The Language of Mathematics: The Importance of Teaching and Learning Mathematical Vocabulary

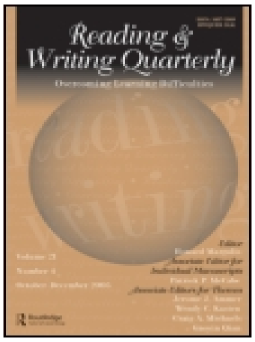
Article *in* Reading and Writing Quarterly · July 2015

DOI: 10.1080/10573569.2015.1030995

Paul J Riccomini
Pennsylvania State University

Elizabeth Hughes
Pennsylvania State University

Karen Fries
Francis Marion University



Routledge
Taylor & Francis Group

Reading & Writing Quarterly

Overcoming Learning Difficulties

ISSN: 1057-3569 (Print) 1521-0693 (Online) Journal homepage: http://www.tandfonline.com/loi/urwl20

The Language of Mathematics: The Importance of Teaching and Learning Mathematical Vocabulary

Paul J. Riccomini, Gregory W. Smith, Elizabeth M. Hughes & Karen M. Fries

Reading & Writing Quarterly, 31: 235–252, 2015

ISSN: 1057-3569 print/1521-0693 online
DOI: 10.1080/10573569.2015.1030995

Routledge
Taylor & Francis Group

The Language of Mathematics: The Importance of Teaching and Learning Mathematical Vocabulary

PAUL J. RICCOMINI
The Pennsylvania State University, University Park, Pennsylvania, USA

GREGORY W. SMITH
Hartwick College, Oneonta, New York, USA

ELIZABETH M. HUGHES
Duquesne University, Pittsburgh, Pennsylvania, USA

KAREN M. FRIES
Francis Marion University, Florence, South Carolina, USA

Vocabulary understanding is a major contributor to overall comprehension in many content areas, including mathematics. Effective methods for teaching vocabulary in all content areas are diverse and long standing. Teaching and learning the language of mathematics is vital for the development of mathematical proficiency. Students' mathematical vocabulary learning is a very important part of their language development and ultimately mathematical proficiency. This article draws on current research-based evidence to (a) provide a rationale for teaching vocabulary, (b) offer a review of research that supports the importance of teaching mathematics vocabulary, and (c) describe specific strategies for teaching mathematics vocabulary. It also addresses implications and the need for future research.

Developing the language of mathematics is an essential aspect of teaching mathematics to young children; this process continues throughout an individual's mathematics education. Because the understanding of

Gregory W. Smith is now at The University of Southern Mississippi in Hattiesburg, MS.
Address correspondence to Paul J. Riccomini, Department of Education Psychology, Counseling, and Special Education, The Pennsylvania State University, 214 CEDAR, University Park, PA 16802, USA. E-mail: pjr146@psu.edu

Color versions of one or more of the figures in the article can be found online at www.tandfonline.com/urwl.

mathematical vocabulary affords access to concepts, mathematical instruction in the areas of language is imperative (Monroe, 1998). The term *language* is defined as "the words, their pronunciation, and the methods of combining them used and understood by a community" ("Language," 2013). Although this definition simplifies a rather complex idea, it highlights the importance of vocabulary development within language. Specifically, in relationship to the language of mathematics, the ability to use words (i.e., vocabulary) to explain, justify, and otherwise communicate mathematically is important to the overall development of mathematical proficiency. In addition, research shows that language is a pivotal component of mathematics success (Seethaler, Fuchs, Star, & Bryant, 2011), and a student's general knowledge of mathematical vocabulary can predict mathematical performance (van der Walt, 2009).

Proficiency in mathematics depends on a continuous growth and blend of intricate combinations of critical component skills such as concepts, procedures, algorithms, computation, problem solving, and language (Riccomini, Sanders, & Jones, 2008). The National Research Council (2001) further described proficiency through five interconnected strands: (a) understanding mathematics, (b) computing fluently, (c) applying concepts to solve problems, (d) reasoning logically, and (e) engaging and communicating with mathematics. Clearly, the importance of students learning the language of mathematics is highlighted in both descriptions. In addition, the National Council of Teachers of Mathematics (2006) placed an emphasis on language development through the adaptive reasoning strand, which the National Research Council (2001) described as the "capacity for logical thought, reflection, explanation, and justification" (p. 116). Mathematical proficiency includes the ability to communicate and reason through written and spoken language.

Further emphasis on the importance of language in the development of mathematical proficiency is evident in the newly formed Common Core State Standards in Mathematics (CCSSM; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In addition to the comprehensive set of grade-level standards described in the CCSSM, there are eight Standards for Mathematical Practice that have clearly embedded the importance of language to mathematical proficiency. The language focus is described in the sixth Standard for Mathematical Practice, "Attend to precision," which includes the following description: "In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions" (p. 7). Undoubtedly, language development and specifically vocabulary are now new points of emphasis and important aspects for teachers to begin to address as per the newly adopted CCSSM.

Although the language of mathematics can be confusing (Rubenstein & Thompson, 2002), it is necessary for the communication of higher order mathematics reasoning (Sloyer, 2003). Goals requiring the use of higher order mathematics reasoning are unmistakably present in national mathematics organizations such as the National Research Council and National Council of Teachers of Mathematics and in the CCSSM. In order to meet these goals, students must effortlessly use, understand, and apply mathematical words, symbols, and diagrams routinely during mathematics activities. If students' language development is weak or underdeveloped, their overall mathematics learning will become slowed (van der Walt, Maree, & Ellis, 2008). Schwartz and Kenney (1995) organized the language of mathematics into more commonly utilized language terms; for example, mathematical nouns or objects were classified as numbers, measurements, and functions, whereas verbs were actions associated with problem solving and reasoning. This organizational framework not only represents the process that individuals go through when they problem solve but also provides a way to assess mathematical development (Kenney, 2005).

In an effort to improve students' overall mathematical performance, educators need to recognize the importance of, and use research-validated instructional methods to teach, important mathematical vocabulary. The purpose of article is to provide teachers with an overall understanding of the impact of mathematical vocabulary on proficiency and specific evidence-based instructional strategies to promote the learning of essential vocabulary in mathematics.

DIFFICULTIES STUDENTS EXPERIENCE WITH MATHEMATICAL VOCABULARY

There are many challenges for students in their learning of the language of mathematics. Communicating mathematically is a complex task for even the most mathematically advanced student. The ability to effectively communicate (expressively and receptively) through the language of mathematics requires mathematical understanding; a robust vocabulary knowledge base; flexibility; fluency and proficiency with numbers, symbols, words, and diagrams; and comprehension skills. Many students struggle with some or all of the important mathematical concepts, especially students with learning disabilities (Morin & Franks, 2010). Every day in their mathematics classrooms, students encounter a text-centered instructional setting that creates unintended barriers to their learning (Anderson-Inman & Horney, 1998). It is important to recognize the many and varied difficulties that present challenges for students; finding instructional strategies and activities to help students overcome these difficulties is imperative.

According to the research of Rubenstein and Thompson (2002), there are at least 11 categories of difficulties associated with learning the language of mathematics. The categories are defined in the following manner: (a) meanings are context dependent (e.g., *foot* as in 12 inches vs. the *foot of the bed*), (b) mathematical meanings are more precise (e.g., *product* as the solution to a multiplication problem vs. the *product* of a company), (c) terms specific to mathematical contexts (e.g., *polygon, parallelogram, imaginary number*), (d) multiple meanings (e.g., *side* of a triangle vs. *side* of a cube), (e) discipline-specific technical meanings (e.g., *cone* as in the shape vs. *cone* as in what one eats), (f) homonyms with everyday words (e.g., *pi* vs. *pie*), (g) related but different words (e.g., *circumference* vs. *perimeter*), (h) specific challenges with translated words (e.g., *mesa* vs. *table*), (i) irregularities in spelling (e.g., *obelus* [÷] vs. *obeli*), (j) concepts may be verbalized in more than one way (e.g., *15 minutes past* vs. *quarter after*), and (k) students and teachers adopt informal terms instead of mathematical terms (e.g., *diamond* vs. *rhombus,* or*in the house* vs. *in the division bracket*). Undoubtedly, the many difficulties that students face when learning the language of mathematics are complex and can negatively impact their language development.

An important first step in helping students to learn and use the language of mathematics is for teachers to understand the many difficulties that vocabulary presents students (Monroe & Orme, 2002). It is only with this recognition and understanding of the specific difficulties that teachers can then begin to address the instructional needs of their students from a language perspective. Effectively designing and delivering vocabulary instruction is a needed course of action. Although a common belief with many teachers is that simply exposing students to new vocabulary words through rich context-specific interactions is the best way to teach vocabulary, many students will require more systematic and explicit instructional techniques and purposeful instructional activities to facilitate their learning (Marzano, 2004).

Providing appropriate academic language support is important for all learners, especially in the mathematics classroom, where the ongoing development of explicit mathematical vocabulary is essential (Bay-Williams & Livers, 2009). There are three main purposes to teaching essential vocabulary in mathematics class to increase students' effective use of mathematical language. First and most obvious is to provide initial instruction to promote the understanding and storage of word meanings in long-term memory. Second, and only after students have developed an understanding, the goal of instruction becomes to help students become fluent and maintain the word meaning over time. Third, the end result of achieving the first two goals is that students are able to easily and accurately use the language of mathematics to explain and justify mathematical concepts and relationships. Without the instructor first teaching basic understanding and facilitating fluency with vocabulary words, the purposeful and effective use of the language of mathematics will likely not occur.

GENERAL APPROACHES AND TECHNIQUES FOR TEACHING MATHEMATICAL VOCABULARY

One would assume that mathematical vocabulary is taught at some level during mathematics classes; however, language development is often overlooked by math teachers (Adams, 2003; Riccomini & Witzel, 2010). It is important that teachers apply general vocabulary instructional techniques to mathematical vocabulary on a regular basis. Developing and then using a systematic plan for teaching vocabulary throughout the year will maximize and facilitate improved understanding of essential vocabulary for students (Manzo, Manzo, & Thomas, 2006).

Marzano's (2004) six steps for educators to maximize student learning of essential vocabulary incorporates components of the following evidence-based instructional strategies that aid in achieving positive academic outcomes across content areas: (a) explicit instruction (Bottge, Heinrichs, Mehta, & Hung, 2002; Test & Ellis, 2005), (b) stimulating prior knowledge (Stroud & Schwartz, 2010; Yeh et al., 2012), (c) repetition (Joseph, Eveleigh, Konrad, Neef, & Volpe, 2012; Kluge, Ritzman, Burkolter, & Sauer, 2011), (d) differentiating instruction (Geisler, Hessler, Gardner, & Lovelace, 2009; Jones, Yssel, & Grant, 2012), and (e) cooperative learning (Ryve, Nilsson, & Patterson, 2013; Wang, 2012). Although his recommendations cut across content areas, they form the basis for the specific strategies and techniques that are described. First, teachers should begin vocabulary instruction by providing students with an informal description, explanation, or example of the new term or phrase either directly or through indirect means. This will help students begin the process of connecting the new meaning to their prior knowledge. Second, it is important to provide students with opportunities to restate the teacher-provided descriptions, explanation, or examples in their own words. This opportunity reinforces the connections to their prior knowledge. Third, to help strengthen the linkage to prior knowledge, students are asked to construct a picture, symbol, or graphic representation of the term or phrase. This is especially important for younger children who have less prior knowledge specific to mathematics.

Fourth, as students become more familiar and comfortable with the language (i.e., learning and using the terms), it is vital for teachers to provide students with periodic opportunities to reengage in a variety of activities to help them further develop and enrich their knowledge. Students often only develop surface-level understanding of the material, and without opportunities for further engagement students will not gain the desired deep understanding necessary for mathematical reasoning and communication. Fifth, involving small-group and/or peer-to-peer discussions on specific terms further develops a deeper understanding and reduces misconceptions that may have formed. Sixth, to facilitate long-term retention, teachers must

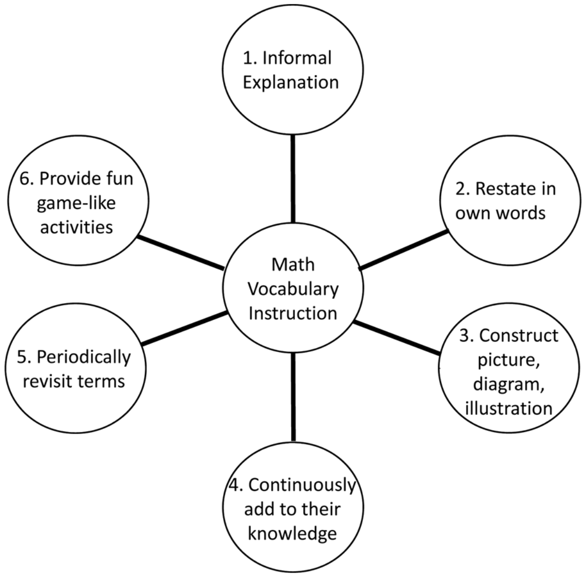


FIGURE 1 Concept map based on six recommendations by Marzano (2004) for effective vocabulary instruction.

provide opportunities for the students to revisit these essential and already-learned terms through such things as game-like activities that students will find enjoyable (see Figure 1).

By grounding their vocabulary instruction in Marzano's six steps, teachers are likely to see an improvement in mathematics language develop. These six steps articulated are neither new nor innovative but frequently get pushed to the side during mathematics instruction for a host of reasons (e.g., time constraints, not valued, lack of teacher training). Learning mathematical vocabulary through daily mathematics instruction that emphasizes the six general recommendations is important and essential for many students, especially struggling students and students with disabilities. Because mathematics naturally progresses from less complex to more complex skills, mastery of vocabulary is essential for long-term success in mathematics (Monroe & Panchyshyn, 2005); hence, the use of specific instructional strategies supported by research is necessary.

ACTIVITIES FOR TEACHING MATHEMATICS VOCABULARY

When students with poor language skills struggle with learning important mathematical vocabulary terms, educators should consider using strategies specifically developed for learning content vocabulary. Although there are many methods of facilitating the learning of vocabulary, five specific techniques for helping students learn and remember essential mathematical vocabulary are described in this article: (a) explicit vocabulary instruction, (b) mnemonic strategies, (c) fluency building through multiple exposures, (d) game-like activities, and (e) technology applications.

Explicit Vocabulary Instruction

Educators recognize that children may naturally learn vocabulary through incidental or embedded learning experiences; however, for many students these types of mathematics learning encounters are not sufficient. Instead of simply exposing students to mathematics vocabulary, it is necessary to directly teach vocabulary (sometimes in isolation) and provide opportunities for numerous and meaningful practices across contexts. The language of mathematics consists not only of words and text but also of symbols and diagrams; explicit instruction can help build the connections between these elements of mathematics language (Van de Walle, 2001).

Explicit articulation of vocabulary terms, definitions, and uses takes the guesswork out of making meaning of unfamiliar terms and focuses the students' learning on correct use and application. Research suggests that explicit instruction of new vocabulary with opportunities for use through incidental learning is more effective than incidental learning in isolation (Sonbul & Schmitt, 2010) and across ages and grade levels (Taylor, Mraz, Nichols, Rickelman, & Wood, 2009). Explicit instruction is an established, highly effective instructional approach that can be used independently or in conjunction with other teaching strategies and techniques (Archer & Hughes, 2011). Explicit vocabulary instruction requires teachers to introduce and teach a new word and its meaning through a systematic and purposeful presentation. This direct presentation highlights the importance of the new word, connects to prior knowledge, and allows students to engage with the multiple uses of the word (Lee & Jung, 2004). Common elements of explicit instruction include logically sequencing key skills, reviewing prior skills and knowledge, providing step-by-step teacher models of new skills along with opportunities for guided and independent practice, and assisting students with connections between new and existing knowledge (Archer & Hughes, 2011). There is a strong literature base supporting explicit instruction for teaching vocabulary in content areas such as reading, science, and social studies (e.g., Harmon, Hedrick, & Wood, 2005; Hong & Diamond, 2012; Jitendra, Edwards, Sacks, & Jacobson, 2004; McKeown & Beck, 2002; Stahl & Fairbanks, 1986; White, Graves, & Slater, 1990).

Concurrent with instruction, the teacher is checking for student understanding and encouraging active learning through frequent questioning and guided activities to promote student independence. After explicit instruction in new vocabulary terms, students could also (a) create concept maps; (b) keep individual math dictionaries of terms, illustrations, and examples; and (c) develop word walls with new terminology (Van de Walle, 2001). This type of instruction is neither incidental nor accidental; the teacher plans and carefully directs all aspects of the lesson.

Mnemonic Strategies

Mnemonic instruction refers to strategies and techniques used to improve learning in memorable and motivating formats. Mnemonic strategies help students learn new information by connecting it to their prior knowledge (Mastropieri & Scruggs, 2007). Mnemonic instructional practices have 30 years of research support including a diverse set of learners and across multiple content areas supporting their use as an evidence-based technique (Forness, Kavale, Blum, & Lloyd, 1997; Jitendra et al., 2004; Mastropieri & Scruggs, 1989). In addition to enhancing the academic performance of low-performing, as well as average- and above-average-achieving, students, mnemonic instruction benefits students with disabilities (Kavale & Forness, 1999).

One specific mnemonic instructional practice, the keyword strategy, has the greatest application to teaching mathematical vocabulary. Overwhelmingly positive evidence exists for the use of the keyword mnemonic technique to teach content vocabulary to students with disabilities (e.g., Mastropieri, Scruggs, & Fulk, 1990; Scruggs & Mastropieri, 2000). Unfortunately, few mathematics-specific examples of the keyword mnemonic strategies are available (see Sanders, 2007).

Educators using the keyword strategy teach students meanings of new vocabulary terms by selecting a similar-sounding word and a picture, drawing, or computer graphic that represents the essential information to learn (Atkinson, 1975; Kavale & Forness, 1999; Mastropieri & Scruggs, 2007). By providing students with a tool to anchor a new term with a similar-sounding word already known by the student, teachers enable students to better recall the meaning of the new term. Further strengthening the effectiveness of the keyword strategy is the use of a picture representation that highlights the critical attributes of the new term. Either this illustration can be created by the student, or to save time the teacher can create the illustration. The last part of the keyword technique is to create a sentence that connects the keyword and the desired definition. This is a powerful memory-aiding device to help students learn and remember essential mathematical vocabulary.

The example keyword mnemonics for the terms *parallel lines* and *ray* highlight the three critical aspects for teachers using this strategy (see Figures 2 and 3). First, the unfamiliar terms are anchored to a familiar keyword: *Parallel lines* is anchored to *pair of elves,* and *ray* is anchored to *run away*. Second, a visual image is created that accentuates the key features of the new term and captures the keyword. In the examples provided, the visual images clearly depict the key features of the definitions and the keywords. Third, a sentence is developed to connect the information in a

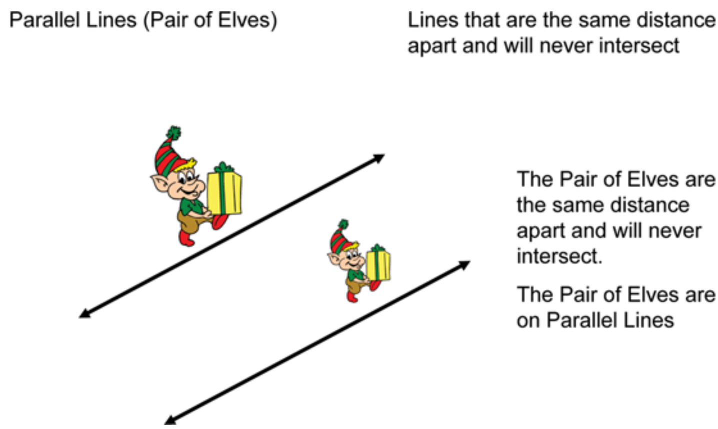


FIGURE 2 Example keyword mnemonics for the term *parallel lines*.

meaningful and memorable fashion. For *parallel line:* "The pair of elves are the same distance apart and will never intersect. The pair of elves are on parallel lines." For *ray:* "Start here!!! Run away and never stop running ray!"

The key to maximizing the effectiveness of the keyword mnemonic is to incorporate the developed keywords mnemonics into the regular classroom instructional routine. Simply presenting students with a keyword mnemonic will not likely result in the desired learning and remembering on the part of the students. Combining the keyword mnemonic strategy with other instructional activities typical in mathematics classes can maximize its effectiveness.

The keyword mnemonic strategy is easily incorporated into bulletin boards, warm-ups, game activities, SmartBoard presentations, teacher-directed or student-centered instructional time, and even peer tutoring. Combining the keyword mnemonic strategy with other instructional activities typical in mathematics classes can maximize its effectiveness.

Fluency Building Through Multiple Exposures

Fluency in mathematics is often associated with basic arithmetic facts (e.g., $5 \times 5 = 25$) and other computational-type problems (e.g., long division, perfect squares), at times overshadowing the vital role of vocabulary recognition and understanding. The National Reading Panel (National Institute of Child Health and Human Development, 2000) highlighted the importance of repeated and multiple exposures to new vocabulary to build fluency. Being fluent with mathematics vocabulary may allow learners to more readily recognize what is required to solve a problem, therefore having more cognitive energy to dedicate to more laborious tasks, such as calculating solutions that require multiple steps. As with explicit instruction, fluency is achieved through planned, purposeful, and targeted practice of specific content.

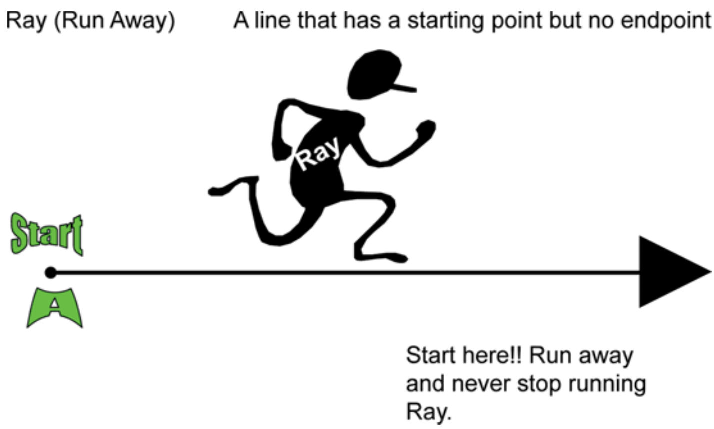


FIGURE 3 Example keyword mnemonic for the term *ray*.

Traditional ways to practice fluency include the use of flashcards, in which one side of an index card has the vocabulary term and the other has the definition and a visual. The creation of the cards also acts as a rehearsal activity and can help with learning and remembering vocabulary or can be paired with other activities to teach sight words (Kaufman, McLaughlin, Derby, & Waco, 2011). Students can rehearse the vocabulary and practice recalling the word or definition. Because the cards contain both the word and the definition, students receive immediate feedback, which has been linked to improved learning (Epstein et al., 2002). Previously mastered vocabulary can be set aside, thus maximizing time spent on learning new material. Flashcards can be used independently (e.g., in reciting) or with a peer or parent. They can be used at home, at school, or in other settings and integrated with other practices (e.g., games, metacognitive strategies). Strengths to using cards include their ease of use and the opportunity for students to practice through repetitive exposures of the vocabulary word; however, this type of practice isolates the word from the context in which it is used.

A variation of the traditional approach was described by Taylor and colleagues (2009), in which one side of the index card is divided into quadrants; the new vocabulary word is listed in the top right quadrant with the definition in the bottom right quadrant. The left two quadrants are used to draw a picture supporting the definition of the word. On the back of the index card, the students describe the relationship between the picture and the new term. This approach, like traditional flashcards, is easy to create and use; but unlike traditional flashcards, this approach includes an example of how the vocabulary word is used in context and incorporates elements highlighted by Marzano (2004).

Although some vocabulary-building activities require dedicated allotments of time, building fluency through multiple exposures to vocabulary can often be accomplished through frequent but brief 5- to 10-min activities (Stump et al., 1992). This versatility allows teachers to incorporate fluency-building activities during brief opportunities of time (e.g., transitions between exchanging classes, at the end of a lesson, while passing out classroom materials) and maximize instructional time. An engaging activity might include passing out mathematics vocabulary cards so that each student has one card. Students then circulate around the classroom to form clusters of related words (students form clusters based on how the words are related). Once the clusters are formed, the teacher can then lead a discussion about how certain terms may fit into more than one category. Another idea is to have students play vocabulary line frog while waiting in line for specials or lunch. The person at the end of the line has an opportunity to jump to the front of the line (or second place, if there is a designated line leader) if the student provides a correct definition or uses the term correctly in context. To increase opportunities to respond, this can be done as a lightning round that the teacher leads at a rapid pace.

Game-Like Activities

Teachers should use a variety of different techniques when teaching vocabulary, which may include game-like activities (Covington, 1992; Johnson, von Hoff Johnson, & Schlichting, 2004). Educational games are ideal for engaging students in motivating activities (Charlton, Williams, & McLaughlin, 2005). Games may be used to improve sight recognition (Berne & Blachowicz, 2008) or to improve and maintain understanding of essential vocabulary (Wells & Narkon, 2011). In addition, using game-like activities is an excellent way to make learning mathematical vocabulary fun and more appealing to students.

In general, teachers have established game activities designed to serve various learning objectives in their classrooms. A common game format used by many teachers is based on the popular television program *Jeopardy!* This game format is used in many classrooms, not just mathematics classrooms, because of its easy-to-learn format and applicability across many different content areas. Because the game is organized into categories (e.g., Geometry, Algebra), it is simple to add an additional category devoted exclusively to vocabulary. There are templates available online to aid in creating and customizing this type of quiz game (e.g., http://www.edtechnetwork.com/powerpoint.html).

Wells and Narkon (2011) explained three games (i.e., Mystery Word, Word-O, and Word Sorts) that can be used to motivate student learning. In Mystery Word, a vocabulary word is selected from a list, and the leader provides clues about the mystery word until the class is able to surmise what the word is. Word-O is an adapted form of Bingo, and Word Sorts allows students to work with a list of words to compare and contrast words in an effort to form categories of words (see Wells & Narkon, 2011). More challenging rounds may include words that are not overtly similar.

Using game-like activities throughout the course of the academic year affords students opportunities to attend to continued vocabulary development in mathematics in a fun, recreational manner. The playful learning opportunities may be both interesting and motivational for students (Charlton et al., 2005; Wells & Narkon, 2011).

Technology Applications

Students with disabilities often struggle with mathematics content in middle school and high school. They are faced with a text-centered world and often lack the skills to read and write at sufficient levels to meet the challenges of secondary education (Anderson-Inman & Horney, 1998). However, teachers can use various instructional techniques and strategies to help their students overcome many of the barriers to learning the language of mathematics. Furthermore, technology applications may become an effective aid for students in the future. As Anderson-Inman and Horney (1998) stated, "Computer-based solutions represent the future in educators' effort to help students with learning disabilities achieve in school up to their potential" (p. 248).

Instructional technology can enhance and support mathematics instruction by offering teachers and their students visual and auditory stimuli and interactive simulations that make mathematics real for students (e.g., demonstrating how data collection can be utilized to find solutions to everyday problems). Although very few studies have specifically addressed vocabulary development with instructional technology, there is evidence that suggests that improved learning outcomes are possible (Hebert & Murdock, 1994; Koury, 1996). Instructional technology can include a range of applications, such as apps, streaming audio and video, software programs, computer simulations, video and audio demonstrations, and graphics programs (e.g., graphing calculators). The Internet now allows students to access real data that can then be used to solve authentic meaningful problems and provide visual representations not easily created or accessible in past mathematics classrooms. Students can learn through interactive computer games that can be highly motivating and challenge students at their optimal learning levels (Gee, 2004). As the technology continues to improve in both access and learner effectiveness, instructional technology has great potential to be a powerful teaching tool for educators and learning aid for students.

Empirical research supports the use of instructional technology (e.g., calculators, graphing calculators, video discs, software applications) by educators in the areas of basic facts as well as problem solving, telling time, ratios and proportions, fractions, and decimals (e.g., Bouck, 2010; Cawthon, Beretvas, Kaye, & Lockhart, 2012; Hofmeister, 1989); however, a minimal amount of empirical research is available regarding the efficacy of instructional technology for teaching vocabulary specific to mathematics for low-achieving students. It is logical that technology applications can and should be developed and applied to enhance instruction of essential vocabulary in mathematics.

Learning technical mathematical vocabulary may require much more than the status quo for mathematics instruction. With ever-increasing advancements in instructional technology, the possibility of significantly impacting the overall mathematical performance of low-achieving students and students with disabilities through the application of technological advances is substantial. Unfortunately, very few researchers have examined the effectiveness of using instructional technological applications to teach mathematical vocabulary specifically, an obvious gap in the knowledge base on evidence-based vocabulary instruction. This is an area that should be explored and further developed.

LIMITATIONS AND THE NEED FOR FUTURE RESEARCH

Although the need for vocabulary instruction in mathematics is great, there is limited published research that focuses specifically on interventions for developing vocabulary in mathematics. Therefore, scholars are tapping into the rich vocabulary research available through literacy research and extending it across content areas such as mathematics, as seen in the framework described by Marzano (2004). Building content-specific vocabulary research from literacy research, as sometimes done here, is a natural extension; however, there are some limitations within this bridge, including the way mathematics vocabulary is often presented with limited context clues (e.g., "Find the *slope*") and the 11 caveats described by Rubenstein and Thompson (2002). Research is needed to identify and analyze instruction with particular attention to these characteristics that distinguish mathematics vocabulary from other expressions of vocabulary. Equally important to how mathematics vocabulary is taught is the question of *when* mathematics vocabulary should be taught and *how* it should be assessed. Mathematics is a content area that builds from prerequisite skills to more advanced skills, calling teachers' attention to when students should be expected to master vocabulary and how to distinguish between limited skills and limited vocabulary, and thereby access to the skills, when the two are intertwined.

Given the limited availability of intervention research specific to mathematics vocabulary, generalization of these suggestions should be made with caution. Like all classroom instruction, instructional decisions should be made based on data supporting students' response to instruction. Therefore, teachers should collect data on the effectiveness of mathematics vocabulary interventions being implemented and make continued educational decisions based on those data.

IMPLICATIONS FOR PRACTICE

Given the large number of terms encountered throughout the course of a year and the varying ability and readiness of students to learn new vocabulary, teachers must judiciously select words to teach and help students not only to learn the new terms as they are encountered but also to continue to remember previously learned terms from year to year. Although using and encountering terms in naturalistic contexts facilitates vocabulary development, for many students, especially struggling students, this development may be fragmented and disjointed; therefore, the consistent and purposeful use of vocabulary building can greatly assist students. Although there is not one right way to build vocabulary skills, a theme that ran through all supports described is clear: purposeful word instruction with multiple opportunities for students to respond and practice vocabulary in multiple contexts. From explicitly introducing a vocabulary word to playing a word game while waiting in line to go to lunch, the instruction should be methodically planned and executed with purpose and precision. Capitalizing on instructional time and providing multiple opportunities for students to successfully learn, use, and practice new and critical vocabulary is important.

CONCLUSION

As the language of mathematics continues to become an emphasis in the development of mathematical proficiency, there is no question about the importance of spending instructional time to teach mathematics vocabulary. van der Walt (2009) emphasized that vocabulary within the language of mathematics is an aspect of instruction that requires specific attention. While vocabulary continues to emerge as an essential aspect of language development in mathematics, resources supporting mathematics vocabulary need to become more prevalent in mathematics literacy. This article has presented an overview of the impact of mathematical vocabulary on proficiency and evidence-supported instructional strategies for incorporating mathematics vocabulary instruction into classroom learning. Rich development and understanding of mathematics vocabulary is essential for students to become actively engaged in mathematics past mundane computational requirements to thorough understanding and meaning making. Educators have the responsibility to provide students with instruction that best supports learning, academic success, and lifelong success. The strategies and techniques described in this article can help teachers accomplish this responsibility once they recognize the importance of the language of mathematics.

REFERENCES

Adams, T. L. (2003). Reading mathematics: More than words can say. *The Reading Teacher*, *56*, 786–795.

Anderson-Inman, L., & Horney, M. (1998). Transforming text for at-risk readers. In D. Reinking, L. D. Labbo, M. C. McKenna, & R. D. Kieffer (Eds.), *Handbook of literacy and technology: Transformations in a post-typographic world* (pp. 15–43). Mahwah, NJ: Erlbaum.

Archer, A. L., & Hughes, C. A. (2011). *Explicit instruction: Effective and efficient teaching*. New York, NY: Guilford Press.

Atkinson, R. C. (1975). Mnemotechnics in second-language learning. *American Psychologist*, *30*, 821–828.

Bay-Williams, J. M., & Livers, S. (2009). Supporting math vocabulary acquisition. *Teaching Children Mathematics*, *16*, 238–245.

Berne, J. I., & Blachowicz, C. L. Z. (2008). What reading teachers say about vocabulary instruction: Voices from the classroom. *The Reading Teacher*, *62*, 452–477.

Bottge, B. A., Heinrichs, M., Mehta, Z. D., & Hung, Y. H. (2002). Weighing the benefits of anchored math instruction for students with disabilities in general education classes. *Journal of Special Education*, *35*, 186–200.

Bouck, E. C. (2010). The impact of calculator type and instructional exposure for students with a disability: A pilot study. *Learning Disabilities*, *16*, 141–148.

Cawthon, S. W., Beretvas, S., Kaye, A. D., & Lockhart, L. (2012). Factor structure of opportunity to learn for students with and without disabilities. *Education Policy Analysis Archives*, *20*(41), 1–30.

Charlton, B., Williams, R. L., & McLaughlin, T. F. (2005). Educational games: A technique to accelerate the acquisition of reading skills of children with learning disabilities. *The International Journal of Special Education*, *20*, 66–72.

Covington, M. V. (1992). *Making the grade: A self-worth perspective on motivation and school reform*. New York, NY: Cambridge University Press.

Epstein, M. L., Lazarus, A. D., Calvano, T. B., Matthews, K. A., Hendel, R. A., Epstein, B. B., & Brosvic, G. M. (2002). Immediate feedback assessment technique promotes learning and corrects inaccurate first responses. *Psychological Record*, *52*, 187–201.

Forness, S. R., Kavale, K. A., Blum, I. M., & Lloyd, J. W. (1997). Mega-analysis of meta analysis: What works in special education and related services. *Teaching Exceptional Children*, *29*(6), 4–9.

Gee, J. P. (2004). Learning by design: Games as learning machines. *Interactive Educational Multimedia*, *8*, 15–23.

Geisler, J. H., Hessler, T., Gardner, I., & Lovelace, T. S. (2009). Differentiating writing interventions for high-achieving urban African American elementary students. *Journal of Advanced Academics*, *20*, 214–247.

Harmon, J. M., Hedrick, W. B., & Wood, K. D. (2005). Research on vocabulary instruction in the content areas: Implications for struggling readers. *Reading & Writing Quarterly*, *21*, 261–280.

Hebert, B. M., & Murdock, J. Y. (1994). Comparing three computer-aided instruction output modes to teach vocabulary words to students with learning disabilities. *Learning Disabilities Research & Practice*, *9*, 136–141.

Hofmeister, A. M. (1989). Mainstreaming students with learning disabilities for videodisc math instruction. *Teaching Exceptional Children*, *21*(3), 52–60.

Hong, S. Y., & Diamond, K. E. (2012). Two approaches to teaching young children science concepts, vocabulary, and scientific problem-solving skills. *Early Childhood Research Quarterly*, *27*, 295–305.

Jitendra, A. K., Edwards, L. L., Sacks, G., & Jacobson, L. A. (2004). What research says about vocabulary instruction for students with learning disabilities. *Exceptional Children*, *70*, 299–321.

Johnson, D. D., von Hoff Johnson, B., & Schlichting, K. (2004). Logology: Word and language play. In J. F. Baumann & E. J. Kame'enui (Eds.), *Vocabulary instruction: Research to practice gap* (pp. 179–200). New York, NY: Guilford Press.

Jones, R. E., Yssel, N., & Grant, C. (2012). Reading instruction in Tier 1: Bridging the gaps by nesting evidenced-based interventions within differentiated instruction. *Psychology in the Schools*, *49*, 210–218.

Joseph, L., Eveleigh, E., Konrad, M., Neef, N., & Volpe, R. (2012). Comparison of the efficiency of two flashcard drill methods on children's reading performance. *Journal of Applied School Psychology*, *28*, 317–337.

Kaufman, L., McLaughlin, T. F., Derby, K. M., & Waco, T. (2011). Employing reading racetracks and DI flashcards with and without cover, copy, and compare and rewards to teach site words to three students with learning disabilities in reading. *Educational Research Quarterly*, *43*(4), 27–50.

Kavale, K. A., & Forness, S. R. (1999). *Efficacy of special education and related service*. Washington, DC: American Association of Mental Retardation.

Kenney, J. M. (2005). Mathematics as language. In *Literacy strategies for improving mathematics instruction* (pp. 1–6). Alexandria, VA: Association for Supervision and Curriculum Development.

Kluge, A., Ritzman, S., Burkolter, D., & Sauer, J. (2011). The interaction of drill and practice and error training with individual differences. *Cognition, Technology & Work*, *13*, 103–120.

Koury, K. A. (1996). The impact of preteaching science content vocabulary using integrated media for knowledge acquisition in a collaborative classroom. *Journal of Computing in Childhood Education*, *7*, 179–197.

Language. (2013). In *Merriam-Webster's online dictionary*. Retrieved from http://www.merriam-webster.com/dictionary/language?show=0&t=1370883898

Lee, H., & Jung, W. S. (2004). Limited English-proficient (LEP) students and mathematical understanding. *Mathematics Teaching in the Middle School*, *9*, 269–272.

Manzo, A. V., Manzo, U. C., & Thomas, M. M. (2006). Rationale for systematic vocabulary development: Antidote for state mandates. *Journal of Adolescent & Adult Literacy*, *49*, 610–619.

Marzano, R. J. (2004). *Building background knowledge for academic achievement*. Alexandria, VA: Association for Supervision and Curriculum Development.

Mastropieri, M. A., & Scruggs, T. E. (1989). Constructing more meaningful relationships: Mnemonic instruction for special populations. *Educational Psychology Review*, *1*, 83–111.

Mastropieri, M. A., & Scruggs, T. E. (2007). *The inclusive classroom: Strategies for effective instruction*. Columbus, OH: Merrill Prentice Hall.

Mastropieri, M. A., Scruggs, T. E., & Fulk, B. J. M. (1990). Teaching abstract vocabulary with the keyword method: Effects on recall and comprehension. *Journal of Learning Disabilities*, *23*, 69–74.

McKeown, M. G., & Beck, I. L. (2002). Direct and rich vocabulary instruction. In J. F. Baumann & E. J. Kame'enui (Eds.), *Vocabulary instruction: Research to practice* (pp. 13–27). New York, NY: Guilford Press.

Monroe, E. E. (1998). Using graphic organizers to teach vocabulary: Does available research inform mathematics instruction. *Education*, *118*, 538–542.

Monroe, E. E., & Orme, M. P. (2002). Developing mathematical vocabulary. *Preventing School Failure*, *46*, 139–142.

Monroe, E., & Panchyshyn, R. (2005). Helping children with words in word problems. *Australian Primary Mathematics Classroom*, *10*(4), 27–29.

Morin, J. E., & Franks, D. J. (2010). Why do some children have difficulty learning mathematics? Looking at language for answers. *Preventing School Failure*, *54*, 111–118.

National Council of Teachers of Mathematics. (2006). *Principles and standards for school mathematics*. Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Authors.

National Institute of Child Health and Human Development. (2000). *Report of the National Reading Panel: Teaching children to read: An evidence-based assessment of the scientific research literature on reading and its implications for reading instruction*. Retrieved from http://www.nichd.nih.gov/publications/nrp/smallbook.htm

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Mathematics Learning Study Committee; J. Kilpatrick, J. Swafford, & B. Findell (Eds.); Center for Education, Division of Behavioral and Social Sciences and Education.Washington, DC: National Academies Press.

Riccomini, P. J., Sanders, S., & Jones, J. (2008). The key to enhancing students' mathematical vocabulary knowledge. *Journal on School Educational Technology*, *4*(1), 1–7.

Riccomini, P. J., & Witzel, B. S. (2010). *Response to intervention in mathematics*. Thousand Oaks, CA: Corwin Press.

Rubenstein, R., & Thompson, D. (2002). Understanding and supporting children's mathematical vocabulary development. *Teaching Children Mathematics*, *9*, 107–112.

Ryve, A., Nilsson, P., & Patterson, K. (2013). Analyzing effective communication in mathematics group work: The role of visual mediators and technical terms. *Educational Studies in Mathematics*, *82*, 497–514.

Sanders, P. S. (2007). *Embedded strategies in mathematics vocabulary instruction: A quasi-experimental study*. Retrieved from http://tigerprints.clemson.edu/cgi/viewcontent.cgi?article=1163&context=all_dissertations

Schwartz, J. L., & Kenney, J. M. (1995). *Assessing mathematical understanding and skills effectively* (Interim report of the Balanced Assessment Program). Cambridge, MA: Harvard Graduate School of Education.

Scruggs, T. E., & Mastropieri, M. A. (2000). The effectiveness of mnemonic instruction for students with learning and behavior problems: An update and research synthesis. *Journal of Behavioral Education*, *10*, 163–173.

Seethaler, P. M., Fuchs, L. S., Star, J. R., & Bryant, J. (2011). The cognitive predictors of computational skill with whole versus rational numbers: An exploratory study. *Learning and Individual Differences*, *21*, 536–542.

Sloyer, C. W. (2003). Mathematical insight: Changing perspective. *Mathematics Teacher*, *96*, 238–242.

Sonbul, S., & Schmitt, N. (2010). Direct teaching of vocabulary after reading: Is it worth the effort? *English Language Teachers Journal*, *64*, 253–260.

Stahl, S. A., & Fairbanks, M. M. (1986). The effects of vocabulary instruction: A model based meta-analysis. *Review of Educational Research*, *56*(1), 72–110.

Stroud, M. J., & Schwartz, N. H. (2010). Summoning prior knowledge through metaphorical graphics: An example chemistry instruction. *Journal of Educational Research*, *103*, 351–366.

Stump, C. S., Lovitt, T. C., Fister, S., Kemp, K., Moore, R., & Schroeder, B. (1992). Vocabulary intervention for secondary level youth. *Learning Disabilities Quarterly*, *15*, 207–222.

Taylor, D. B., Mraz, M., Nichols, W. D., Rickelman, R. J., & Wood, K. (2009). Using explicit instruction to promote vocabulary learning for struggling readers. *Reading & Writing Quarterly*, *25*, 205–220.

Test, D. W., & Ellis, M. F. (2005). The effects of LAP fractions on addition and subtraction of fractions with students with mild disabilities. *Education & Treatment of Children*, *28*(1), 11–24.

Van de Walle, J. A. (2001). *Elementary and middle school mathematics: Teaching developmentally* (4th ed.). New York, NY: Addison Wesley Longman.

van der Walt, M. (2009). Study orientation and basic vocabulary in mathematics in primary school. *South African Journal of Science and Technology*, *28*, 378–392.

van der Walt, M., Maree, K., & Ellis, S. (2008). A mathematics vocabulary questionnaire for immediate use in the intermediate phase. *South African Journal of Education*, *28*, 489–504.

Wang, M. (2012). Effects of cooperative learning on achievement motivation of female university students. *Asian Social Science*, *8*(15), 108–114.

Wells, J. C., & Narkon, D. E. (2011). Motivate students to engage in word study using vocabulary games. *Intervention in School and Clinic*, *47*, 45–49.

White, T. G., Graves, M. F., & Slater, W. H. (1990). Growth of reading vocabulary in diverse elementary schools: Decoding and word meaning. *Journal of Educational Psychology*, *82*, 281–290.

Yeh, T., Tseng, K., Cho, C., Barufaldi, J., Lin, M., & Chang, C. (2012). Exploring the impact of prior knowledge and appropriate feedback on students' perceived cognitive load and learning outcomes: Animation-based earthquakes instruction. *International Journal of Science Education*, *34*, 1555–1570.