Curriculum Guide

For

Mathematics

* (Revised June 2017)

SJ

SCHOOL &

EARLY LEARNING CENTER

Table of Contents

Pg #

The Purpose of Mathematics in St. John's Lutheran School Curriculum

At St. John's Lutheran School we value the study of mathematics as an important tool in implementing our vision and philosophy statements. We believe that mathematics helps us understand and perceive the unlimited power, perfection, creativity and wisdom of God in His creation. The study of mathematics serves as a tool by which man can study the quantitative properties of God's creation.

We also believe that responsible and productive members of today's technological society need to have a broad, connected, and useful knowledge of mathematics. Our students live (and sometimes work) in the 21st century, an era dominated by computers, worldwide communication, and a global economy. Our world and economy today require workers who are prepared to absorb new ideas, to perceive patterns, and to solve unconventional problems. By building a mathematics foundation for our students here at St. John's we believe this will allow them to develop the mathematical literacy needed for citizenship and employment in the 21st century.

Adopted Textbooks for Mathematics

- Saxon Primary Mathematics K-3, © 2008
- Saxon Mathematics Intermediate Math 4, © 2008
- Saxon Mathematics Intermediate Math 5, © 2008
- Saxon Mathematics Math Course 1, © 2008
- Saxon Mathematics Math Course 2, © 2008
- Saxon Mathematics Math Course 3, © 2008
- Saxon Mathematics Algebra I, © 2008

St. Johns Mathematics Curriculum Standards

(Adapted from the Colorado Academic Standards in Mathematics and The Common Core State Standards for Mathematics)

These St. John's curriculum standards in mathematics are the topical organization of the concepts and skills every St. John's student should know and be able to do throughout their Kindergarten through eighth-grade experience.

1. **Number Sense, Properties, and Operations**
   Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties and understanding these properties leads to fluency with operations.

2. **Patterns, Functions, and Algebraic Structures**
   Pattern sense gives students a lens with which to understand trends and commonalities. Students recognize and represent mathematical relationships and analyze change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

3. **Data Analysis, Statistics, and Probability**
   Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

4. **Shape, Dimension, and Geometric Relationships**
   Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

5. **Students will understand that mathematics is a tool by which we can study the quantitative properties of God's creation.**

Mathematics Curriculum Guide
Organization and Format

The standards are presented in a format which follows each standard from one grade level to the next starting from Kindergarten and working up to eighth-grade. Included in each grade levels standards and curriculum are sections which show how 21st century skills and the elements of school readiness and postsecondary and workforce readiness indicators give depth and context to essential learning.

The elements of the revised standards are:

Prepared Graduate Competencies: The Kindergarten through eighth-grade concepts and skills that all students who complete their education at St. John's must master to ensure their success in a secondary, postsecondary, and workforce setting.

Standard: The topical organization of an academic content area.

Grade Level Expectations: The articulation (at each grade level), concepts, and skills of a standard that indicate a student is making progress toward being ready for high school. *What do students need to know from Kindergarten through eighth grade?*

Evidence Outcomes: The indication that a student is meeting an expectation at the mastery level. *How do we know that a student can do it?*

21st Century Skills and Readiness Competencies: Includes the following:

- ***Inquiry Questions:***
  Sample questions are intended to promote deeper thinking, reflection and refined understandings precisely related to the grade level expectation.

- ***Relevance and Application:***
  Examples of how the grade level expectation is applied at home, on the job or in a real-world, relevant context.

- ***Nature of the Discipline:***
  The characteristics and viewpoint one keeps as a result of mastering the grade level expectation.

Prepared Graduate Competencies in Mathematics

The prepared graduate competencies are the Kindergarten through eighth-grade concepts and skills that all students who complete their education at St. John's must master to ensure their success in a secondary, postsecondary, and workforce setting

Prepared graduates in mathematics:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities.
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error.
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency.
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning.
- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts.
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data.
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations.
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data.
- Apply transformation to numbers, shapes, functional representations, and data.
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics.
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking.
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions.

21st Century Skills and Readiness Competencies in Mathematics

Mathematics in St. John's description of 21st century skills is a synthesis of the essential abilities students must apply in our rapidly changing world. Today's mathematics students need a repertoire of knowledge and skills that are more diverse, complex, and integrated than any previous generation. Mathematics is inherently demonstrated in each of St. John's 21st century skills, as follows:

<u>Critical Thinking and Reasoning</u>
Mathematics is a discipline grounded in critical thinking and reasoning. Doing mathematics involves recognizing problematic aspects of situations, devising and carrying out strategies, evaluating the reasonableness of solutions, and justifying methods, strategies, and solutions. Mathematics provides the grammar and structure that make it possible to describe patterns that exist in nature and society.

<u>Information Literacy</u>
The discipline of mathematics equips students with tools and habits of mind to organize and interpret quantitative data. Informationally literate mathematics students effectively use learning tools, including technology, and clearly communicate using mathematical language.

<u>Collaboration</u>
Mathematics is a social discipline involving the exchange of ideas. In the course of doing mathematics, students offer ideas, strategies, solutions, justifications, and proofs for others to evaluate. In turn, the mathematics student interprets and evaluates the ideas, strategies, solutions, justifications and proofs of others.

<u>Self-Direction</u>
Doing mathematics requires a productive disposition and self-direction. It involves monitoring and assessing one's mathematical thinking and persistence in searching for patterns, relationships, and sensible solutions.

<u>Invention</u>
Mathematics is a dynamic discipline, ever expanding as new ideas are contributed. Invention is the key element as students make and test conjectures, create mathematical models of real-world phenomena, generalize results, and make connections among ideas, strategies and solutions.

Mathematics Curriculum Framework

Standard 1: Number Sense, Properties, and Operations

Number sense provides students with a firm foundation in mathematics. Students build a deep understanding of quantity, ways of representing numbers, relationships among numbers, and number systems. Students learn that numbers are governed by properties, and understanding these properties leads to fluency with operations.

Prepared Graduate Competencies in the Number Sense, Properties, and Operations Standard are:

- Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities
- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Apply transformation to numbers, shapes, functional representations, and data

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Kindergarten

Concepts and skills students master:
1. Whole numbers can be used to name, count, represent, and order quantity

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Use objects and pictures to represent whole numbers from 0-30 (introduce # 0-100). - Sequence whole numbers from 0-20. - Identify, read, and write numbers to 100. - Read, write, and order numerals from 0-20 in meaningful contexts. - Count from 1-100. - Count by 2's, 5's, and 10's,. - Use one-to one correspondence to count and compare sets of objects containing 0-10 members. - Count backwards. - Identify and write numbers on hundred number chart. - Use two or more sets of objects to demonstrate and explain which set is equal to, less than, or greater than. - Use concrete materials to demonstrate and explain the meaning of whole half, and fourth. - Compares simple fractions. - Learn value of penny, nickel, dime, quarter, and dollar. - Match sets and numbers. - Write money amounts to$1.00 using dollar and cent symbols - Select coins for given amount in pretend store. - Group objects in sets of ten. - Use ordinal positions for first through sixth. - Identify more than 1 number or less than 1 number. - Use a number line. - Describe the concept of zero. - Estimate reasonable quantity for a given number of objects. - Use expanded form to represent numbers. - Identify even and odd numbers.	**Inquiry Questions:** 1. Why do we count things? 2. Is there a wrong way to count? Why? 3. How do you know when you have more or less? 4. What does it mean to be second and how is it different than two? **Relevance and Application:** 1. Counting is used constantly in everyday life such as counting plates for the dinner table, people on a team, pets in the home, or trees in a yard. 2. Numerals are used to represent quantities. 3. People use numbers to communicate with others such as two more forks for the dinner table, one less sister than my friend, or six more dollars for a new toy. **Nature of Mathematics:** 1. Mathematics involves visualization and representation of ideas. 2. Numbers are used to count and order both real and imaginary objects. 3. Mathematicians attend to precision. 4. Mathematicians look for and make use of structure.

Standard: 1 Number Sense, Properties, and Operations

<table>
<tr><td colspan="2">Prepared Graduates:
Apply transformation to numbers, shapes, functional representations, and data</td></tr>
<tr><td colspan="2">Grade Level Expectation: Kindergarten</td></tr>
<tr><td colspan="2">Concepts and skills students master:
2. Composing and decomposing quantity forms the foundation for addition and subtraction and other operations</td></tr>
<tr><td>Evidence Outcomes</td><td>21st Century Skills and Readiness Competencies</td></tr>
<tr><td rowspan="3">Students can:
- Add and subtract whole numbers by combining and separating objects and explain the process.
- Tell simple stories using addition and subtraction.
- Act out subtraction and addition story problems.
- Draw pictures for addition and addition story problems.
- Add and subtract whole numbers by combining and separating objects.
- Draw pictures for story problems with addition and subtraction.
- Combine sets with story pictures.
- For any number from 1 to 9, find the number that makes 10 when added to the given number.
- Write number sentences to show addition.
- Fluently add and subtract within 5.
- Shows the meaning of division (separating into groups).
- Acts out to show division situations.
- Uses manipulatives to model and solve division problems.
- Solves problems involving dividing into groups (division).</td><td>Inquiry Questions:
1. What happens when two quantities are combined?
2. What happens when a set of objects is separated into different sets?</td></tr>
<tr><td>Relevance and Application:
1. People combine quantities to find a total such as number of boys and girls in a classroom or coins for a purchase.
2. People use subtraction to find what is left over such as coins left after a purchase, number of toys left after giving some away.</td></tr>
<tr><td>Nature of Mathematics:
1. Mathematicians create models of problems that reveal relationships and meaning.
2. Mathematics involves the creative use of imagination.
3. Mathematicians reason abstractly and quantitatively.
4. Mathematicians model with mathematics.</td></tr>
</table>

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: First Grade

Concepts and skills students master:

1. The whole number system describes place value relationships within and beyond 1000 and forms the foundation for efficient algorithms

Evidence Outcomes

Students can:

- Count to 1,000
  - i. Count starting at any number less than 1,000
  - ii. Within 1,000, read and write numerals and represent a number of objects with a written numeral.
- Represent and use the digits of a two-digit number.
  - i. Represent the digits of a three-digit number as hundreds, tens and ones.[i]
  - ii. Compare two three-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
  - iii. Compare two sets of objects, including coins (up to quarters), up to at least 25 using language such as "three more or three fewer"
- Use place value and properties of operations to add and subtract.
  - i. Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of ten, using concrete models or drawings, and/or the relationship between addition and subtraction.
  - ii. Identify coins and find the value of a collection of coins.
  - iii. Mentally find 10 more or 10 less than any two-digit number, without counting; explain the reasoning used.
  - iv. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
  - v. Relate addition and subtraction strategies to a written method and explain the reasoning used.

21st Century Skills and Readiness Competencies

Inquiry Questions:

1. Can numbers always be related to tens?
2. Why not always count by one?
3. Why was a place value system developed?
4. How does a position of a digit affect its value?
5. How big is 100?

Relevance and Application:

1. The comparison of numbers helps to communicate and to make sense of the world. (For example, if someone has two more dollars than another, gets four more points than another, or takes out three fewer forks than needed.
2. Understanding monetary values helps in decision making in purchasing a comparing price amounts.

Nature of Mathematics:

1. Mathematics involves visualization and representation of ideas.
2. Numbers are used to count and order both real and imaginary objects.
3. Mathematicians reason abstractly and quantitatively.
4. Mathematicians look for and make use of structure.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: First Grade

Concepts and skills students master:
2. Number relationships can be used to solve addition and subtraction problems

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Represent and solve problems involving addition and subtraction. i. Use addition and subtraction within 20 to solve word problems. ii. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20. - Apply properties of operations and the relationship between addition and subtraction. i. Apply properties of operations as strategies to add and subtract. ii. Relate subtraction to unknown-addend problem. - Add and subtract within 20. i. Relate counting to addition and subtraction. ii. Add and subtract within 20 using multiple strategies. iii. Demonstrate fluency for addition and subtraction within 10. - Use addition and subtraction equations to show number relationships. i. Use the equal sign to demonstrate equality in number relationships. ii. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.	**Inquiry Questions:** 1. What is addition and how is it used? 2. What is subtraction and how is it used? 3. How are addition and subtraction related? **Relevance and Application:** 1. Addition and subtraction are used to model real-world situations such as computing saving or spending, finding the number of days until a special day, or determining an amount needed to earn a reward. 2. Fluency with addition and subtraction facts helps to quickly find answers to important questions. **Nature of Mathematics:** 1. Mathematicians use addition and subtraction to take numbers apart and put them back together in order to understand number relationships. 2. Mathematicians make sense of problems and persevere in solving them. 3. Mathematicians look for and make use of structure.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Second Grade

Concepts and skills students master:

1. The whole number system describes place value relationships within and beyond 1000 and forms the foundation for efficient algorithms

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Use place value to read, write, count, compare, and represent numbers. i. Represent the digits of a three-digit number as hundreds, tens, and ones. ii. Count within 1000. iii. Skip-count by 5s, 10s, and 100s. iv. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. v. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. - Use place value understanding and properties of operations to add and subtract. i. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. ii. Add up to four two-digit numbers using strategies based on place value and properties of operations. iii. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. iv. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. vi. Explain why addition and subtraction strategies work, using place value and the properties of operations. - Show equivalency of coins, and combine coins up to $0.99 - Identifies dozen and half dozen	**Inquiry Questions:** 1. How big is 1,000? 2. How does the position of a digit in a number affect its value? **Relevance and Application:** 1. The ability to read and write numbers allows communication about quantities such as the cost of items, number of students in a school, or number of people in a theatre. 2. Place value allows people to represent large quantities. For example, 725 can be thought of as 700 + 20 + 5. **Nature of Mathematics:** 1. Mathematicians use place value to represent many numbers with only ten digits. 2. Mathematicians construct viable arguments and critique the reasoning of others. 3. Mathematicians look for and make use of structure. 5. Mathematicians look for and express regularity in repeated reasoning.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:

Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Second Grade

Concepts and skills students master:

2. Formulate, represent, and use strategies to add and subtract within 100 with flexibility, accuracy, and efficiency

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Represent and solve problems involving addition and subtraction. i. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. ii. Apply addition and subtraction concepts to financial decision-making - Fluently add and subtract within 20 using mental strategies. - Know from memory all sums of two one-digit numbers. - Use equal groups of objects to gain foundations for multiplication. i. Determine whether a group of objects (up to 20) has an odd or even number of members. ii. Write an equation to express an even number as a sum of two equal addends. iii. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns and write an equation to express the total as a sum of equal addends. - Demonstrate understanding of basic multiplication facts of 1's, 2's, 3's, 4's, 5's, and 10's. - Determine whether an estimate or exact answer is acceptable and reasonable.	**Inquiry Questions:** 1. What are the ways numbers can be broken apart and put back together? 2. What could be a result of not using pennies (taking them out of circulation)? **Relevance and Application:** 1. Addition is used to find the total number of objects such as total number of animals in a zoo, total number of students in first and second grade. 2. Subtraction is used to solve problems such as how many objects are left in a set after taking some away, or how much longer one line is than another. 3. The understanding of the value of a collection of coins helps to determine how many coins are used for a purchase or checking that the amount of change is correct. **Nature of Mathematics:** 1. Mathematicians use visual models to understand addition and subtraction. 2. Mathematicians make sense of problems and persevere in solving them. 3. Mathematicians reason abstractly and quantitatively. 4. Mathematicians look for and express regularity in repeated reasoning.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. The whole number system describes place value relationships and forms the foundation for efficient algorithms.

<table>
<tr><th>Evidence Outcomes</th><th>21st Century Skills and Readiness Competencies</th></tr>
<tr><td rowspan="3">Students can:
- Use place value and properties of operations to perform multi-digit arithmetic.
i. Use place value to round whole numbers to the nearest 10 or 100.
ii. Identify place value through 0 – 1,000,000’s place.
iii. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
iv. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 using strategies based on place value and properties of operations.
v. Compare whole numbers using words or symbols (<, >, or =).
- Locate positive and negative numbers on a number line.</td><td>Inquiry Questions:
1. How do patterns in our place value system assist in comparing whole numbers?
2. How might the most commonly used number system be different if humans had twenty fingers instead of ten?</td></tr>
<tr><td>Relevance and Application:
1. Knowledge and use of place value for large numbers provides context for distance in outer space, prehistoric timelines, and ants in a colony.
2. The building and taking apart of numbers provide a deep understanding of the base 10 number system.</td></tr>
<tr><td>Nature of Mathematics:
1. Mathematicians use numbers like writers use letters to express ideas.
2. Mathematicians look for and make use of structure.
3. Mathematicians look for and express regularity in repeated reasoning.</td></tr>
</table>

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Third Grade

Concepts and skills students master:
2. Parts of a whole can be modeled and represented in different ways

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Develop understanding of fractions as numbers. i. Describe a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; describe a fraction a/b as the quantity formed by a parts of size $1/b$. ii. Describe a fraction as a number on the number line; represent fractions on a number line diagram. iii. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. 1. Identify two fractions as equivalent (equal) if they are the same size, or the same point on a number line. 2. Identify and generate simple equivalent fractions. Explain why the fractions are equivalent. 3. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. 4. Compare two fractions with the same numerator or the same denominator by reasoning about their size. 5. Explain why comparisons are valid only when the two fractions refer to the same whole. 6. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions. - Locate and label ½'s, and multiples of ¼'s and ½'s between whole numbers on a number line.	**Inquiry Questions:** 1. How many ways can a whole number be represented? 2. How can a fraction be represented in different, equivalent forms? 3. How do we show part of unit? **Relevance and Application:** 1. Fractions are used to share fairly with friends and family such as sharing an apple with a sibling, and splitting the cost of lunch. 2. Equivalent fractions demonstrate equal quantities even when they are presented differently such as knowing that 1/2 of a box of crayons is the same as 2/4, or that 2/6 of the class is the same as 1/3. **Nature of Mathematics:** 1. Mathematicians use visual models to solve problems. 2. Mathematicians make sense of problems and persevere in solving them. 3. Mathematicians reason abstractly and quantitatively.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Third Grade

Concepts and skills students master:
3. Multiplication and division are inverse operations and can be modeled in a variety of ways.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Represent and solve problems involving multiplication and division. i. Interpret products of whole numbers. ii. Interpret whole-number quotients of whole numbers. iii. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. iv. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. v. Model strategies to achieve a personal financial goal using arithmetic operations. - Apply properties of multiplication and the relationship between multiplication and division. i. Apply properties of operations as strategies to multiply and divide. ii. Interpret division as an unknown-factor problem. - Multiply and divide within 100. i. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. ii. Recall from memory all products of two one-digit numbers. - Solve problems involving the four operations, and identify and explain patterns in arithmetic. i. Solve two-step word problems using the four operations. ii. Represent two-step word problems using equations with a letter standing for the unknown quantity. iii. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. iv. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.	**Inquiry Questions:** 1. How are multiplication and division related? 2. How can you use a multiplication or division fact to find a related fact? 3. Why was multiplication invented? Why not just add? 4. Why was division invented? Why not just subtract? **Relevance and Application:** 1. Many situations in daily life can be modeled with multiplication and division such as how many tables to set up for a party, how much food to purchase for the family, or how many teams can be created. 2. Use of multiplication and division helps to make decisions about spending allowance or gifts of money such as how many weeks of saving an allowance of $5 per week to buy a soccer ball that costs $32?. **Nature of Mathematics:** 1. Mathematicians often learn concepts on a smaller scale before applying them to a larger situation. 2. Mathematicians construct viable arguments and critique the reasoning of others. 3. Mathematicians model with mathematics. 4. Mathematicians look for and make use of structure.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Generalize place value understanding for multi-digit whole numbers i. Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. ii. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. iii. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. iv. Use place value understanding to round multi-digit whole numbers to any place. v. Read, write, and order numerals and number words from 0 – 100,000,000's place. - Use decimal notation to express fractions, and compare decimal fractions i. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. ii. Use decimal notation for fractions with denominators 10 or 100. iii. Compare two decimals to hundredths by reasoning about their size.	**Inquiry Questions:** 1. Why isn't there a "oneths" place in decimal fractions? 2. How can a number with greater decimal digits be less than one with fewer decimal digits? 3. Is there a decimal closest to one? Why? **Relevance and Application:** 1. Decimal place value is the basis of the monetary system and provides information about how much items cost, how much change should be returned, or the amount of savings that has accumulated. 2. Knowledge and use of place value for large numbers provides context for population, distance between cities or landmarks, and attendance at events. **Nature of Mathematics:** 1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems. 2. Mathematicians reason abstractly and quantitatively. 3. Mathematicians look for and make use of structure.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations.

Grade Level Expectation: Fourth Grade

Concepts and skills students master:
2. Different models and representations can be used to compare fractional parts.

Evidence Outcomes

Students can:

- Use ideas of fraction equivalence and ordering to:
    i. Explain equivalence of fractions using drawings and models.
    ii. Use the principle of fraction equivalence to recognize and generate equivalent fractions.
    iii. Compare two fractions with different numerators and different denominators, and justify the conclusions.
- Build fractions from unit fractions by applying understandings of operations on whole numbers.
    i. Apply previous understandings of addition and subtraction to add and subtract fractions.
        1. Compose and decompose fractions as sums and differences of fractions with the same denominator in more than one way and justify with visual models.
        2. Add and subtract mixed numbers with like denominators.
        3. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.
    ii. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
        1. Express a fraction *a*/*b* as a multiple of 1/*b*.
        2. Use a visual fraction model to express a/b as a multiple of 1/b, and apply to multiplication of whole number by a fraction.
        3. Solve word problems involving multiplication of a fraction by a whole number.
- Locate and label halves and multiples of thirds and fourths between whole numbers on a number line.

21st Century Skills and Readiness Competencies

Inquiry Questions:

1. How can different fractions represent the same quantity?
2. How are fractions used as models?
3. Why are fractions so useful?
4. What would the world be like without fractions?

Relevance and Application:

1. Fractions and decimals are used any time there is a need to apportion such as sharing food, cooking, making savings plans, creating art projects, timing in music, or portioning supplies.
2. Fractions are used to represent the chance that an event will occur such as randomly selecting a certain color of shirt or the probability of a certain player scoring a soccer goal.
3. Fractions are used to measure quantities between whole units such as number of meters between houses, the height of a student, or the diameter of the moon.

Nature of Mathematics:

1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems.
2. Mathematicians construct viable arguments and critique the reasoning of others.
3. Mathematicians model with mathematics.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Are fluent with basic numerical, symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency.

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

3. Formulate, represent, and use algorithms to compute with flexibility, accuracy, and efficiency.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Use place value understanding and properties of operations to perform multi-digit arithmetic. i. Fluently add and subtract multi-digit whole numbers using standard algorithms. ii. Multiply a whole number of up to four digits by a one-digit whole number, and multiply a whole number of up to three digits by a two-digit number, using strategies based on place value and the properties of operations. iii. Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. iv. Illustrate and explain multiplication and division calculation by using equations, rectangular arrays, and/or area models. - Use the four operations with whole numbers to solve problems. i. Interpret a multiplication equation as a comparison. ii. Represent verbal statements of multiplicative comparisons as multiplication equations. iii. Multiply or divide to solve word problems involving multiplicative comparison. iv. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. v. Represent multistep word problems with equations using a variable to represent the unknown quantity. vi. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. vii. Using the four operations analyze the relationship between choice and opportunity cost.	**Inquiry Questions:** 1. Is it possible to make multiplication and division of large numbers easy? 2. What do remainders mean and how are they used? 3. When is the "correct" answer not the most useful answer? **Relevance and Application:** 1. Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra. **Nature of Mathematics:** 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians develop simple procedures to express complex mathematical concepts. 3. Mathematicians make sense of problems and persevere in solving them. 4. Mathematicians construct viable arguments and critique the reasoning of others. 5. Mathematicians look for and express regularity in repeated reasoning.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. The decimal number system to the hundredths place describes place value patterns and relationships that are repeated in large and small numbers and forms the foundation for efficient algorithms

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Explain that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. i. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. ii. Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. iii. Use whole-number exponents to denote powers of 10. - Read, write, and compare decimals to thousandths. i. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form. ii. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. - Use place value understanding to round decimals to any place. - Convert like measurement units within a given measurement system. i. Convert among different-sized standard measurement units within a given measurement system. ii. Use measurement conversions in solving multi-step, real world problems. - Demonstrate equivalent relationships of fractions, decimals, and percents.	**Inquiry Questions:** 1. What is the benefit of place value system? 2. What would it mean if we did not have a place value system? 3. What is the purpose of a place value system? 4. What is the purpose of zero in a place value system? **Relevance and Application:** 1. Place value is applied to represent a myriad of numbers using only ten symbols. **Nature of Mathematics:** 1. Mathematicians use numbers like writers use letters to express ideas. 2. Mathematicians look closely and make use of structure by discerning patterns. 3. Mathematicians make sense of problems and persevere in solving them. 4. Mathematicians reason abstractly and quantitatively. 5. Mathematicians construct viable arguments and critique the reasoning of others.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

2. Formulate, represent, and use algorithms with multi-digit whole numbers and decimals with flexibility, accuracy, and efficiency

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Fluently multiply multi-digit whole numbers using standard algorithms. - Find whole-number quotients of whole numbers. i. Use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. ii. Illustrate and explain calculations by using equations, rectangular arrays, and/or area models. - Add, subtract, multiply, and divide decimals to hundredths. i. Use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. ii. Relate strategies to a written method and explain the reasoning used. - Write and interpret numerical expressions. i. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. ii. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. - Demonstrate the ability to find square roots, and solve multiplication and division problems involving square roots.	**Inquiry Questions:** 1. How are mathematical operations related? 2. What makes one strategy or algorithm better than another? **Relevance and Application:** 1. Multiplication is an essential component of mathematics. Knowledge of multiplication is the basis for understanding division, fractions, geometry, and algebra. 2. There are many models of multiplication and division such as the area model for tiling a floor and the repeated addition to group people for games. **Nature of Mathematics:** 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians develop simple procedures to express complex mathematical concepts. 3. Mathematicians construct viable arguments and critique the reasoning of others. 4. Mathematicians model with mathematics.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

3. Formulate, represent, and use algorithms to add and subtract fractions with flexibility, accuracy, and efficiency

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Use equivalent fractions as a strategy to add and subtract fractions. i. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. ii. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions with like denominators. iii. Solve word problems involving addition and subtraction of fractions referring to the same whole.	**Inquiry Questions:** 1. How do operations with fractions compare to operations with whole numbers? 2. Why are there more fractions than whole numbers? 3. Is there a smallest fraction? **Relevance and Application:** 1. Computational fluency with fractions is necessary for activities in daily life such as cooking and measuring for household projects and crafts. 2. Estimation with fractions enables quick and flexible decision-making in daily life. For example, determining how many batches of a recipe can be made with given ingredients, the amount of carpeting needed for a room, or fencing required for a backyard. **Nature of Mathematics:** 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians make sense of problems and persevere in solving them. 3. Mathematicians reason abstractly and quantitatively. 4. Mathematicians look for and make use of structure.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:

Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

4. The concepts of multiplication and division can be applied to multiply and divide fractions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). - Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. - Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. In general, (a/b) × (c/d) = ac/bd. - Interpret multiplication as scaling (resizing). i. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. ii. Apply the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. - Solve real world problems involving multiplication of fractions and mixed numbers. - Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. - Interpret division of a whole number by a unit fraction, and compute such quotients. - Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions.	**Inquiry Questions:** 1. Do adding and multiplying always result in an increase? Why? 2. Do subtracting and dividing always result in a decrease? Why? 3. How do operations with fractional numbers compare to operations with whole numbers? **Relevance and Application:** 1. Rational numbers are used extensively in measurement tasks such as home remodeling, clothes alteration, graphic design, and engineering. 2. Situations from daily life can be modeled using operations with fractions, decimals, and percents such as determining the quantity of paint to buy or the number of pizzas to order for a large group. 3. Rational numbers are used to represent data and probability such as getting a certain color of gumball out of a machine, the probability that a batter will hit a home run, or the percent of a mountain covered in forest. **Nature of Mathematics:** 1. Mathematicians explore number properties and relationships because they enjoy discovering beautiful new and unexpected aspects of number systems. They use their knowledge of number systems to create appropriate models for all kinds of real-world systems. 2. Mathematicians make sense of problems and persevere in solving them. 3. Mathematicians model with mathematics. 4. Mathematicians look for and express regularity in repeated reasoning.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning.

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Quantities can be expressed and compared using ratios and rates

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Apply the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. - Apply the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. - Use ratio and rate reasoning to solve real-world and mathematical problems. i. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. ii. Use tables to compare ratios. iii. Solve unit rate problems including those involving unit pricing and constant speed. iv. Find a percent of a quantity as a rate per 100. v. Solve problems involving finding the whole, given a part and the percent. vi. Use common fractions and percents to calculate parts of whole numbers in problem situations including comparisons of savings rates at different financial institutions. vii. Express the comparison of two whole number quantities using differences, part-to-part ratios, and part-to-whole ratios in real contexts, including investing and saving. viii. Use ratio reasoning to convert measurement units.	**Inquiry Questions:** 1. How are ratios different from fractions? 2. What is the difference between quantity and number? **Relevance and Application:** 1. Knowledge of ratios and rates allows sound decision-making in daily life such as determining best values when shopping, creating mixtures, adjusting recipes, calculating car mileage, using speed to determine travel time, or making saving and investing decisions. 2. Ratios and rates are used to solve important problems in science, business, and politics. For example developing more fuel-efficient vehicles, understanding voter registration and voter turnout in elections, or finding more cost-effective suppliers. 3. Rates and ratios are used in mechanical devices such as bicycle gears, car transmissions, and clocks. **Nature of Mathematics:** 1. Mathematicians develop simple procedures to express complex mathematical concepts. 2. Mathematicians make sense of problems and persevere in solving them. 3. Mathematicians reason abstractly and quantitatively.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Sixth Grade

Concepts and skills students master:
2. Formulate, represent, and use algorithms with positive rational numbers with flexibility, accuracy, and efficiency

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Fluently divide multi-digit numbers using standard algorithms. - Fluently add, subtract, multiply, and divide multi-digit decimals using standard algorithms for each operation. - Find the greatest common factor of two whole numbers less than or equal to 100. - Find the least common multiple of two whole numbers less than or equal to 12. - Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. - Interpret and model quotients of fractions through the creation of story contexts. - Compute quotients of fractions. - Solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.	**Inquiry Questions:** 1. Why might estimation be better than an exact answer? 2. How do operations with fractions and decimals compare to operations with whole numbers? **Relevance and Application:** 1. Rational numbers are an essential component of mathematics. Understanding fractions, decimals, and percentages is the basis for probability, proportions, measurement, money, algebra, and geometry. **Nature of Mathematics:** 1. Mathematicians envision and test strategies for solving problems. 2. Mathematicians model with mathematics. 3. Mathematicians look for and make use of structure.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Sixth Grade

Concepts and skills students master:
3. In the real number system, rational numbers have a unique location on the number line and in space

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Explain why positive and negative numbers are used together to describe quantities having opposite directions or values. i. Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. - Use number line diagrams and coordinate axes to represent points on the line and in the plane with negative number coordinates. ii. Describe a rational number as a point on the number line. iii. Use opposite signs of numbers to indicate locations on opposite sides of 0 on the number line. iv. Identify that the opposite of the opposite of a number is the number itself. v. Explain when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. vi. Find and position integers and other rational numbers on a horizontal or vertical number line diagram. vii. Find and position pairs of integers and other rational numbers on a coordinate plane b. Order and find absolute value of rational numbers. i. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. ii. Write, interpret, and explain statements of order for rational numbers in real-world contexts. iii. Define the absolute value of a rational number as its distance from 0 on the number line and interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. iv. Distinguish comparisons of absolute value from statements about order. - Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane including the use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	**Inquiry Questions:** 1. Why are there negative numbers? 2. How do we compare and contrast numbers? 3. Are there more rational numbers than integers? **Relevance and Application:** 1. Communication and collaboration with others is more efficient and accurate using rational numbers. For example, negotiating the price of an automobile, sharing results of a scientific experiment with the public, and planning a party with friends. 2. Negative numbers can be used to represent quantities less than zero or quantities with an associated direction such as debt, elevations below sea level, low temperatures, moving backward in time, or an object slowing down **Nature of Mathematics:** 1. Mathematicians use their understanding of relationships among numbers and the rules of number systems to create models of a wide variety of situations. 2. Mathematicians construct viable arguments and critique the reasoning of others. 3. Mathematicians attend to precision.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Make both relative (multiplicative) and absolute (arithmetic) comparisons between quantities. Multiplicative thinking underlies proportional reasoning.

Grade Level Expectation: Seventh Grade

Concepts and skills students master:
1. Proportional reasoning involves comparisons and multiplicative relationships among ratios

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Analyze proportional relationships and use them to solve real-world and mathematical problems. - Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. - Identify and represent proportional relationships between quantities. i. Determine whether two quantities are in a proportional relationship. ii. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. iii. Represent proportional relationships by equations. iv. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate. - Use proportional relationships to solve multistep ratio and percent problems. i. Estimate and compute unit cost of consumables (to include unit conversions if necessary) sold in quantity to make purchase decisions based on cost and practicality. ii. Solve problems involving percent of a number, discounts, taxes, simple interest, percent increase, and percent decrease.	**Inquiry Questions:** 1. What information can be determined from a relative comparison that cannot be determined from an absolute comparison? 2. What comparisons can be made using ratios? 3. How do you know when a proportional relationship exists? 4. How can proportion be used to argue fairness? 5. When is it better to use an absolute comparison? 6. When is it better to use a relative comparison? **Relevance and Application:** 1. The use of ratios, rates, and proportions allows sound decision-making in daily life such as determining best values when shopping, mixing cement or paint, adjusting recipes, calculating car mileage, using speed to determine travel time, or enlarging or shrinking copies. 2. Proportional reasoning is used extensively in the workplace. For example, determine dosages for medicine; develop scale models and drawings; adjusting salaries and benefits; or prepare mixtures in laboratories. 3. Proportional reasoning is used extensively in geometry such as determining properties of similar figures, and comparing length, area, and volume of figures. **Nature of Mathematics:** 1. Mathematicians look for relationships that can be described simply in mathematical language and applied to a myriad of situations. Proportions are a powerful mathematical tool because proportional relationships occur frequently in diverse settings. 2. Mathematicians reason abstractly and quantitatively. 3. Mathematicians construct viable arguments and critique the reasoning of others.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Seventh Grade

Concepts and skills students master:
2. Formulate, represent, and use algorithms with rational numbers flexibly, accurately, and efficiently

Evidence Outcomes

Students can:

- Apply understandings of addition and subtraction to add and subtract rational numbers including integers.
  i. Represent addition and subtraction on a horizontal or vertical number line diagram.
  ii. Describe situations in which opposite quantities combine to make 0.
  iii. Demonstrate $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative.
  iv. Show that a number and its opposite have a sum of 0 (are additive inverses).
  v. Interpret sums of rational numbers by describing real-world contexts.
  vi. Demonstrate subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$.
  vii. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
  viii. Apply properties of operations as strategies to add and subtract rational numbers.
- Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers including integers.
  i. Apply properties of operations to multiplication of rational numbers.
  ii. Interpret products of rational numbers by describing real-world contexts.
  iii. Apply properties of operations to divide integers.
  iv. Apply properties of operations as strategies to multiply and divide rational numbers.
  v. Convert a rational number to a decimal using long division.
  vi. Show that the decimal form of a rational number terminates in 0s or eventually repeats.
  vii. Solve real-world and mathematical problems involving the four operations with rational numbers.

21st Century Skills and Readiness Competencies

Inquiry Questions:

1. How do operations with rational numbers compare to operations with integers?
2. How do you know if a computational strategy is sensible?
3. Is $0.\overline{9}$ equal to one?
4. How do you know whether a fraction can be represented as a repeating or terminating decimal?

Relevance and Application:

1. The use and understanding algorithms help individuals spend money wisely. For example, compare discounts to determine best buys and compute sales tax.
2. Estimation with rational numbers enables individuals to make decisions quickly and flexibly in daily life such as estimating a total bill at a restaurant, the amount of money left on a gift card, and price markups and markdowns.
3. People use percentages to represent quantities in real-world situations such as amount and types of taxes paid, increases or decreases in population, and changes in company profits or worker wages).

Nature of Mathematics:

1. Mathematicians see algorithms as familiar tools in a tool chest. They combine algorithms in different ways and use them flexibly to accomplish various tasks.
2. Mathematicians make sense of problems and persevere in solving them.
3. Mathematicians construct viable arguments and critique the reasoning of others.
4. Mathematicians look for and make use of structure.

Standard: 1 Number Sense, Properties, and Operations

Prepared Graduates:
Understand the structure and properties of our number system. At their most basic level numbers are abstract symbols that represent real-world quantities

Grade Level Expectation: Eighth Grade

Concepts and skills students master:
1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Define irrational numbers. - Demonstrate informally that every number has a decimal expansion. i. For rational numbers show that the decimal expansion repeats eventually. ii. Convert a decimal expansion which repeats eventually into a rational number. - Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions. - Apply the properties of integer exponents to generate equivalent numerical expressions. - Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. - Evaluate square roots of small perfect squares and cube roots of small perfect cubes. - Use numbers expressed in the form of a single digit times a whole-number power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. - Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. i. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. ii. Interpret scientific notation that has been generated by technology. - Understand and evaluate negative integer exponents.	**Inquiry Questions:** 1. Why are real numbers represented by a number line and why are the integers represented by points on the number line? 2. Why is there no real number closest to zero? 3. What is the difference between rational and irrational numbers? **Relevance and Application:** 1. Irrational numbers have applications in geometry such as the length of a diagonal of a one by one square, the height of an equilateral triangle, or the area of a circle. 2. Different representations of real numbers are used in contexts such as measurement (metric and customary units), business (profits, network down time, productivity), and community (voting rates, population density). 3. Technologies such as calculators and computers enable people to order and convert easily among fractions, decimals, and percents. **Nature of Mathematics:** 1. Mathematics provides a precise language to describe objects and events and the relationships among them. 2. Mathematicians reason abstractly and quantitatively. 3. Mathematicians use appropriate tools strategically. 4. Mathematicians attend to precision.

2. Patterns, Functions, and Algebraic Structures

Pattern sense gives students a lens with which to understand trends and commonalities. Being a student of mathematics involves recognizing and representing mathematical relationships and analyzing change. Students learn that the structures of algebra allow complex ideas to be expressed succinctly.

Prepared Graduate Competencies in the 2. Patterns, Functions, and Algebraic Structures Standard are:

- Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency
- Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Standard: 2. Patterns, Functions, and Algebraic Structures

<table>
<tr><td colspan="2">Prepared Graduates:
Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics</td></tr>
<tr><td colspan="2">Grade Level Expectation: Kindergarten</td></tr>
<tr><td colspan="2">Concepts and skills students master:
1. Reproduce, extend, create, use and describe patterns and sequences using a variety of materials and methods</td></tr>
<tr><td>Evidence Outcomes</td><td>21st Century Skills and Readiness Competencies</td></tr>
<tr><td rowspan="3">Students can:
- Recognize, construct, extend and describe patterns in a variety of motions, colors, designs, sounds, rhythms, music, positions, sizes, and quantities.
- Identify the missing number in a sequence.
- Sort, classify, describe, and order collections of objects in a variety of ways (example: sort buttons into 2 groups and explain why they were sorted that way)
- Explore a balance scale.
- Recognize when patterns exist, describe patterns, reproduce pattern, and create new patterns.
- Give ABAB as name to patterns.
- Explore and discuss addition and subtraction.</td><td>Inquiry Questions:
1. What characteristics can be used to classify numbers into different groups?
2. How can we predict the next element in a pattern?</td></tr>
<tr><td>Relevance and Application:
1. Understanding patterns and relationships among numbers, shapes, etc. help the student in everyday tasks even down to correct coloring.</td></tr>
<tr><td>Nature of Mathematics:
1. Mathematics involves pattern seeking.
2. Mathematicians use patterns to simplify calculations.
3. Mathematicians model with mathematics.</td></tr>
</table>

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: First Grade

Concepts and skills students master:
1. Reproduce, extend, create, use and describe patterns and sequences using a variety of materials and methods

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Create and extend patterns using concrete materials. - Begin with concrete patterns extend, revisit, many times in varied format. - Determine a missing or inaccurate piece in a number pattern or sequence. - Sort, classify, describe, order objects in a variety of ways. - Represent equivalent forms of the same number. - Compare sets of objects and ID sets with more, fewer, and the same. - ID sets with the greatest and least number of objects. - ID missing items in an array or matrix. - Conjecture and test ideas about patterns (e.g., 2, 4__8). - Continue the pattern in a given table of data using number or concrete materials and verbally describe the pattern. - Explore input/output functions (1 child, 2 eyes, 2 have 4, etc.).	**Inquiry Questions:** 1. What characteristics can be used to classify numbers into different groups? 2. How can we predict the next element in a pattern? **Relevance and Application:** 1. Understanding patterns and relationships among numbers, shapes, etc. help the student in everyday tasks and in interacting with the world. **Nature of Mathematics:** 1. Mathematics involves pattern seeking. 2. Mathematicians use patterns to simplify calculations. 3. Mathematicians model with mathematics.

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Second Grade

Concepts and skills students master:

1. Reproduce, extend, create, use and describe patterns and sequences using a variety of materials and methods

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Verbally describe, create, and extend patterns using symbols, words, numbers. - Use a pattern to find missing elements. - Determine missing or inaccurate pieces of a pattern. - Determine missing addends in open sentences. - Match tables and graphs of points on a coordinate plane. - Recognize, generate, and analyze number sequences. - Use charts, tables, and graphs, to describe patterns. - Describe patterns and other relationships using tables, graphs, and open sentences. - Create and test rules that describe patterns in operations. - Create and extend using symbols. - Investigate and record concrete patterns to explain how the change in one variable affects the change in another.	**Inquiry Questions:** 1. What characteristics can be used to classify numbers into different groups? 2. How can we predict the next element in a pattern? 3. How can using graphs, tables, charts, etc. help us to better interpret data? **Relevance and Application:** 1. Understanding patterns and relationships among numbers, shapes, etc. help the student in everyday tasks and in interacting with the world. 2. Symbols help to represent situations from everyday life with simple equations such as finding how much additional money is needed to buy an item, determining the number of players missing from a soccer team, or calculating the number of students absent from school. **Nature of Mathematics:** 1. Mathematics involves pattern seeking. 2. Mathematicians use patterns to simplify calculations. 3. Mathematicians model with mathematics.

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. Number patterns and relationships can be represented by symbols

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Reproduce, extend, create, and describe patterns using pictures, money, measurement, addition, subtraction, multiplication or geometric shapes. - Generate patterns of increasing complexity. - Use a pattern to find missing elements. - Use patterns to generate a rule. - Identify and describe patterns and other relationships using charts, tables, graphs, and open sentences. - Compare patterns using graphs. - Identify a rule using addition or subtraction patterns and solve a new problem using their rule. - Determine how the change in one quantity affects the change in the other by addition, subtraction, or multiplication. - Identifies and writes a function rule. - Uses a function rule to complete a table. - Represents an unknown using a symbol. - Simplifies expressions containing parentheses. - Simplifies expressions containing exponents.	**Inquiry Questions:** 3. What characteristics can be used to classify numbers into different groups? 4. How can we predict the next element in a pattern? 5. Why do we use symbols to represent missing numbers? 4. Why is finding an unknown quantity important? **Relevance and Application:** 1. Use of an input/output table helps to make predictions in everyday contexts such as the number of beads needed to make multiple bracelets or number of inches of expected growth. 2. Symbols help to represent situations from everyday life with simple equations such as finding how much additional money is needed to buy a skateboard, determining the number of players missing from a soccer team, or calculating the number of students absent from school. **Nature of Mathematics:** 1. Mathematics involves pattern seeking. 2. Mathematicians use patterns to simplify calculations. 3. Mathematicians model with mathematics.

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. Number patterns and relationships can be represented by symbols

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Generate and analyze patterns and identify apparent features of the pattern that were not explicit in the rule itself. i. Use number relationships to find the missing number in a sequence ii. Use a symbol to represent and find an unknown quantity in a problem situation iii. Complete input/output tables iv. Find the unknown in simple equations - Apply concepts of squares, primes, composites, factors, and multiples to solve problems i. Find all factor pairs for a whole number in the range 1–100. ii. Recognize that a whole number is a multiple of each of its factors. iii. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. iv. Determine whether a given whole number in the range 1–100 is prime or composite.	**Inquiry Questions:** 1. What characteristics can be used to classify numbers into different groups? 2. How can we predict the next element in a pattern? 3. Why do we use symbols to represent missing numbers? 4. Why is finding an unknown quantity important? **Relevance and Application:** 1. Use of an input/output table helps to make predictions in everyday contexts such as the number of beads needed to make multiple bracelets or number of inches of expected growth. 2. Symbols help to represent situations from everyday life with simple equations such as finding how much additional money is needed to buy a skateboard, determining the number of players missing from a soccer team, or calculating the number of students absent from school. 3. Comprehension of the relationships between primes, composites, multiples, and factors develop number sense. The relationships are used to simplify computations with large numbers, algebraic expressions, and division problems, and to find common denominators. **Nature of Mathematics:** 1. Mathematics involves pattern seeking. 2. Mathematicians use patterns to simplify calculations. 3. Mathematicians model with mathematics.

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data

Grade Level Expectation: Fifth Grade

Concepts and skills students master:
1. Number patterns are based on operations and relationships

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Generate two numerical patterns using given rules. - Identify apparent relationships between corresponding terms. - Form ordered pairs consisting of corresponding terms from the two patterns, and graphs the ordered pairs on a coordinate plane. - Explain informally relationships between corresponding terms in the patterns. - Use patterns to solve problems including those involving saving and checking accounts. - Explain, extend, and use patterns and relationships in solving problems, including those involving saving and checking accounts such as understanding that spending more means saving less. - Describe how a change in one quantity results in a change in another quantity. - Recognize that a variable is used to represent an unknown quantity.	**Inquiry Questions:** 1. How do you know when there is a pattern? 2. How are patterns useful? **Relevance and Application:** 1. The use of a pattern of elapsed time helps to set up a schedule. For example, classes are each 50 minutes with 5 minutes between each class. 2. The ability to use patterns allows problem-solving. For example, a rancher needs to know how many shoes to buy for his horses, or a grocer needs to know how many cans will fit on a set of shelves. **Nature of Mathematics:** 1. Mathematicians use creativity, invention, and ingenuity to understand and create patterns. 2. The search for patterns can produce rewarding shortcuts and mathematical insights. 3. Mathematicians construct viable arguments and critique the reasoning of others. 4. Mathematicians model with mathematics. 5. Mathematicians look for and express regularity in repeated reasoning.

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

1. Algebraic expressions can be used to generalize properties of arithmetic

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Write and evaluate numerical expressions involving whole-number exponents. - Write, read, and evaluate expressions in which letters stand for numbers. i. Write expressions that record operations with numbers and with letters standing for numbers. ii. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient) and describe one or more parts of an expression as a single entity. iii. Evaluate expressions at specific values of their variables including expressions that arise from formulas used in real-world problems. iv. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). - Apply the properties of operations to generate equivalent expressions. - Identify when two expressions are equivalent.	**Inquiry Questions:** 1. If we didn't have variables, what would we use? 2. What purposes do variable expressions serve? 3. What are some advantages to being able to describe a pattern using variables? 4. Why does the order of operations exist? 5. What other tasks/processes require the use of a strict order of steps? **Relevance and Application:** 1. The simplification of algebraic expressions allows one to communicate mathematics efficiently for use in a variety of contexts. 2. Using algebraic expressions we can efficiently expand and describe patterns in spreadsheets or other technologies. **Nature of Mathematics:** 1. Mathematics can be used to show that things that seem complex can be broken into simple patterns and relationships. 2. Mathematics can be expressed in a variety of formats. 3. Mathematicians reason abstractly and quantitatively. 4. Mathematicians look for and make use of structure. 5. Mathematicians look for and express regularity in repeated reasoning.

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:

Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:

2. Variables are used to represent unknown quantities within equations and inequalities

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Describe solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? - Use substitution to determine whether a given number in a specified set makes an equation or inequality true. - Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. i. Recognize that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. - Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. - Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. - Show that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams. - Represent and analyze quantitative relationships between dependent and independent variables. i. Use variables to represent two quantities in a real-world problem that change in relationship to one another. ii. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. iii. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.	**Inquiry Questions:** 1. Do all equations have exactly one unique solution? Why? 2. How can you determine if a variable is independent or dependent? **Relevance and Application:** 1. Variables allow communication of big ideas with very few symbols. For example, d = r * t is a simple way of showing the relationship between the distance one travels and the rate of speed and time traveled, and $C = \pi d$ expresses the relationship between circumference and diameter of a circle. 2. Variables show what parts of an expression may change compared to those parts that are fixed or constant. For example, the price of an item may be fixed in an expression, but the number of items purchased may change. **Nature of Mathematics:** 1. Mathematicians use graphs and equations to represent relationships among variables. They use multiple representations to gain insights into the relationships between variables. 2. Mathematicians can think both forward and backward through a problem. An equation is like the end of a story about what happened to a variable. By reading the story backward, and undoing each step, mathematicians can find the value of the variable. 3. Mathematicians model with mathematics.

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Seventh Grade

Concepts and skills students master:
1. Properties of arithmetic can be used to generate equivalent expressions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Use properties of operations to generate equivalent expressions. i. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. ii. Demonstrate that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. - Identify the algebraic terms >expression=, >term=, >variable=, >coefficient=, and >constant=.	**Inquiry Questions:** 1. How do symbolic transformations affect an equation or expression? 2. How is it determined that two algebraic expressions are equivalent? **Relevance and Application:** 1. The ability to recognize and find equivalent forms of an equation allows the transformation of equations into the most useful form such as adjusting the density formula to calculate for volume or mass. **Nature of Mathematics:** 1. Mathematicians abstract a problem by representing it as an equation. They travel between the concrete problem and the abstraction to gain insights and find solutions. 2. Mathematicians reason abstractly and quantitatively. 3. Mathematicians look for and express regularity in repeated reasoning.

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Seventh Grade

Concepts and skills students master:
2. Equations and expressions model quantitative relationships and phenomena

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form, using tools strategically. - Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies. - Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. i. Fluently solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. ii. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. iii. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. iv. Graph the solution set of the inequality and interpret it in the context of the problem.	**Inquiry Questions:** 1. Do algebraic properties work with numbers or just symbols? Why? 2. Why are there different ways to solve equations? 3. How are properties applied in other fields of study? 4. Why might estimation be better than an exact answer? 5. When might an estimate be the only possible answer? **Relevance and Application:** 1. Procedural fluency with algebraic methods allows use of linear equations and inequalities to solve problems in fields such as banking, engineering, and insurance. For example, it helps to calculate the total value of assets or find the acceleration of an object moving at a linearly increasing speed. 2. Comprehension of the structure of equations allows one to use spreadsheets effectively to solve problems that matter such as showing how long it takes to pay off debt, or representing data collected from science experiments. 3. Estimation with rational numbers enables quick and flexible decision-making in daily life. For example, determining how many batches of a recipe can be made with given ingredients, how many floor tiles to buy with given dimensions, the amount of carpeting needed for a room, or fencing required for a backyard. **Nature of Mathematics:** 1. Mathematicians model with mathematics.

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
Understand that equivalence is a foundation of mathematics represented in numbers, shapes, measures, expressions, and equations

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Describe the connections between proportional relationships, lines, and linear equations. - Graph proportional relationships, interpreting the unit rate as the slope of the graph. - Compare two different proportional relationships represented in different ways. - Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. - Derive the equation y = mx for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at *b*.	**Inquiry Questions:** 1. How can different representations of linear patterns present different perspectives of situations? 2. How can a relationship be analyzed with tables, graphs, and equations? 3. Why is one variable dependent upon the other in relationships? **Relevance and Application:** 1. Fluency with different representations of linear patterns allows comparison and contrast of linear situations such as service billing rates from competing companies or simple interest on savings or credit. 2. Understanding slope as rate of change allows individuals to develop and use a line of best fit for data that appears to be linearly related. 3. The ability to recognize slope and y-intercept of a linear function facilitates graphing the function or writing an equation that describes the function. **Nature of Mathematics:** 1. Mathematicians represent functions in multiple ways to gain insights into the relationships they model. 2. Mathematicians model with mathematics.

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
Are fluent with basic numerical and symbolic facts and algorithms, and are able to select and use appropriate (mental math, paper and pencil, and technology) methods based on an understanding of their efficiency, precision, and transparency

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

2. Properties of algebra and equality are used to solve linear equations and systems of equations

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Solve linear equations in one variable. i. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. ii. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. - Analyze and solve pairs of simultaneous linear equations. i. Explain that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. ii. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. iii. Solve real-world and mathematical problems leading to two linear equations in two variables.	**Inquiry Questions:** 1. What makes a solution strategy both efficient and effective? 2. How is it determined if multiple solutions to an equation are valid? 3. How does the context of the problem affect the reasonableness of a solution? 4. Why can two equations be added together to get another true equation? **Relevance and Application:** 1. The understanding and use of equations, inequalities, and systems of equations allows for situational analysis and decision-making. For example, it helps people choose cell phone plans, calculate credit card interest and payments, and determine health insurance costs. 2. Recognition of the significance of the point of intersection for two linear equations helps to solve problems involving two linear rates such as determining when two vehicles traveling at constant speeds will be in the same place, when two calling plans cost the same, or the point when profits begin to exceed costs. **Nature of Mathematics:** 1. Mathematics involves visualization. 2. Mathematicians use tools to create visual representations of problems and ideas that reveal relationships and meaning. 3. Mathematicians make sense of problems and persevere in solving them. 4. Mathematicians use appropriate tools strategically.

Standard: 2. Patterns, Functions, and Algebraic Structures

Prepared Graduates:
Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

3. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: a. Define, evaluate, and compare functions. i. Define a function as a rule that assigns to each input exactly one output. ii. Show that the graph of a function is the set of ordered pairs consisting of an input and the corresponding output. iii. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). iv. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line. v. Give examples of functions that are not linear. b. Use functions to model relationships between quantities. i. Construct a function to model a linear relationship between two quantities. ii. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. iii. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. iv. Describe qualitatively the functional relationship between two quantities by analyzing a graph. v. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**Inquiry Questions:** 1. How can change best be represented mathematically? 2. Why are patterns and relationships represented in multiple ways? 3. What properties of a function make it a linear function? **Relevance and Application:** 1. Recognition that non-linear situations is a clue to non-constant growth over time helps to understand such concepts as compound interest rates, population growth, appreciations, and depreciation. 2. Linear situations allow for describing and analyzing the situation mathematically such as using a line graph to represent the relationships of the circumference of circles based on diameters. **Nature of Mathematics:** 1. Mathematics involves multiple points of view. 2. Mathematicians look at mathematical ideas arithmetically, geometrically, analytically, or through a combination of these approaches. 3. Mathematicians look for and make use of structure. 4. Mathematicians look for and express regularity in repeated reasoning.

3. Data Analysis, Statistics, and Probability

Data and probability sense provides students with tools to understand information and uncertainty. Students ask questions and gather and use data to answer them. Students use a variety of data analysis and statistics strategies to analyze, develop and evaluate inferences based on data. Probability provides the foundation for collecting, describing, and interpreting data.

Prepared Graduate Competencies in the 3. Data Analysis, Statistics, and Probability Standard are:

- Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts
- Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data
- Communicate effective logical arguments using mathematical justification and proof. Mathematical argumentation involves making and testing conjectures, drawing valid conclusions, and justifying thinking
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:
Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Kindergarten

Concepts and skills students master:

1. Visual displays of information can used to answer questions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Represent and interpret data. i. Organize, represent, and interpret data with up to two categories. ii. Ask and answer questions about the total number of data points how many in each category, and how many more or less are in one category than in another.	**Inquiry Questions:** 1. What kinds of questions generate data? 2. What questions can be answered by a data representation? **Relevance and Application:** 1. People use graphs and charts to communicate information and learn about a class or community such as the kinds of cars people drive, or favorite ice cream flavors of a class. **Nature of Mathematics:** 1. Mathematicians organize and explain random information. 2. Mathematicians model with mathematics.

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:
Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: First Grade

Concepts and skills students master:
1. Visual displays of information can used to answer questions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Represent and interpret data. ii. Organize, represent, and interpret data with up to two categories. ii. Ask and answer questions about the total number of data points how many in each category, and how many more or less are in one category than in another. - Determine the mode from a given set of numbers.	**Inquiry Questions:** 1. What kinds of questions generate data? 2. What questions can be answered by a data representation? **Relevance and Application:** 1. People use graphs and charts to communicate information and learn about a class or community such as the kinds of cars people drive, or favorite ice cream flavors of a class. **Nature of Mathematics:** 1. Mathematicians organize and explain random information 2. Mathematicians model with mathematics.

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:
Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Second Grade

Concepts and skills students master:

1. Visual displays of information can used to answer questions

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Represent and interpret data. i. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. ii. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. iii. Solve simple put together, take-apart, and compare problems using information presented in picture and bar graphs. iv. Design a survey and collect data and use this data to make predictions.	**Inquiry Questions:** 1. What are the ways data can be displayed? 2. What can data tell you about the people you survey? 3. What makes a good survey question? **Relevance and Application:** 1. People use data to describe the world and answer questions such as how many classmates are buying lunch today, how much it rained yesterday, or in which month are the most birthdays. **Nature of Mathematics:** 1. Mathematics can be displayed as symbols. 2. Mathematicians make sense of problems and persevere in solving them. 3. Mathematicians model with mathematics. 3. Mathematicians attend to precision.

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:
Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Third Grade

Concepts and skills students master:

1. Visual displays are used to describe data

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Represent and interpret data. i. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. ii. Organize and display data using bar graphs, pictographs, and tables. iii. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. iv. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. - Determine the mode from a set of numbers. - Given pictures, determine all the possible combinations of matching a set containing two elements, and with a set containing three elements. - Determine which outcomes are the most likely, least likely, or equally likely when using a chance device.	**Inquiry Questions:** 1. What can data tell you about your class or school? 2. How do data displays help us understand information? **Relevance and Application:** 1. The collection and use of data provides better understanding of people and the world such as knowing what games classmates like to play, how many siblings friends have, or personal progress made in sports. **Nature of Mathematics:** 1. Mathematical data can be represented in both static and animated displays. 2. Mathematicians model with mathematics. 3. Mathematicians use appropriate tools strategically. 4. Mathematicians attend to precision.

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:
Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. Visual displays are used to describe data

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). - Solve problems involving addition and subtraction of fractions by using information presented in line plots. - Organize, construct, read, and interpret a table, line plot, bar graph, and/or pictograph from given data. - Determine and support which outcomes are most likely, least likely, or equally likely when using a chance device.	**Inquiry Questions:** 1. What can you learn by collecting data? 2. What can the shape of data in a display tell you? **Relevance and Application:** 1. The collection and analysis of data provides understanding of how things work. For example, measuring the weather every day for a year helps to better understand weather. **Nature of Mathematics:** 1. Mathematics helps people use data to learn about the world. 2. Mathematicians model with mathematics. 3. Mathematicians use appropriate tools strategically. 4. Mathematicians attend to precision.

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:
Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. Visual displays are used to interpret data

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Represent and interpret data. i. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). ii. Use operations on fractions for this grade to solve problems involving information presented in line plots. iii. From a given scenario, choose the correct graph from possible graph representations. iv. Make convincing arguments based on data analysis. - Distinguish between the median, mean, mode, and range of a set of data. - Differentiate between categorical and numerical data. - Use zero to represent the probability of an impossible event, and one to represent the probability of a certain event. - Use common fractions to represent the probability of events that are neither certain nor impossible.	**Inquiry Questions:** 1. How can you make sense of the data you collect? **Relevance and Application:** 1. The collection and analysis of data provides understanding of how things work. For example, measuring the temperature every day for a year helps to better understand weather. **Nature of Mathematics:** 1. Mathematics helps people collect and use information to make good decisions. 2. Mathematicians model with mathematics. 3. Mathematicians use appropriate tools strategically. 4. Mathematicians attend to precision.

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:
Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Sixth Grade

Concepts and skills students master:
1. Visual displays and summary statistics of one-variable data condense the information in data sets into usable knowledge

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Identify a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. - Demonstrate that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. - Explain that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. - Summarize and describe distributions. i. Display numerical data in plots on a number line, including dot plots, histograms, stem & leaf, and box plots. ii. Summarize numerical data sets in relation to their context. 1. Report the number of observations. 2. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. 3. Give quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. 4. Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. - Demonstrate the meaning of random sampling and biased versus unbiased samples. - Assign 0% to an impossible event, and 100% to a certain event.	**Inquiry Questions:** 1. Why are there so many ways to describe data? 2. When is one data display better than another? 3. When is one statistical measure better than another? 4. What makes a good statistical question? **Relevance and Application:** 1. Comprehension of how to analyze and interpret data allows better understanding of large and complex systems such as analyzing employment data to better understand our economy, or analyzing achievement data to better understand our education system. 2. Different data analysis tools enable the efficient communication of large amounts of information such as listing all the student scores on a state test versus using a box plot to show the distribution of the scores. **Nature of Mathematics:** 1. Mathematicians leverage strategic displays to reveal data. 2. Mathematicians model with mathematics. 3. Mathematicians use appropriate tools strategically. 4. Mathematicians attend to precision.

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:
Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions.

Grade Level Expectation: Seventh Grade

Concepts and skills students master:
1. Statistics can be used to gain information about populations by examining samples

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Use random sampling to draw inferences about a population. i. Explain that generalizations about a population from a sample are valid only if the sample is representative of that population. ii. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. iii. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. - Draw informal comparative inferences about two populations. i. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. ii. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. - Determine the quartiles of a set of data.	**Inquiry Questions:** 1. How might the sample for a survey affect the results of the survey? 2. How do you distinguish between random and bias samples? 3. How can you declare a winner in an election before counting all the ballots? **Relevance and Application:** 1. The ability to recognize how data can be biased or misrepresented allows critical evaluation of claims and avoids being misled. For example, data can be used to evaluate products that promise effectiveness or show strong opinions. 2. Mathematical inferences allow us to make reliable predictions without accounting for every piece of data. **Nature of Mathematics:** 1. Mathematicians are informed consumers of information. They evaluate the quality of data before using it to make decisions. 2. Mathematicians use appropriate tools strategically.

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:
Recognize and make sense of the many ways that variability, chance, and randomness appear in a variety of contexts.

Grade Level Expectation: Seventh Grade

Concepts and skills students master:
2. Mathematical models are used to determine probability.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Explain that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. - Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. - Develop a probability model and use it to find probabilities of events. i. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. ii. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. iii. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. - Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. i. Explain that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. ii. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. iii. For an event described in everyday language identify the outcomes in the sample space which compose the event. iv. Design and use a simulation to generate frequencies for compound events.	**Inquiry Questions:** 1. Why is it important to consider all of the possible outcomes of an event? 2. Is it possible to predict the future? How? 3. What are situations in which probability cannot be used? **Relevance and Application:** 1. The ability to efficiently and accurately count outcomes allows systemic analysis of such situations as trying all possible combinations when you forgot the combination to your lock or deciding to find a different approach when there are too many combinations to try; or counting how many lottery tickets you would have to buy to play every possible combination of numbers. 2. The knowledge of theoretical probability allows the development of winning strategies in games involving chance such as knowing if your hand is likely to be the best hand or is likely to improve in a game of cards. **Nature of Mathematics:** 1. Mathematicians approach problems systematically. When the number of possible outcomes is small, each outcome can be considered individually. When the number of outcomes is large, a mathematician will develop a strategy to consider the most important outcomes such as the most likely outcomes, or the most dangerous outcomes. 2. Mathematicians construct viable arguments and critique the reasoning of others. 3. Mathematicians model with mathematics.

Standard: 3. Data Analysis, Statistics, and Probability

Prepared Graduates:
Solve problems and make decisions that depend on understanding, explaining, and quantifying the variability in data

Grade Level Expectation: Eighth Grade

Concepts and skills students master:
1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. - Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. - For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. - Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. - Explain patterns of association seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. i. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. ii. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.	**Inquiry Questions:** 1. How is it known that two variables are related to each other? 2. How is it known that an apparent trend is just a coincidence? 3. How can correct data lead to incorrect conclusions? 4. How do you know when a credible prediction can be made? **Relevance and Application:** 1. The ability to analyze and interpret data helps to distinguish between false relationships such as developing superstitions from seeing two events happen in close succession versus identifying a credible correlation. 2. Data analysis provides the tools to use data to model relationships, make predictions, and determine the reasonableness and limitations of those predictions. For example, predicting whether staying up late affects grades, or the relationships between education and income, between income and energy consumption, or between the unemployment rate and GDP. **Nature of Mathematics:** 1. Mathematicians discover new relationship embedded in information. 2. Mathematicians construct viable arguments and critique the reasoning of others. 3. Mathematicians model with mathematics.

4. Shape, Dimension, and Geometric Relationships

Geometric sense allows students to comprehend space and shape. Students analyze the characteristics and relationships of shapes and structures, engage in logical reasoning, and use tools and techniques to determine measurement. Students learn that geometry and measurement are useful in representing and solving problems in the real world as well as in mathematics.

Prepared Graduate Competencies in the 4. Shape, Dimension, and Geometric Relationships standard are:

- Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error
- Make sound predictions and generalizations based on patterns and relationships that arise from numbers, shapes, symbols, and data
- Apply transformation to numbers, shapes, functional representations, and data
- Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics
- Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates: Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Kindergarten

Concepts and skills students master:

1. Shapes can be described by characteristics and position and created by composing and decomposing

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). i. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. ii. Correctly name shapes regardless of their orientations or overall size. iii. Identify shapes as two-dimensional or three dimensional. - Analyze, compare, create, and compose shapes. i. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts and other attributes. ii. Model shapes in the world by building shapes from components and drawing shapes. - Compose simple shapes to form larger shapes.	**Inquiry Questions:** 1. What are the ways to describe where an object is? 2. What are all the things you can think of that are round? What is the same about these things? 3. How are these shapes alike and how are they different? 4. Can you make one shape with other shapes? **Relevance and Application:** 1. Shapes help people describe the world. For example, a box is a cube, the Sun looks like a circle, and the side of a dresser looks like a rectangle. 2. People communicate where things are by their location in space using words like next to, below, or between. **Nature of Mathematics:** 1. Geometry helps discriminate one characteristic from another. 2. Geometry clarifies relationships between and among different objects. 3. Mathematicians model with mathematics. 5. Mathematicians look for and make use of structure.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates: Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Kindergarten

Concepts and skills students master:

2. Measurement is used to compare and order objects

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Identify date - Compare yesterday, today, and tomorrow - Identify morning, afternoon, evening, and night - Identify days of the week and months of the year - Identify seasons - Compare differences in hourly, daily, and/or seasonal temperature. - Identify cold, cool, warm, and/or hot - Compare situations and objects by relative temperature - Tell time to the hour and half hour - Use digital and analog clocks to tell and show time - Measure with cup, pint, and quart - Follow recipe and measure - Describe and compare measurable attributes. i. Describe measurable attributes of objects, such as length or weight. ii. Describe several measurable attributes of a single object. iii. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. iv. Order several objects by length, height, weight, or price - Classify objects and count the number of objects in each category. i. Classify objects into given categories. ii. Count the numbers of objects in each category. iii. Sort the categories by count.	**Inquiry Questions:** 1. How can you tell when one thing is bigger than another? 2. How is height different from length? **Relevance and Application:** 1. Measurement helps to understand and describe the world such as in cooking, playing, or pretending. 2. People compare objects to communicate and collaborate with others. For example, we describe items like the long ski, the heavy book, the expensive toy. **Nature of Mathematics:** 1. A system of measurement provides a common language that everyone can use to communicate about objects. 2. Mathematicians use appropriate tools strategically. 3. Mathematicians attend to precision.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates: Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: First Grade

Concepts and skills students master:

1. Shapes can be described by characteristics and position and created by composing and decomposing

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Distinguish between defining attributes versus non-defining attributes. - Build and draw shapes to possess defining attributes. - Compose two-dimensional shapes or three-dimensional shapes to create a composite shape, and compose new shapes from the composite shape. - Partition circles and rectangles into two and four equal shares. i. Describe shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. ii. Describe the whole as two of, or four of the equal shares. - Explores and identifies transformations: translations (slides), rotations (turns), and reflections (flips).	**Inquiry Questions:** 1. What shapes can be combined to create a square? 2. What shapes can be combined to create a circle? **Relevance and Application:** 1. Many objects in the world can be described using geometric shapes and relationships such as architecture, objects in your home, and things in the natural world. Geometry gives us the language to describe these objects. 2. Representation of ideas through drawing is an important form of communication. Some ideas are easier to communicate through pictures than through words such as the idea of a circle, or an idea for the design of a couch. **Nature of Mathematics:** 1. Geometers use shapes to represent the similarity and difference of objects. 2. Mathematicians model with mathematics. 3. Mathematicians look for and make use of structure.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates: Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: First Grade

Concepts and skills students master:

2. Measurement is used to compare and order objects and events

<table>
<tr><th>Evidence Outcomes</th><th>21st Century Skills and Readiness Competencies</th></tr>
<tr><td rowspan="3">Students can:
- Measure lengths indirectly and by iterating length units.
i. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
ii. Express the length of an object as a whole number of length units.
- Tell and write time.
i. Tell and write time in hours and half-hours using analog and digital clocks.
- Identify dates on a calendar.
- Identify and use calendar terms and concepts such as yesterday, today, tomorrow, morning, afternoon, evening, and night.
- Sequence daily events.
- Compare and order objects by temperature, length, weight, capacity, and size using customary and non-standard units.</td><td>Inquiry Questions:
1. How can you tell when one thing is bigger than another?
2. Why do we measure objects and time?
3. How are length and time different? How are they the same?</td></tr>
<tr><td>Relevance and Application:
1. Time measurement is a means to organize and structure each day and our lives, and to describe tempo in music.
2. Measurement helps to understand and describe the world such as comparing heights of friends, describing how heavy something is, or how much something holds.</td></tr>
<tr><td>Nature of Mathematics:
1. With only a few words, mathematicians use measurable attributes to describe countless objects.
2. Mathematicians use appropriate tools strategically.
3. Mathematicians attend to precision.</td></tr>
</table>

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Second Grade

Concepts and skills students master:
1. Shapes can be described by their attributes and used to represent part/whole relationships.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. - Identify triangles, quadrilaterals, pentagons, hexagons, circles, and cubes. - Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. - Recognize that equal shares of identical wholes need not have the same shape. - Recognize congruent & similar shapes. - Describe symmetry and identify a line of symmetry in squares and rectangles. - Identify right angles and non-right angles. - Explore and discuss the concepts of parallel, perpendicular, horizontal, and oblique (diagonal), - Use geometric concepts as a means for describing the world.	**Inquiry Questions:** 1. How can we describe geometric figures? 2. Is a half always the same size and shape? **Relevance and Application:** 1. Fairness in sharing depends on equal quantities, such as sharing a piece of cake, candy bar, or payment for a chore. 2. Shapes are used to communicate how people view their environment. 3. Geometry provides a system to describe, organize, and represent the world around us. **Nature of Mathematics:** 1. Geometers use shapes to describe and understand the world. 2. Mathematicians reason abstractly and quantitatively. 3. Mathematicians model with mathematics.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Second Grade

Concepts and skills students master:
2. Some attributes of objects are measurable and can be quantified using different tools.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Measure and estimate lengths in standard units. i. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. ii. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. iii. Estimate lengths using units of inches, feet, centimeters, and meters. iv. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. v. Order objects according to length, capacity, weight, and temperature. vi. Measure temperature to nearest 2 degrees and 10 degrees Fahrenheit. - Relate addition and subtraction to length. i. Represent whole numbers as lengths from 0 on a number line diagram and represent whole-number sums and differences within 100 on a number line diagram. - Solve problems time and money. i. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. ii. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.	**Inquiry Questions:** 1. What are the different things we can measure? 2. How do we decide which tool to use to measure something? 3. What would happen if everyone created and used their own rulers? **Relevance and Application:** 1. Measurement is used to understand and describe the world including sports, construction, and explaining the environment. **Nature of Mathematics:** 1. Mathematicians use measurable attributes to describe countless objects with only a few words. 2. Mathematicians use appropriate tools strategically. 3. Mathematicians attend to precision.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Third Grade

Concepts and skills students master:
1. Geometric figures are described by their attributes

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Reason with shapes and their attributes. i. Explain that shapes in different categories may share attributes and that the shared attributes can define a larger category. 1. Identify rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. ii. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. - Identify a line of symmetry for regular polygons and other familiar objects. - Identify points, lines, segments, and angles (right, acute, obtuse). - Identify the characteristics of a two dimensional figure, and a three dimensional figure. - Discuss and Identify line segments (including parallel and perpendicular, horizontal, vertical, and oblique). - Identify triangles by their sides and angles. - Use geometric concepts as a means for describing the world.	**Inquiry Questions:** 1. What words in geometry are also used in daily life? 2. Why can different geometric terms be used to name the same shape? **Relevance and Application:** 1. Recognition of geometric shapes allows people to describe and change their surroundings such as creating a work of art using geometric shapes, or design a pattern to decorate. **Nature of Mathematics:** 1. Mathematicians use clear definitions in discussions with others and in their own reasoning. 2. Mathematicians construct viable arguments and critique the reasoning of others. 3. Mathematicians look for and make use of structure.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:

Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Third Grade

Concepts and skills students master:

2. Linear and area measurement are fundamentally different and require different units of measure.

<table>
<tr><th>Evidence Outcomes</th><th>21st Century Skills and Readiness Competencies</th></tr>
<tr><td rowspan="3">Students can:
- Use concepts of area and relate area to multiplication and to addition.
i. Recognize area as an attribute of plane figures and apply concepts of area measurement.
ii. Find area of rectangles with whole number side lengths using a variety of methods.
iii. Relate area to the operations of multiplication and addition and recognize area as additive.
- Describe perimeter as an attribute of plane figures and distinguish between linear and area measures.
- Solve real world and mathematical problems involving perimeters of polygons.
i. Find the perimeter given the side lengths.
ii. Find an unknown side length given the perimeter.
iii. Find rectangles with the same perimeter and different areas or with the same area and different perimeters.</td><td>Inquiry Questions:
1. What kinds of questions can be answered by measuring?
2. What are the ways to describe the size of an object or shape?
3. How does what we measure influence how we measure?
3. What would the world be like without a common system of measurement?</td></tr>
<tr><td>Relevance and Application:
1. The use of measurement tools allows people to gather, organize, and share data with others such as sharing results from science experiments, or showing the growth rates of different types of seeds.
2. A measurement system allows people to collaborate on building projects, mass produce goods, make replacement parts for things that break, and trade goods.</td></tr>
<tr><td>Nature of Mathematics:
1. Mathematicians use tools and techniques to accurately determine measurement.
2. People use measurement systems to specify attributes of objects with enough precision to allow collaboration in production and trade.
3. Mathematicians make sense of problems and persevere in solving them.
4. Mathematicians model with mathematics.</td></tr>
</table>

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Third Grade

Concepts and skills students master:
3. Time and attributes of objects can be measured with appropriate tools.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Solve problems involving measurement and estimation of intervals of time, temperature, liquid volumes, and masses of objects. i. Tell and write time to the nearest minute. ii. Measure time intervals in minutes. iii. Solve word problems involving addition and subtraction of time intervals in minutes using a number line diagram. iv. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). v. Use models to add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. - Measure objects using tools from their environment.	**Inquiry Questions:** 1. Why do we need standard units of measure? 2. Why do we measure time? **Relevance and Application:** 1. A measurement system allows people to collaborate on building projects, mass produce goods, make replacement parts for things that break, and trade goods. **Nature of Mathematics:** 1. People use measurement systems to specify the attributes of objects with enough precision to allow collaboration in production and trade. 2. Mathematicians use appropriate tools strategically. 3. Mathematicians attend to precision.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

1. Appropriate measurement tools, units, and systems are used to measure different attributes of objects and time.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. i. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec (US customary units and metric units). ii. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. iii. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. iv. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. v. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. - Use concepts of angle and measure angles. i. Describe angles as geometric shapes that are formed wherever two rays share a common endpoint, and explain concepts of angle measurement. ii. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. iii. Demonstrate that angle measure as additive. iv. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems. - Choose appropriate tool to measure familiar objects in situations that contain length, weight, capacity, time, and temperature.	**Inquiry Questions:** 1. How do you decide when close is close enough? 2. How can you describe the size of geometric figures? **Relevance and Application:** 1. Accurate use of measurement tools allows people to create and design projects around the home or in the community such as flower beds for a garden, fencing for the yard, wallpaper for a room, or a frame for a picture. **Nature of Mathematics:** 1. People use measurement systems to specify the attributes of objects with enough precision to allow collaboration in production and trade. 2. Mathematicians make sense of problems and persevere in solving them. 3. Mathematicians use appropriate tools strategically. 4. Mathematicians attend to precision.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Fourth Grade

Concepts and skills students master:

2. Geometric figures in the plane and in space are described and analyzed by their attributes.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. - Identify points, line segments, angles, and perpendicular and parallel lines in two-dimensional figures. - Classify and identify two-dimensional figures according to attributes of line relationships or angle size. - Identify a line of symmetry for a two-dimensional figure. - Identify and give examples of congruency. - Recognize common attributes of squares, rectangles, triangles, and circles. - Locate objects on a coordinate grid and label ordered pairs. - Recognize geometry in the student's everyday world.	**Inquiry Questions:** 1. How do geometric relationships help us solve problems? 2. Is a square still a square if it's tilted on its side? 3. How are three-dimensional shapes different from two-dimensional shapes? 4. What would life be like in a two-dimensional world? 5. Why is it helpful to classify things like angles or shapes? **Relevance and Application:** 1. The understanding and use of spatial relationships helps to predict the result of motions such as how articles can be laid out in a newspaper, what a room will look like if the furniture is rearranged, or knowing whether a door can still be opened if a refrigerator is repositioned. 2. The application of spatial relationships of parallel and perpendicular lines aid in creation and building. For example, hanging a picture to be level, building windows that are square, or sewing a straight seam. **Nature of Mathematics:** 1. Geometry is a system that can be used to model the world around us or to model imaginary worlds. 2. Mathematicians look for and make use of structure. 3. Mathematicians look for and express regularity in repeated reasoning.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Fifth Grade

Concepts and skills students master:

1. Properties of multiplication and addition provide the foundation for volume an attribute of solids.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Model and justify the formula for volume of rectangular prisms. i. Model the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes. ii. Show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. iii. Represent threefold whole-number products as volumes to represent the associative property of multiplication. - Find volume of rectangular prisms using a variety of methods and use these techniques to solve real world and mathematical problems. i. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. ii. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths. iii. Use the additive nature of volume to find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts.	**Inquiry Questions:** 1. Why do you think a unit cube is used to measure volume? **Relevance and Application:** 1. The ability to find volume helps to answer important questions such as which container holds more. **Nature of Mathematics:** 1. Mathematicians create visual and physical representations of problems and ideas that reveal relationships and meaning. 2. Mathematicians make sense of problems and persevere in solving them. 3. Mathematicians model with mathematics.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Fifth Grade

Concepts and skills students master:
2. Geometric figures can be described by their attributes and specific locations in the plane.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Graph points on the coordinate plane to solve real-world and mathematical problems. - Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. - Classify two-dimensional figures and three-dimensional figures into categories based on their properties. i. Explain that attributes belonging to a category of two-dimensional & three-dimensional figures also belong to all subcategories of that category. ii. Classify two-dimensional figures and three-dimensional figures in a hierarchy based on properties.	**Inquiry Questions:** 1. How does using a coordinate grid help us solve real world problems? 2. What are the ways to compare and classify geometric figures? 3. Why do we classify shapes? **Relevance and Application:** 1. The coordinate grid is a basic example of a system for mapping relative locations of objects. It provides a basis for understanding latitude and longitude, GPS coordinates, and all kinds of geographic maps. 2. Symmetry is used to analyze features of complex systems and to create worlds of art. For example symmetry is found in living organisms, the art of MC Escher, and the design of tile patterns, and wallpaper. **Nature of Mathematics:** 1. Geometry's attributes give the mind the right tools to consider the world around us. 2. Mathematicians model with mathematics. 3. Mathematicians look for and make use of structure.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Make claims about relationships among numbers, shapes, symbols, and data and defend those claims by relying on the properties that are the structure of mathematics

Grade Level Expectation: Sixth Grade

Concepts and skills students master:
1. Objects in space and their parts and attributes can be measured and analyzed

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Develop and apply formulas and procedures for area of plane figures. - Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. - Apply these techniques in the context of solving real-world and mathematical problems. - Develop and apply formulas and procedures for volume of regular prisms. - Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths. - Show that volume is the same as multiplying the edge lengths of a rectangular prism. - Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. - Draw polygons in the coordinate plan to solve real-world and mathematical problems. - Draw polygons in the coordinate plane given coordinates for the vertices. - Use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. - Develop and apply formulas and procedures for the surface area. i. Represent three-dimensional figures using nets made up of rectangles and triangles. ii. Use nets to find the surface area of figures. iii. Apply techniques for finding surface area in the context of solving real-world and mathematical problems. - Use a protractor to measure and draw angles. - Find the circumference of a circle and begin to use the formula. - Find and compare the relationship between the circumference and diameter or a circle. e.g. the value of pi.	**Inquiry Questions:** 1. Can two shapes have the same volume but different surface areas? Why? 2. Can two figures have the same surface area but different volumes? Why? 3. What does area tell you about a figure? 4. What properties affect the area of figures? **Relevance and Application:** 1. Knowledge of how to find the areas of different shapes helps do projects in the home and community. For example how to use the correct amount of fertilizer in a garden, buy the correct amount of paint, or buy the right amount of material for a construction project. 2. The application of area measurement of different shapes aids with everyday tasks such as buying carpeting, determining watershed by a center pivot irrigation system, finding the number of gallons of paint needed to paint a room, decomposing a floor plan, or designing landscapes. **Nature of Mathematics:** 1. Mathematicians realize that measurement always involves a certain degree of error. 2. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning. 3. Mathematicians make sense of problems and persevere in solving them. 4. Mathematicians reason abstractly and quantitatively.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Seventh Grade

Concepts and skills students master:

1. Modeling geometric figures and relationships leads to informal spatial reasoning and proof.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Draw construct, and describe geometrical figures and describe the relationships between them. i. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. ii. Draw (freehand, with ruler and protractor or compass, and with technology) geometric shapes with given conditions. iii. Construct triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. iv. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. - Show the difference between the concepts of congruence and similarity. - Use transformations; *e.g., reflections, translations, rotations;* to find relationships among geometric figures.	**Inquiry Questions:** 1. Is there a geometric figure for any given set of attributes? 2. How does scale factor affect length, perimeter, angle measure, area and volume? 3. How do you know when a proportional relationship exists? **Relevance and Application:** 1. The understanding of basic geometric relationships helps to use geometry to construct useful models of physical situations such as blueprints for construction, or maps for geography. 2. Proportional reasoning is used extensively in geometry such as determining properties of similar figures, and comparing length, area, and volume of figures. **Nature of Mathematics:** 1. Mathematicians create visual representations of problems and ideas that reveal relationships and meaning. 2. The relationship between geometric figures can be modeled 3. Mathematicians look for relationships that can be described simply in mathematical language and applied to a myriad of situations. Proportions are a powerful mathematical tool because proportional relationships occur frequently in diverse settings. 4. Mathematicians use appropriate tools strategically. 5. Mathematicians attend to precision.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Understand quantity through estimation, precision, order of magnitude, and comparison. The reasonableness of answers relies on the ability to judge appropriateness, compare, estimate, and analyze error

Grade Level Expectation: Seventh Grade

Concepts and skills students master:
2. Linear measure, angle measure, area, and volume are fundamentally different and require different units of measure.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - State the formulas for the area and circumference of a circle and use them to solve problems. - Give an informal derivation of the relationship between the circumference and area of a circle. - Use properties of supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. - Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. - Demonstrate the relationship of the circumference to the diameter of a circle as approximating pi in geometry.	**Inquiry Questions:** 1. How can geometric relationships among lines and angles be generalized, described, and quantified? 2. How do line relationships affect angle relationships? 3. Can two shapes have the same volume but different surface areas? Why? 4. Can two shapes have the same surface area but different volumes? Why? 5. How are surface area and volume like and unlike each other? 6. What do surface area and volume tell about an object? 7. How are one-, two-, and three-dimensional units of measure related? 8. Why is pi an important number? **Relevance and Application:** 1. The ability to find volume and surface area helps to answer important questions such as how to minimize waste by redesigning packaging, or understanding how the shape of a room affects its energy use. **Nature of Mathematics:** 1. Geometric objects are abstracted and simplified versions of physical objects. 2. Geometers describe what is true about all cases by studying the most basic and essential aspects of objects and relationships between objects. 3. Mathematicians make sense of problems and persevere in solving them. 4. Mathematicians construct viable arguments and critique the reasoning of others.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Apply transformation to numbers, shapes, functional representations, and data

Grade Level Expectation: Eighth Grade

Concepts and skills students master:

1. Transformations of objects can be used to define the concepts of congruence and similarity.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Verify experimentally the properties of rotations, reflections, and translations. - Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. - Demonstrate that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. - Given two congruent figures, describe a sequence of transformations that exhibits the congruence between them. - Demonstrate that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. - Given two similar two-dimensional figures, describe a sequence of transformations that exhibits the similarity between them. - Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.	**Inquiry Questions:** 1. What advantage, if any, is there to using the Cartesian coordinate system to analyze the properties of shapes? 2. How can you physically verify that two lines are really parallel? **Relevance and Application:** 1. Dilations are used to enlarge or shrink pictures. 2. Rigid motions can be used to make new patterns for clothing or architectural design. **Nature of Mathematics:** 1. Geometry involves the investigation of invariants. Geometers examine how some things stay the same while other parts change to analyze situations and solve problems. 2. Mathematicians construct viable arguments and critique the reasoning of others. 3. Mathematicians model with mathematics.

Standard: 4. Shape, Dimension, and Geometric Relationships

Prepared Graduates:
Use critical thinking to recognize problematic aspects of situations, create mathematical models, and present and defend solutions

Grade Level Expectation: Eighth Grade

Concepts and skills students master:
2. Direct and indirect measurement can be used to describe and make comparisons.

Evidence Outcomes	21st Century Skills and Readiness Competencies
Students can: - Explain a proof of the Pythagorean Theorem and its converse. - Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. - Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. - State the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	**Inquiry Questions:** 1. Why does the Pythagorean Theorem only apply to right triangles? 2. How can the Pythagorean Theorem be used for indirect measurement? 3. How are the distance formula and the Pythagorean theorem the same? Different? 4. How are the volume formulas for cones, cylinders, prisms and pyramids interrelated? 5. How is volume of an irregular figure measured? 6. How can cubic units be used to measure volume for curved surfaces? **Relevance and Application:** 1. The understanding of indirect measurement strategies allows measurement of features in the immediate environment such as playground structures, flagpoles, and buildings. 2. Knowledge of how to use right triangles and the Pythagorean Theorem enables design and construction of such structures as a properly pitched roof, handicap ramps to meet code, structurally stable bridges, and roads. 3. The ability to find volume helps to answer important questions such as how to minimize waste by redesigning packaging or maximizing volume by using a circular base. **Nature of Mathematics:** 1. Mathematicians use geometry to model the physical world. Studying properties and relationships of geometric objects provides insights in to the physical world that would otherwise be hidden. 2. Geometric objects are abstracted and simplified versions of physical objects 3. Mathematicians make sense of problems and persevere in solving them. 4. Mathematicians construct viable arguments and critique the reasoning of others.

Standard 5: Students will understand that mathematics is a tool by which we can study the quantitative properties of God's creation

**** As students in grades K-4 extend their knowledge, they will . . . .***

Kindergarten	First Grade	Second Grade	Third Grade	Fourth Grade
Benchmark 1: Grow in their understanding of God's infinite wisdom, and the way He has ordered our world.				
Benchmark 2: Develop an awareness of the important role which mathematics plays in God's creation by becoming informed citizens who can deal with social and technological problems rooted in mathematics and technology				

**** As students in grades 5-8 extend their knowledge, they will . . . .***

Fifth Grade	Sixth Grade	Seventh Grade	Eighth Grade/ Algebra ½
Benchmark 1: Grow in their understanding of God's infinite wisdom, and the way He has ordered our world.			
Benchmark 2: Develop an awareness of the important role which mathematics plays in God's creation by becoming informed citizens who can deal with social and technological problems rooted in mathematics and technology			

**** As students in Algebra extend their knowledge, they will . . . .***

Algebra
Benchmark 1: Grow in their understanding of God's infinite wisdom, and the way He has ordered our world.
Benchmark 2: Develop an awareness of the important role which mathematics plays in God's creation by becoming informed citizens who can deal with social and technological problems rooted in mathematics and technology

(Algebra)

By Content Strand

Algebra Foundations

- Understanding Variables and Expressions
- Simplifying Expressions Using the Product Property of Exponents
- Using Order of Operations
- Finding Absolute Value and Adding Real Numbers
- Simplifying and Comparing Expressions with Symbols of Inclusion
- Evaluating and Comparing Algebraic Expressions
- Calculating and Comparing Square Roots
- Using the Distributive Property to Simplify Expressions
- Simplifying and Evaluating Variable Expressions
- Translating Between Words and Algebraic Expressions
- Simplifying and Evaluating Expressions with Integer and Zero Exponents
- Simplifying and Evaluating Expressions Using the Power Property for Exponents
- Simplifying Expressions with Square Roots and Higher-Order Roots

Functions & Relations

- Graphing on a Coordinate Plane
- Graphing Functions
- Recognizing and Extending Geometric Sequences
- Identifying and Graphing Exponential Functions
- Graphing Cubic Functions
- Graphing and Comparing Linear, Quadratic, and Exponential Functions

Equations

- Combining Like Terms
- Solving Two-Step Equations
- Solving Decimal Equations
- Solving Multi-Step Equations
- Solving Equations with Variables on Both Sides
- Solving Literal Equations
- Using Rates, Ratios, and Proportions
- Solving Percent Problems
- Solving Problems Involving the Percent of Change
- Using Trigonometric Ratios

Linear Equations & Functions

- Locating and Using Intercepts
- Finding Rates of Change and Slope
- Finding Slope Using the Slope Formula
- Writing Equations in Slope-Intercept Form
- Determining the Equation of a Line Given Two Points
- Identifying, Writing, and Graphing Direct Variation
- Identifying, Writing, and Graphing Inverse Variation
- Writing Equations of Parallel and Perpendicular Lines

Polynomials

- Simplifying Expressions Using the GCF
- Adding and Subtracting Polynomials
- Multiplying Polynomials
- Finding Special Products of Binomials
- Factoring Trinomials: $x2 + bx + c$
- Factoring Trinomials: $ax2 + bx + c$
- Factoring Trinomials by Using the GCF
- Factoring Special Products
- Factoring Polynomials by Grouping
- Dividing Polynomials

Rational Expressions & Functions

- Using the Distributive Property to Simplify Rational Expressions
- Simplifying Rational Expressions
- Simplifying Rational Expressions with Like Denominators
- Finding the Least Common Multiple
- Graphing Rational Functions
- Multiplying and Dividing Rational Expressions
- Adding and Subtracting Rational Expressions
- Simplifying Complex Fractions
- Combining Rational Expressions with Unlike Denominators

Inequalities

- Translating Between Words and Inequalities
- Graphing Inequalities
- Solving Inequalities by Adding or Subtracting
- Solving Inequalities by Multiplying or Dividing
- Solving Compound Inequalities
- Solving Two-Step and Multi-Step Inequalities
- Solving Inequalities with Variables on Both Sides

Systems of Equations & Inequalities

- Solving Systems of Linear Equations by Graphing
- Solving Systems of Linear Equations by Substitution
- Solving Systems of Linear Equations by Elimination
- Solving and Classifying Special Systems of Linear Equations
- Graphing Linear Inequalities
- Graphing Systems of Linear Inequalities
- Graphing and Solving Systems of Linear and Quadratic Equations

Radical Expressions & Functions

- Simplifying Radical Expressions
- Adding and Subtracting Radical Expressions
- Multiplying Radical Expressions
- Graphing Square-Root Functions

Absolute-Value Equations & Inequalities

- Solving Absolute-Value Equations
- Solving Absolute-Value Inequalities
- Solving Multi-Step Absolute-Value Equations
- Solving Multi-Step Absolute-Value Inequalities
- Graphing Absolute-Value Functions

Quadratic Equations & Functions

- Identifying Quadratic Functions
- Solving Problems Using the Pythagorean Theorem
- Calculating the Midpoint and Length of a Segment
- Graphing Calculator: Characteristics of Parabolas
- Identifying Characteristics of Quadratic Functions
- Graphing Quadratic Functions
- Solving Quadratic Equations by Factoring
- Solving Quadratic Equations by Graphing

Probability & Data Analysis

- Determining the Theoretical Probability of an Event
- Analyzing and Comparing Statistical Graphs
- Identifying Misleading Representations of Data
- Finding the Probability of Independent and Dependent Events
- Analyzing Measures of Central Tendency
- Displaying Data in Stem-and-Leaf Plots and Histograms