

### Addition: Partial Sums

Many times it is easier to break apart addends. Often it makes sense to break them apart by their place value. Consider  $248 + 345$

$$\begin{aligned}
 248 &= 200 + 40 + 8 \\
 345 &= 300 + 40 + 5 \\
 500 + 80 + 13 &= 593
 \end{aligned}$$

Sometimes we might use partial sums in different ways to make an easier problem. Consider  $484 + 276$

$$\begin{aligned}
 484 &= 400 + 84 \\
 276 &= 260 + 16 \\
 660 + 100 &= 760
 \end{aligned}$$

### Addition: Adjusting

We can adjust addends to make them easier to work with. We can adjust by giving a value from one addend to another. Consider  $326 + 274$ . We can take 1 from 326 and give it to 274.

$$\begin{array}{r}
 326 + 274 \\
 \quad -1 \quad +1 \\
 \hline
 \text{More Friendly Problem} \longrightarrow 325 + 275 = 600
 \end{array}$$

Consider  $173 + 389$ . We can take 27 from 389 and give it to 173 to make 200.

$$\begin{array}{r}
 173 + 389 \\
 +27 \quad -27 \\
 \hline
 \text{More Friendly Problem} \longrightarrow 200 + 362 = 562
 \end{array}$$

### Addition: Traditional Algorithm

This algorithm is useful for adding large numbers. We add place values and regroup when needed.

$$\begin{array}{r}
 \phantom{0}^1 \phantom{0}^1 \\
 13,089 \\
 + 4,684 \\
 \hline
 17,773
 \end{array}$$

### Subtraction: Count Up or Count Back

When subtracting, we can count back to find the difference of 2 numbers. In many situations, it is easier to count up. Consider  $536 - 179$ .

#### Counting Up

$$\begin{aligned}
 179 + 21 &= 200 \\
 200 + 300 &= 500 \\
 500 + 36 &= 536
 \end{aligned}$$

The total of our **counting up** is 357. So,  $536 - 179 = 357$

#### Counting Back

$$\begin{aligned}
 536 - 36 &= 500 \\
 500 - 300 &= 200 \\
 200 - 21 &= 179 \\
 (-) 357
 \end{aligned}$$

The total of our **counting back** is 357. So,  $536 - 179 = 357$

### Subtraction: Adjusting

We can use "friendlier numbers" to solve problems.  $4,000 - 563$  can be challenging to regroup. But the difference between these numbers is the same as the difference between  $3,999 - 562$ . Now, we don't need to regroup.

$$\begin{array}{r}
 \text{(Original problem)} \quad 4,000 \quad - \quad 563 = \\
 \text{(Compensation)} \quad \quad -1 \quad \quad \quad -1 \\
 \hline
 3,999 \quad - \quad 562 = 3,437
 \end{array}$$

### Subtraction: Traditional Algorithm

This algorithm is useful for subtracting large numbers. We regroup when necessary.

$$\begin{array}{r}
 \phantom{0}^8 \phantom{0}^1 \\
 14,290 \\
 - 3,236 \\
 \hline
 11,054
 \end{array}$$

### Multiplication: Partial Products

Students move from area/array models to working with numbers.

Consider  $26 \times 45$ , we can break apart each factor by its place value.

$26 = (20 + 6)$  We can then multiply each  
 $45 = (40 + 5)$  of the "parts" and add them  
back together.

$$\begin{aligned}
 (20 \times 40) + (20 \times 5) + (40 \times 6) + (6 \times 5) \\
 800 \quad + \quad 100 \quad + \quad 240 \quad + \quad 30 \\
 900 \quad + \quad 240 \quad + \quad 30 \\
 1,140 \quad + \quad 30 \\
 \hline
 \text{So, } 26 \times 45 = 1,170 \quad 1,170
 \end{aligned}$$

It might seem like a lot of numbers above. But, when we think about it, the multiplication is quite simple. This understanding develops mental math, the traditional algorithm, and algebraic concepts including factoring polynomials.

Sometimes, it makes sense to work with different parts. Consider  $51 \times 21$ . We might think of 21 as  $10 + 10 + 1$ :

$$\begin{aligned}
 (51 \times 10) + (51 \times 10) + (51 \times 1) \\
 510 \quad + \quad 510 \quad + \quad 51 \\
 1,020 \quad + \quad 51 \\
 \hline
 1,071
 \end{aligned}$$

So,  $51 \times 21 = 1,071$

Another example, consider  $4 \times 327$ . We can break 327 into  $(300 + 20 + 7)$  then multiply.

$$\begin{aligned}
 4 \times 300 &= 1,200 \\
 4 \times 20 &= 80 \\
 + 4 \times 7 &= 28 \\
 \hline
 \text{So, } 4 \times 327 &= 1,308 \quad 1,308
 \end{aligned}$$

