

Accelerated Pre-calculus Curriculum for Summer Institute

Saint Joseph High School, Metuchen, New Jersey

TEXTBOOK: Pre-calculus: Graphing, Numerical, Algebraic (Eighth Edition) by Demana, Waits, Foley and Kennedy

Chapter P Prerequisites

P.1 Real Numbers

Objective: The student will be able to:

- convert between decimals and fractions
- write inequalities
- apply the basic properties of algebra
- work with exponents and scientific notation

P.2 Cartesian Coordinate System

Objective: The student will be able to:

- graph points
- find distances and midpoints on a number line and in a coordinate plane
- write standard-form equations of circles

P. 3 Linear Equations and Inequalities

Objective: The student will be able to:

- solve linear equations in one variable
- solve linear inequalities in one variable

P. 4 Lines in the Plane

Objective: The student will be able to:

- use the concepts of slope and y-intercept to graph and write linear equations in two variables

P. 5 Solving Equations Graphically, Numerically, and Algebraically

Objective: The student will be able to:

- solve equations involving quadratic, absolute value, and fractional expressions by finding x-intercepts or intersections on graphs, by using algebraic technologies, or by using numerical technologies (graphing calculator generated graphs, tables, etc.)

P. 6 Solving Inequalities Algebraically and Graphically

Objective: The student will be able to:

- solve inequalities involving absolute value, quadratic polynomials, and expressions involving fractions

Chapter 1 Functions and Graphs

1.1 Modeling and Equation Solving

Objective: The student will be able to:

- use numerical, algebraic, and graphical methods to solve problems
- be able to translate from one model to another

1.2 Functions and Their Properties

Objective: The student will be able to:

- represent functions numerically, algebraically, and graphically
- determine the domain and range for functions
- analyze function characteristics such as extreme values, symmetry, asymptotes, and end behavior

1.3 Ten Basic Functions

Objective: The student will be able to:

- recognize graphs of ten basic functions
- determine domains of functions related to the ten basic functions
- combine the ten basic functions in various ways to create new functions

1.4 Building Functions from Functions

Objective: The student will be able to:

- build new functions from old functions in several ways:
by: adding, subtracting, multiplying, and dividing functions, defining functions parametrically, composing functions, and computing inverses of functions

1.5 Graphical Transformations

Objective: The student will be able to:

- algebraically and graphically represent translations, reflections, stretches, and shrinks of functions and parametric relations

1.6 Modeling with Functions

Objective: The student will be able to:

- identify appropriate basic functions with which to model real world problems
- produce specific functions to model data, formulas, graphs, and verbal descriptions

Chapter 2 Polynomial, Power, and Rational Functions

2.1 Linear and Quadratic Functions with Modeling

Objective: The student will be able to:

- recognize and graph linear and quadratic functions
- use these functions to model application problems

2.2 Power Functions with Modeling

Objective: The student will be able to:

- sketch power functions of the form $f(x) = kx^a$ (where k and a are rational numbers)

2.3 Polynomial Functions of Higher Degree with Modeling

Objective: The student will be able to:

- graph polynomial functions
- predict their end behavior
- find their real zeros using a grapher or an algebraic method

2.4 Real Zeros of Polynomial Functions

Objective: The student will be able to:

- divide polynomials using long division or synthetic division'
- apply the Remainder Theorem, Factor Theorem, and Rational Zeros Theorem
- find upper and lower bounds for zeros of polynomials

2.5 Complex Numbers

Objective: The student will be able to:

- add, subtract, and divide complex numbers
- find complex zeros of quadratic functions

2.6 Complex Zeros and the Fundamental Theorem of Algebra

Objective: The student will be able to:

- factor polynomials with real coefficients

2.7 Rational Functions and Equations

Objective: The student will be able to:

- describe the graphs of rational functions
- identify horizontal and vertical asymptotes
- predict the end behavior of rational functions

2.8 Solving Inequalities in One Variable

Objective: The student will be able to:

- solve inequalities involving polynomials and rational functions by using both algebraic and graphical techniques

Chapter 3 Exponential, Logistic, and Logarithmic Functions

3.1 Exponential and Logistic Functions

Objective: The student will be able to:

- evaluate exponential expressions
- identify and graph exponential and logistic functions

3.2 Exponential and Logistic Modeling

Objective: The student will be able to:

- use exponential growth, decay, and regression to model real-life problems

3.3 Logarithmic Functions and Their Graphs

Objective: The student will be able to:

- convert equations between logarithmic form and exponential form
- evaluate common and natural logarithms
- graph common and natural logarithms

3.4 Properties of Logarithmic Functions

Objective: The student will be able to:

- apply the properties of logarithms to evaluate expressions and graph functions and be able to re-express data

3.5 Equation Solving and Modeling

Objective: The student will be able to:

- apply the properties of logarithms to solve exponential and logarithmic equations algebraically
- solve applications problems using these equations

Chapter 4 Trigonometric Functions

4.1 Angles and Their Measures

Objective: The student will be able to:

- convert between radians and degrees
- find arc lengths
- convert to nautical miles
- solve problems involving angular speed

4.2 Trigonometric Functions of Acute Angles

Objective: The student will be able to:

- define the six trigonometric functions using the lengths of the sides of a right triangle

4.3 Trigonometry Extended: The Circular Functions

Objective: The student will be able to:

- solve problems involving the trigonometric functions of real numbers and the properties of the sine and cosine as periodic functions

4.4 Graphs of Sine and Cosine: Sinusoids

Objective: The student will be able to:

- generate the graphs of the sine and cosine functions
- explore various transformations of these graphs

4.5 Graphs of Tangent, Cotangent, Secant, and Cosecant

Objective: The student will be able to:

- generate the graphs for the tangent, cotangent, secant, and cosecant functions
- explore various transformations of these graphs

4.6 Graphs of Composite Trigonometric Functions

Objective: The student will be able to:

- graph sums, differences, and other combinations of trigonometric and algebraic functions

4.7 Inverse Trigonometric Functions

Objective: The student will be able to:

- relate the concept of inverse function to trigonometric functions

4.8 Solving Problems with Trigonometry

Objective: The student will be able to:

- apply the concepts of trigonometry to solve real-world problems

Chapter 5 Analytic Trigonometry

5.1 Fundamental Identities

Objective: The student will be able to:

- use the fundamental identities to simplify trigonometric expressions and solve trigonometric equations

5.2 Proving Trigonometric Identities

Objective: The student will be able to:

- decide whether an equation is an identity
- confirm identities analytically

5.3 Sum and Difference Identities

Objective: The student will be able to:

- apply the identities for the cosine, sine, and tangent of a difference or sum

5.4 Multiple-Angle Identities

Objective: The student will be able to:

- apply the double-angle identities, power-reducing identities, and half-angle identities

5.5 The Law of Sines

Objective: The student will be able to:

- understand the proof of the Law of Sines
- use the computational applications of the Law of Sines to solve a variety of problems

5.6 The Law of Cosines

Objective: The student will be able to:

- apply the Law of Cosines to solve acute and obtuse triangles and to determine the area of a triangle in terms of the measures of the sides and angles
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Chapter 6 Vectors, Parametric Equations, and Polar Equations (if time permits)

6.1 Vectors in the Plane

Objective: The student will be able to:

- apply the arithmetic of vectors
- use vectors to solve real-world problems

6.2 Dot Product of Vectors

Objective: The student will be able to:

- calculate dot products
- calculate projections of vectors

6.3 Parametric Equations and Motion

Objective: The student will be able to:

- define parametric equations
- graph curves parametrically
- solve application problems using parametric equations

6.4 Polar Coordinates (optional)

Objective: The student will be able to:

- convert points and equations from polar to rectangular coordinates and vice versa

6.5 Graphs of Polar Equations (optional)

Objective: The student will be able to:

- graph polar equations
- determine the maximum r-value and the symmetry of a graph

6.6 DeMoivre's Theorem and nth Roots (optional)

Objective: The student will be able to:

- represent complex numbers in trigonometric form and perform operations on them

Chapter 7 Systems and Matrices

7.1 Solving Systems of Two Equations

Objective: The student will be able to:

- solve systems of equations graphically and algebraically

7.2 Matrix Algebra

Objective: The student will be able to:

- find sums, differences, products, and inverses of matrices

7.3 Multivariate Linear Systems and Row Operations

Objective: The student will be able to:

- solve systems of linear equations using Gaussian elimination, the reduced row echelon form of a matrix, or an inverse matrix

7.4 Partial Fractions

Objective: The student will be able to:

- decompose rational expressions into partial fractions

7.5 Systems of Inequalities in Two Variables

Objective: The student will be able to:

- solve linear programming problems and systems of inequalities using graphical methods

Chapter 8 Analytic Geometry in Two and Three Dimensions

8.1 Conic Sections and Parabolas

Objective: The student will be able to:

- find the standard form equation, focus, and directrix of a parabola

8.2 Ellipses

Objective: The student will be able to:

- find the standard form equation, vertices, and foci of an ellipse

8.3 Hyperbolas

Objective: The student will be able to:

- find the standard form equation, vertices, and foci of a hyperbola

8.4 Translation and Rotation of Axes (if time permits)

Objective: The student will be able to:

- graph conic sections that have been rotated

8.5 Polar Equations of Conics (if time permits)

Objective: The student will be able to:

- understand the general focus-directrix definition of a conic section
- write equations of conic sections in polar form

8.6 Three-Dimensional Cartesian Coordinate System (if time permits)

Objective: The student will be able to:

- draw three-dimensional figures and analyze vectors in space

Chapter 9 Discrete Mathematics (optional, as time permits)

9.1 Basic Combinatorics

Objective: The student will be able to:

- use the multiplication principle of counting, permutations, or combinations to count the number of ways that a task can be done

9.2 The Binomial Theorem

Objective: The student will be able to:

- expand a power of a binomial using the binomial theorem or Pascal's triangle
- find the coefficients of a given term of a binomial expansion

9.3 Probability

Objective: The student will be able to:

- identify a sample space
- calculate probabilities and conditional probabilities in sample spaces with equally likely or unequally likely outcomes

9.4 Sequences and Series

Objective: The student will be able to:

- express arithmetic and geometric sequences explicitly and recursively
- use sigma notation and basic summation formulas to find the sum of a finite series or a converging infinite geometric series

9.5 Mathematical Induction

Objective: The student will be able to:

- use the principle of mathematical induction to prove mathematical generalizations

9.6 Statistics and Data (Graphical)

Objective: The student will be able to:

- distinguish between categorical and quantitative variables
- use various kinds of graphs to display data

9.7 Statistics and Data (Algebraic)

Objective: The student will be able to:

- use measures of center, the five-number summary, a boxplot, standard deviation, and normal distribution to describe quantitative data

Chapter 10 An Introduction to Calculus: Limits, Derivatives, and Integrals (as time permits)

10.1 Limits and Motion: The Tangent Problem

Objective: The student will be able to:

- calculate instantaneous velocities and derivatives using limits

10.2 Limits and Motion: The Area Problem

Objective: The student will be able to:

- calculate definite integrals using areas

10.3 More on Limits

Objective: The student will be able to:

- use the properties of limits
- evaluate one-sided limits
- evaluate two-sided limits
- evaluate limits involving infinity

10.4 Numerical Derivatives and Integrals

Objective: The student will be able to:

- estimate derivatives and integrals using numerical techniques