes to complete the assessment. Remember to write down any working out.

ASSESSMENT RUBRIC:

| Level | Descriptor Criteria A | Descriptor Criteria B |
|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0 | You do not reach a standard described by any of the descriptors below. | |
| 1-2 | <p>The student is able to:</p> <ol style="list-style-type: none"> I. select appropriate mathematics when solving simple problems in familiar situations II. apply the selected mathematics successfully when solving these problems III. generally solve these problems correctly in a variety of contexts. | <p>The student is able to:</p> <ol style="list-style-type: none"> I. apply, with teacher support, mathematical problem-solving techniques to discover simple patterns II. state predictions consistent with patterns. |
| 3-4 | <p>The student is able to:</p> <ol style="list-style-type: none"> I. select appropriate mathematics when solving more complex problems in familiar situations II. apply the selected mathematics successfully when solving these problems III. generally solve these problems correctly in a variety of contexts. | <p>The student is able to:</p> <ol style="list-style-type: none"> I. apply mathematical problem-solving techniques to discover simple patterns II. suggest general rules consistent with findings. |
| 5-6 | <p>The student is able to:</p> <ol style="list-style-type: none"> I. select appropriate mathematics when solving challenging problems in familiar situations II. apply the selected mathematics successfully when solving these problems III. generally solve these problems correctly in a variety of contexts. | <p>The student is able to:</p> <ol style="list-style-type: none"> I. select and apply mathematical problem-solving techniques to discover complex patterns II. describe patterns as general rules consistent with findings III. verify the validity of these general rules. |
| 7-8 | <p>The student is able to:</p> <ol style="list-style-type: none"> I. select appropriate mathematics when solving challenging problems in both familiar and unfamiliar situations II. apply the selected mathematics successfully when solving these problems III. generally solve these problems correctly in a variety of contexts. | <p>The student is able to:</p> <ol style="list-style-type: none"> I. select and apply mathematical problem-solving techniques to discover complex patterns II. describe patterns as general rules consistent with correct findings III. prove, or verify and justify, these general rules. |

COMMAND TERMS:

Select - Choose from a list or group.

Apply - Use knowledge and understanding in response to a given situation or real circumstances. Use an idea, equation, principle, theory or law in relation to a given problem or issue.

Solve - Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.

Describe - Give a detailed account or picture of a situation, event, pattern or process.

Verify/Justify/Prove - Give valid reasons or evidence to support an answer or conclusion.

State - Give a specific name, value or other brief answer without explanation or calculation.

Suggest - Propose a solution, hypothesis or other possible answer.

Formula Sheet

The n th term of an arithmetic sequence

$$u_n = u_1 + (n - 1)d$$

The sum of n terms of an arithmetic sequence

$$S_n = \frac{n}{2}(2u_1 + (n - 1)d); S_n = \frac{n}{2}(u_1 + u_n)$$

The n th term of a geometric sequence

$$u_n = u_1 r^{n-1}$$

The sum of n terms of a finite geometric sequence

$$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, r \neq 1$$

| n | 1 | 2 | 3 | 4 |
|------------|-----------------|--------------------|---------------------|----------------------|
| u_n | $a + b + c + d$ | $8a + 4b + 2c + d$ | $27a + 9b + 3c + d$ | $64a + 16b + 4c + d$ |
| $\Delta 1$ | | $7a + 3b + c$ | $19a + 5b + c$ | $37a + 7b + c$ |
| $\Delta 2$ | | | $12a + 2b$ | $18a + 2b$ |
| $\Delta 3$ | | | | $6a$ |

| Criteria A Level 1-2: The student is able to: | Criteria B Level 1-2: The student is able to: |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| I. select appropriate mathematics when solving simple problems in familiar situations II. apply the selected mathematics successfully when solving these problems III. generally solve these problems correctly in a variety of contexts. | I. apply , with teacher support, mathematical problem-solving techniques to discover simple patterns II. state predictions consistent with patterns. |

1. Write down a rule to describe the following sequences and then find the next term:

| | |
|------------------|-----------------------------------------|
| a. 2, -1, -4, -7 | b. 6, 3, $1\frac{1}{2}$, $\frac{3}{4}$ |
|------------------|-----------------------------------------|

2. Find the first three terms of the arithmetic sequences with the n^{th} term:

| | |
|---------------------|----------------------|
| a. $u_n = 3n^2 - 2$ | b. $u_n = (n^2 - 3)$ |
|---------------------|----------------------|

3. Find the first three terms of the geometric sequence with the n^{th} term:

| | |
|------------------------|----------------------------------------------------|
| a. $u_n = 4 \cdot 3^n$ | b. $u_n = 5 \cdot \left(-\frac{1}{2}\right)^{n-1}$ |
|------------------------|----------------------------------------------------|

| Criteria A Level 3-4: The student is able to: | Criteria B Level 3-4: The student is able to: |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------|
| I. select appropriate mathematics when solving more complex problems in familiar situations II. apply the selected mathematics successfully when solving these problems III. generally solve these problems correctly in a variety of contexts. | I. apply mathematical problem-solving techniques to discover simple patterns II. suggest general rules consistent with findings. |

4. A new subscription music streaming service gains 1000 subscribers in its first month, 2500 in its second month, 4000 in its third month, and so on for the whole first year, increasing by a constant amount each month.

a. Find the total number of subscribers it would gain in its first year.

5. A small business repairs mobile phones. The managers predict 10 phones will be repaired in the first week, 12 phones in the second week, 14 in the third week and so on, increasing capacity by two phones every week as the business grows by word of mouth.

a. Write down an expression for the number of phones repaired in the n^{th} week. Explain why the number of phones repaired each week forms an arithmetic sequence.

b. Find the total number of phones repaired in the first 10 weeks using the expression you have written down in question a.

6. A movie producer expects that 1 000 000 people will see her new film in its first week in the cinemas. Past experience tells her that in each subsequent week, the number of people who see the film will be 80% of the week before. Use her modeling assumptions to estimate the number of people who will see the film in the first eight weeks. Give your answer to an appropriate degree of accuracy. Explain why your answer is only an estimate.

| Criteria A Level 5-6: The student is able to: | Criteria B A Level 5-6: The student is able to: |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| I. select appropriate mathematics when solving challenging problems in familiar situations II. apply the selected mathematics successfully when solving these problems III. generally solve these problems correctly in a variety of contexts. | I. select and apply mathematical problem-solving techniques to discover complex patterns II. describe patterns as general rules consistent with findings III. verify the validity of these general rules. |

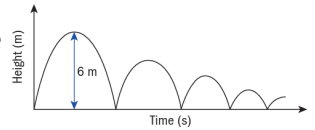
7. Use the difference method to **find** the general term u_n of :

2, 7, 18, 38, 70, 117, 182

8. An arithmetic series has the first term 11, and the sum of the first fifteen terms as 585. Find the common difference.
9. A geometric series has a common ratio 1.5, and the sum of the first six terms is 1995. Find the value of its first term.

| Criteria A Level 7-8: The student is able to: | Criteria B Level 7-8: The student is able to: |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| I. select appropriate mathematics when solving challenging problems in both familiar and unfamiliar situations II. apply the selected mathematics successfully when solving these problems III. generally solve these problems correctly in a variety of contexts. | I. select and apply mathematical problem-solving techniques to discover complex patterns II. describe patterns as general rules consistent with correct findings III. prove, or verify and justify , these general rules. |

10. A ball is kicked into the air from the ground. It reaches a height of 6 m before it falls and hits the ground. It then bounces up again, several times. Each bounce is $\frac{2}{3}$ the height of the previous bounce.



| | |
|--------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
| <p>a. In the first bounce, the ball travels 12 m (6 m up and 6 m down). Write down the total distance it travels in the second bounce.</p> | <p>b. Find the total distance it has traveled as it returns to the ground for the sixth time.</p> |
|--------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|

11. The fifth term of an arithmetic series is 5. The eighth term is 14. Calculate the sum of the first 20 terms of the series.