

Symbolic Expression (SE): The use and manipulation of symbols and expressions provide a variety of representations for solving problems and expressing mathematical concepts, relationships, and reasoning.

(K-4) Elementary School Learning Targets		(5-8) Middle School Learning Targets		(9-12) High School Learning Targets
<p>E.SE-1 Use equations and expressions involving basic operations to represent a given context</p> <ul style="list-style-type: none"> Represent numerical relationships using combinations of symbols ($=$, $>$, $<$) and numbers to form expressions and equations Solve for unknown in simple number binary number sentences (e.g., $____ + 4 = 7$); Write equations showing inverse operations and related operations (e.g., addition-multiplication). 		<p>M.SE-1 Represent relationships and interpret expressions and equations in terms of a given context for determining an unknown value.</p> <ul style="list-style-type: none"> Represent mathematical relationships symbolically and solve for any variable (for 1st degree equations and for common formula (literal equation)); Explain how to manipulate an algebraic expression to create equivalent expressions and provide step-by-step explanations and justifications. 		<p>H.SE-1 Represent relationships and interpret expressions and equations in terms of a given context (including complex and families of functions) for determining unknown values (including two or more variables).</p> <ul style="list-style-type: none"> Represent and interpret multi-step problems; Represent complex numbers and vectors; Demonstrate the relationship between systems of equations and matrix representations; Represent the relationship between functions and modeling.
Grades K-2	Grades 3-4	Grades 5-6	Grades 7-8	Grades 9-12
<p>Students use symbolic expression when...</p> <p>E.SE.1a recognizing that symbols correspond to specific quantities (e.g., matching symbols to sets of quantities above three) K.CC-3</p> <p>E.SE.1b using oral and then written numerals and symbols to express quantities K.CC-3 (0-20), K.CC 4 1.NBT-1 (0-120); 1.MD-3 (time) 2.NBT-3 (0-1000); 2.MD-7, (time), 8 (money)</p> <p>E.SE.1c using symbols ($=$, $>$, $<$) to compare whole number quantities, write equations, and determine if equations are true 1.OA-7; 1.NBT-3 2.OA-3, 4; 2.NBT-4</p> <p>E.SE.1d representing addition and subtraction in multiple formats, including expressions K.OA-1 1.OA-1, 2, 3, 4 2.OA-1, 4</p> <p>E.SE.1e demonstrating understanding of finding the unknown in addition and subtraction equations (e.g., when all but 1 of 3 numbers is known-what makes this true?) K.OA-1, 3, 4 1.OA-7, 8 2.MD-5; 2.OA-4</p>	<p>Students use symbolic expression when...</p> <p>E.SE.1f extending finding unknown numbers in equations using multiplication and division, including using letters for unknown quantities 3.OA-4, 8 4.OA-3</p> <p>E.SE.1g using symbols ($=$, $>$, $<$) to compare whole numbers, fractions, or decimals; write equations; and express inverse or related operations 3.NF-3d 4.OA-1; 4.NF-2, 7</p> <p>E.SE.1h expressing whole numbers as fractions, and fractions as equivalent decimals; recognizing that a fraction is one number, not two 3.NF-3c 4.NF-6</p>	<p>Students use symbolic expression when...</p> <p>M.SE.1a using symbols ($=$, $>$, $<$) to compare whole numbers, fractions, or decimals; write equations; and express inverse or related operations 5.NBT-3b</p> <p>M.SE.1b writing, interpreting, and using expressions, equations, and inequalities (including using brackets, parentheses, or braces) 5.OA-1, 2 6.EE-2a, 2b, 6, 8, 9</p> <p>M.SE.1c maintaining equality between both sides of the equation to solve equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers 6.EE-7</p> <p>M.SE.1d interpreting and using symbols to express relationships (e.g., simple formulas - volume, area; ordered pairs, ratios, percents, positive-negative numbers, exponents) 5.MD-5b; 5.G-1 6.RP-1, 3c; 6.NS-6a, 6b; 6.EE-1, 6</p>	<p>Students use symbolic expression when...</p> <p>M.SE.1e interpreting and using symbols to express relationships or solutions (e.g., formulas; ordered pairs, ratios, exponents, squaring and cubing) 7.RP-2c, 2d; 7.G-4 8.EE-1, 2; 8.G-9</p> <p>M.SE.1f writing and interpreting mathematical expressions, equations, and inequalities that correspond to given situations 7.EE-4a, 4b 8.EE-8c; 8.F-3</p> <p>M.SE.1g evaluating expressions; using expressions, linear equations, inequalities, and formulas to solve problems 7.EE-4a, 4b 8.EE-8c; 8.F-3</p>	<p>Students use symbolic expression when...</p> <p>H.SE.1a interpreting and using symbols to express relationships (e.g., identifying parts of expressions, generating equivalent expressions, formulas; exponents) A.SSE-1, 2, 3 F.IF-8b</p> <p>H.SE.1b creating mathematical models, using rules and relationships to describe and predict objects and events in the real world F.BF-1a S.ID-6a</p> <p>H.SE.1c identifying essential quantitative relationships in a situation, using symbolic expressions to represent it, and drawing reasonable conclusions from it A.SSE-1,2, 3 G.GMD-3</p> <p>H.SE.1d interpreting and using symbols to express relationships and solve problems (e.g., volume formulas; exponents, square and cube roots) F.IF-8b N.RN-1 G.GMD-3</p> <p>H.SE.1e recognizing that there limitations in mathematics models A.CED-3 S.IC-2</p>

The Nature of Numbers and Operations (NO): Understandings of number - “how many” or “how much” – and number types extend applications of arithmetic properties, operations, and number systems and guide the use of computational strategies and algorithms.

(K-4) Elementary School Learning Targets		(5-8) Middle School Learning Targets		(9-12) High School Learning Targets
<p>E.NO-1 Build flexibility using whole numbers, fractions, and decimals to understand the nature of number and number systems:</p> <ul style="list-style-type: none"> Count, model, and estimate quantities; Compare, represent, and order numbers; Apply place value concepts and expanded notation to compose and decompose whole numbers. 		<p>M.NO-1 Build flexibility using rational and irrational numbers to expand understanding of number systems:</p> <ul style="list-style-type: none"> Estimate, compare, and represent numbers (fractions, decimals, and percents; integers); Use exponents to express quantities and relationships; Use integers in problem solving. 		<p>H.NO-1 Demonstrate flexibility using rational and irrational numbers and number systems, including complex numbers and matrices.</p>
Grades K-2	Grades 3-4	Grades 5-6	Grades 7-8	Grades 9--12
<p>Demonstrate an understanding of number and number systems ...</p> <p>E.NO.1a showing mastery of the prerequisite core skills of cardinality, constancy, and 1:1 correspondence K.CC-2, 4</p> <p>E.NO.1b developing an understanding of number and principles of quantity (e.g., hold up 5 fingers at once to show 5, locate things in 2s without counting; using number words to indicate small exact numbers or relative change in quantity – more/less, small/big) K.CC-4b, 4c; K.OA-4</p> <p>E.NO.1c developing number line skills (linear representations) using 0 to 20, and later 0 to 100</p> <p>E.NO.1d identifying numbers (names, symbols, quantity) and the count sequence K.CC-1, 2, 4</p> <p>E.NO.1e reading and writing numbers; counting and estimating (e.g., how many?; skip counting by 2s, 5s, 10s; even/odd) K.CC-1, 2, 3, 4, 5 1.NBT-1; 2.NBT-2, 3; 2.OA-3</p> <p>E.NO.1f representing, ordering, and comparing whole numbers K.CC-6, 7 1.NBT-1, 3; 2.NBT-4</p> <p>E.NO.1g recognizing that numbers can be divided (represented as fractions)</p> <p>E.NO.1h applying place value understanding to compare and order numbers, express number relationships (<, >, =), and express numbers in expanded form 1.NBT-2c, 3; 2.NBT-1, 4</p> <p>E.NO.1i recognizing zero as an additive identity, origin for the number line, and representing no units as a quantity or in place value K.CC-3; 1.NBT-2; 2.NBT-1</p>	<p>Demonstrate an understanding of number and number systems ...</p> <p>E.NO.1j applying place value concepts to: read, write, and compare whole numbers up to 100,000; use expanded form; and round numbers to a given place 3.NBT-1 4.NBT-2, 3</p> <p>E.NO.1k explaining the meaning of place value (that a digit in one place represents 10 times what it represents in the place to its right) 4.NBT-1</p> <p>E.NO.1l identifying and locating fractions on the number line or as regions, or parts of a set or unit, and recognizing that whole numbers are a subset of rational numbers 3.NF-1, 2, 3a, 3c</p> <p>E.NO.1m composing and representing equivalent fractions in the form $\frac{a}{b}$ 3.NF-1, 2, 3a, 3b 4.NF-1, 2</p> <p>E.NO.1n comparing and modeling fractions, including with different denominators 3.NF-2, 3a, 3b, 3c, 3d 4.NF-1, 2</p> <p>E.NO.1o rewriting fractions as equivalent decimals 4.NF-5, 6</p> <p>E.NO.1p using number words to indicate decimal values (tenths, hundredths)</p> <p>E.NO.1q using and comparing decimals to the hundredths 4.NF-7</p>	<p>Demonstrate an understanding of number and number systems ...</p> <p>M.NO.1a explaining the meaning of place value (that a digit in one place represents 10 times what it represents in the place to its right) 5.NBT-1</p> <p>M.NO.1b extending place value understanding to reading (e.g., naming the values with number words, rather than “point four”), writing, comparing, and rounding decimals 5.NBT-3, 4</p> <p>M.NO.1c using a variety of fractional and decimal representations and locating them on a number line 5.NBT-3a; 5.NF-1</p> <p>M.NO.1d representing integers (positive/negative numbers) and locating them on a number line 6.NS-5, 6c, 7a</p> <p>M.NO.1e describing, representing, and comparing absolute value relationships 6.NS-7c, 7d</p> <p>M.NO.1f recognizing equivalence of representations using fractions, decimals, and percents and using them to solve ratio problems 6.RP-1, 3</p>	<p>Demonstrate an understanding of number and number systems ...</p> <p>M.NO.1g representing and using integers; comparing and expressing absolute value and additive inverse relationships 7.NS-1a, 1b, 1c, 2b</p> <p>M.NO.1h recognizing and modeling fractions, decimals, and percents as different representations of rational numbers 7.NS-2d 8.NS-1</p> <p>M.NO.1i using exponents and scientific notation to express very large or very small quantities 8.EE-1, 3, 4</p> <p>M.NO.1j making interpretations and comparisons of scientific notation produced by technology or appearing in various media 8.EE-3, 4</p> <p>M.NO.1k distinguishing rational numbers (terminating and repeating) from irrational numbers (non-terminating and non-repeating), and recognizing that together they form the real number system and that both can be represented on the number line 8.NS-1, 2</p>	<p>Demonstrate an understanding of number and number systems ...</p> <p>H.NO.1a using exponents and scientific notation to represent quantities and expressions (Also addressed at grade 8: 8.EE-1, 2, 3, 4) N.RN-2 A.SSE-2, 3c</p> <p>H.NO.1b distinguishing rational numbers (terminating and repeating) from irrational numbers (non-terminating and non-repeating), and recognizing that together they form the real number system and that both can be represented on the number line (Also addressed at grade 8: 8.NS-1, 2) N.RN-3</p> <p>H.NO.1c modeling and describing that complex numbers augment real numbers</p> <p>H.NO.1d extending operations and properties to work with complex numbers N.CN-2</p> <p>H.NO.1e using matrices to store and manipulate data</p>

The Nature of Numbers and Operations (NO): Understandings of number - “how many” or “how much” – and number types extend applications of arithmetic properties, operations, and number systems and guide use of computational strategies and algorithms.

(K-4) Elementary School Learning Targets		(5-8) Middle School Learning Targets		(9-12) High School Learning Targets
<p>E.NO-2 Build an understanding of computational strategies and algorithms:</p> <ul style="list-style-type: none"> Fluently add, subtract, multiply, divide, and estimate; Perform and represent operations with whole numbers, fractions, and mixed numbers; Identify multiples and factors of whole numbers. 		<p>M.NO-2 Expand use of computational strategies and algorithms to rational numbers:</p> <ul style="list-style-type: none"> Perform operations fluently with rational numbers, including fractions, decimals, and percents; Identify equivalence of indicated division and fractional parts. 		<p>H.NO-2 Build an understanding of computational strategies and algorithms including matrices and irrational and complex numbers:</p> <ul style="list-style-type: none"> Use matrix operations and complex and irrational number operations; Apply exponential expressions (laws and properties).
Grades K-2	Grades 3-4	Grades 5-6	Grades 7-8	Grades 9-12
<p>Build understanding and fluency with operations...</p> <p>E.NO.2a representing addition and subtraction in multiple ways (composing/decomposing numbers, diagrams, using objects, arrays, equations, number lines), including regrouping K.OA-1, 2, 3, 4; K.NBT-1 1.OA-1, 2, 5, 6; 1.NBT-4, 5, 6 2.OA-1, 4; 2.NBT-7</p> <p>E.NO.2b explaining or modeling the relationship between addition and subtraction 1.OA-3, 4 1.NBT-4, 6 2.NBT-5, 7, 9</p> <p>E.NO.2c working flexibly with common addition and subtraction situations K.OA-2 1.OA-3, 5, 6, 8; 1.NBT-5, 6 2.OA-1, 2; 2.NBT-2, 5, 7</p>	<p>E.NO.2d modeling multiplication (equal-sized groups, arrays, area models, equal-sized jumps on number lines, multiplicative comparisons) and division (successive subtraction, partitioning, sharing) of whole numbers 3.OA-1, 2, 3, 4, 5 4.OA-1, 2, 3; 4.NBT-5, 6</p> <p>E.NO.2e describing relationships between addition-multiplication; multiplication-division; addition-subtraction; why commutativity does not apply to subtraction or division 3.OA-5, 7, 9; 3.NBT-2 4.OA-2</p> <p>E.NO.2f identifying factors and multiples of numbers 3.OA-6 4.OA-4</p> <p>E.NO.2g recognizing fractions as one number (one quantity), rather than two numbers (numerator and denominator) and using number lines to represent magnitude of fractions 3.NF-1, 2, 3a, 3c</p> <p>E.NO.2h adding, subtracting, and multiplying fractions, including mixed numbers 4.NF-3, 4</p>	<p>M.NO.2a working flexibility with common addition, subtraction, multiplication, and division situations 5.NBT-5, 6; 5.NF-5a 6.NS-2, 3, 4</p> <p>M.NO.2b recognizing fractions as one number (one quantity), rather than two numbers (numerator and denominator) and using number lines to represent magnitude of fractions and equivalent /non-equivalent fractions 5.NF-3</p> <p>M.NO.2c using operations and standard algorithms with whole numbers, fractions (unlike denominators), and decimals (to hundredths) 5.NBT-5, 6, 7; 5.NF-1, 2, 4, 7 6.NS-1, 3</p> <p>M.NO.2d contrasting situations as additive or multiplicative M.NO.2e ordering/comparing integers and representing them on the number line 6.NS-6a, 6c, 7</p>	<p>Build understanding and fluency with operations...</p> <p>M.NO.2f describing proportional relationships and solving related problems (Also addressed at grade 6: 6.RP-2, 3) 7.RP-1, 2, 3</p> <p>M.NO.2g using operations with complex fractions</p> <p>M.NO.2h using operations involving percents and percent increase/decrease 7.RP-3</p> <p>M.NO.2i using operations with rational numbers; representing rational numbers and approximations of irrational numbers on a number line 7.NS-1, 2; 7.EE-3 8.NS-1, 2</p>	<p>Build understanding and fluency with operations...</p> <p>H.NO.2a using operations with rational numbers; representing rational numbers and approximations of irrational numbers on a number line N.RN-3 A.SSE-3b A.REI-2 A.APR-1</p> <p>H.NO.2b operating with irrational and complex numbers A.REI-2 N.RN-3 N.CN-1, 2</p> <p>H.NO.2c identifying exponential situations and applying the laws and properties of exponents in simplifying expressions and solving equations A.SSE-2, 3 N.RN-1, 2</p> <p>H.NO.2d using matrices to represent situations; perform and interpret basic matrix operations</p>

The Nature of Numbers and Operations (NO): Understandings of number - “how many” or “how much” – and number types extend applications of arithmetic properties, operations, and number systems and guide use of computational strategies and algorithms.

(K-4) Elementary School Learning Targets		(5-8) Middle School Learning Targets		(9-12) High School Learning Targets
<p>E.NO-3 Use reasoning to support solutions and informal arguments and to develop metacognitive skills:</p> <ul style="list-style-type: none"> Use estimation and rounding to support informal arguments; Develop both additive and multiplicative thinking; Test, model, and explain solutions. 		<p>M.NO-3 Develop metacognitive skills through making conjectures and justifying mathematical solutions and arguments:</p> <ul style="list-style-type: none"> Use estimation and rounding to support reasonableness of arguments/justifications; Apply multiplicative and proportional reasoning; Make, test, and justify conjectures using mathematical concepts and models. 		<p>H.NO-3 Develop metacognitive skills through use of mathematical arguments to justify reasonableness of outcomes, to support formal proofs (including technology applications), and to develop metacognitive skills.</p>
Grades K-2	Grades 3-4	Grades 5-6	Grades 7-8	Grades 9 - 12
<p>Students demonstrate mathematical reasoning ...</p> <p>E.NO.3a exploring and explaining answers to questions, such as “Does this answer make sense?” K.CC-4, 5, 6; KOA-1 1.OA-1, 2, 7, 8; 1.G-3 2.OA-1; 2.NBT-7, 9</p> <p>E.NO.3b constructing arguments using concrete referents such as objects, diagrams, tables, actions (e.g., clapping, movement) and estimating K.OA.1, 2; K.G-5 1.OA-1, 2, 7, 8 1.G-3; 1.MD-4 2.OA-1; 2.NBT-7, 9 2.MD-3; 2.G-3</p>	<p>Students demonstrate mathematical reasoning ...</p> <p>E.NO.3c evaluating the reasonableness of answers using mental computation, arithmetic patterns, and estimation strategies, including rounding to the nearest 10 or 100 3.OA-5, 8, 9; 3.NBT-1, 2, 3; 3.MD-2 4.OA-3, 4, 5; 4.NBT-3</p> <p>E.NO.3d constructing arguments and explaining reasonableness of outcomes using a variety of concrete supports (e.g., models, diagrams, tables) 3.OA-8, 9 3.MD-1, 3, 4, 7b, 7c, 7d, 8 4.OA-3, 5 4.NBT- 5, 6 4.NF-1, 2, 3b, 3d, 4a, 6, 7 4.MD-4</p>	<p>Students demonstrate mathematical reasoning ...</p> <p>M.NO.3a using informal and rule-based arguments, evidence, and examples (e.g., estimation, rounding, arrays, visual models, diagrams) to justify mathematical solutions 5.OA-3; 5.NBT-2, 6, 7 5.NF-2, 4a, 5, 6, 7c 6.RP-3a, 3d; 6.NS-1, 6c, 8; 6.EE-5</p> <p>M.NO.3b critiquing the mathematical arguments provided by others 5.OA-3; 5.NBT-2, 6, 7 5.NF-2, 4a, 5 6.RP-3a, 3d; 6.NS-1, 7b, 7d, 8; 6.EE-5</p>	<p>Students demonstrate mathematical reasoning ...</p> <p>M.NO.3c using stated assumptions, definitions, patterns, and previously established results in constructing mathematical arguments 7.RP-2a, 2d; 7.NS-1a, 2a, 3; 7.EE-3 8.NS-2; 8.EE-6; 8.F-5; 8.SP-4</p> <p>M.NO.3d building a logical progression of statements to explore and evaluate the truth of conjectures 7.RP-2a, 2d; 7.NS-2a; 7.EE-3; 7.G-4 8.NS-2; 8.EE-6; 8.F-5; 8.SP-4</p>	<p>Students demonstrate mathematical reasoning ...</p> <p>H.NO.3a comparing the effectiveness of two plausible arguments, distinguishing correct logic or reasoning from that which is flawed, and if there is a flaw in an argument, explaining it</p> <p>F.IF-9 } Functions F.LE-1a } F.TF-8 }</p> <p>N.RN-1, 3 } Number & Quantity N.Q-1 }</p> <p>A.REI-1, 2, 5, 11 } Algebra A.SSE-2, 3c } A.CED-3, 4 } A.APR-4, 6 }</p> <p>G.SRT-4 } Geometry G.C-1 } G.CO-9, 10, 11 } G.GPE-4 }</p> <p>S.ID-3, 4, 9 } Statistics & Probability S.IC-2, 3, 5, 6 }</p>

NOTE: The third N&O strand focuses on mathematical reasoning and problem solving. The grade span learning targets and progress indicators for this strand can be integrated with many CCSS standards at each grade level using problem solving contexts and performance tasks. These CCSS standards have the greatest potential to require deeper understanding ONLY if assessment tasks require explanations and evidence of reasoning and concepts applied. While this strand is listed under the Nature of Numbers and Operations, the skills and concepts described in progress indicators apply to all mathematics strands when constructing arguments and making and supporting conjectures. (For more information about Depth of Knowledge and cognitive rigor see http://www.nciea.org/cgi-bin/pubspage.cgi?sortby=pub_date.)

Measurement (ME): Measurement attributes, processes, and tools help us quantify, compare, and solve problems involving objects, situations, and events.

(K-4) Elementary School Learning Targets		(5-8) Middle School Learning Targets		(9-12) High School Learning Targets
<p>E.ME-1 Explore relationships among units, attributes, and measures within a system of measurement:</p> <ul style="list-style-type: none"> Identify measurement attributes and units; Use measurement attributes to describe and compare objects, situations, or events. 		<p>M.ME-1 Extend understanding of attributes and units:</p> <ul style="list-style-type: none"> Make conversions within measurement systems; Relate measurement attributes, measures, models, and formulas. 		<p>H.ME-1 Explore measurable attributes, measurement systems, and processes of measurement of more complex or abstract quantities.</p>
Grades K-2	Grades 3-4	Grades 5-6	Grades 7-8	Grades 9-12
<p>Students understand measurable attributes by...</p> <p>E.ME.1a recognizing, identifying, and describing the measurable attributes of objects K.MD-1(length/height, weight) 1.MD-2 (l/h), 1.MD-3 (time) 2.MD-1 (l/h/w), 2.MD-7 (time), 2.MD-8 (money)</p> <p>E.ME.1b comparing and ordering objects/events according to their specified attributes (using standard or non-standard units of measure), including indirectly by using a third object, or using common referents to estimate or compare K.MD-2, 3 1.MD-1 2.MD-3, 4</p> <p>E.ME.1c recognizing that the smaller the unit, the more units are needed to measure an object ; and that units can be decomposed/partitioned into smaller units 1.MD-2 2.MD-2, 3</p>	<p>Students understand measurable attributes by ...</p> <p>E.ME.1d describing and demonstrating: unit attributes, iterating, tiling, identical units, number line intervals, standardization, proportionality, additivity, and origin 3.MD-1, 4, 5, 6, 7a, 7c, 7d 4.MD-7</p> <p>E.ME.1e justifying the need for measuring with standard units as compared to non-standard units</p> <p>E.ME.1f selecting the appropriate unit for measuring a given attribute (length, area, mass, liquid volume, size of angle), recognizing that a unit must have the same attributes as the object (e.g., unit of length must measure an object that has length) 3.MD-2,5 4.MD-1, 2, 6</p> <p>E.ME.1g exploring what happens to 2-dimensional measurements (perimeter or area) when the dimensions of the figure are changed 3.MD-7, 8 4.MD-3</p>	<p>Students understand measurable attributes by ...</p> <p>M.ME.1a identifying and describing measurable attributes (including area, surface area, volume, fractional units, absolute value with temperature), and selecting appropriate customary or metric units of measure when solving problems 5.MD-1, 3 6.NS-7b</p> <p>M.ME.1b recognizing relationships among units and using proportional reasoning to convert measurements from one unit to another within the same system 5.MD-1 6.RP-3d</p> <p>M.ME.1c recognizing how the formulas for area and volume for a variety of shapes and solids are related 6.G-1, 2, 4</p>	<p>Students understand measurable attributes by ...</p> <p>M.ME.1d applying proportional reasoning to problems with ratios of length, area, and quantities measured in like or different units 7.RP-1, 2b; 7.G-1</p> <p>M.ME.1e exploring what happens to 2- and 3-dimensional measurements (such as surface area, area, and volume) when the figure is changed in some way (e.g., scale drawings) 7.G-1 8.G-4, 9</p>	<p>Students understand measurable attributes by...</p> <p>H.ME.1a making decisions about units and scales that are appropriate for problem-solving situations within or across mathematics disciplines or real-world contexts N.Q-1, 2 G.CO-12 G.MG-1, 2, 3</p> <p>H.ME.1b investigating the results when linear dimensions of objects change by some factor (e.g., area and volume change disproportionately: area in proportion to the square of the factor and volume in proportion to its cube) A.REI-3 F.BF-1a</p> <p>H.ME.1c exploring quantifications of real-world applications of abstract units such as sound (decibels, pitch), gigabytes, Richter scale, acceleration, and other less tangible units of measure N.Q-1, 2</p>

Measurement (ME): Measurement attributes, processes, and tools help us quantify, compare, and solve problems involving objects, situations, and events.

(K-4) Elementary School Learning Targets					(5-8) Middle School Learning Targets		(9-12) High School Learning Targets	
<i>E.ME-2 Apply appropriate techniques (iteration and tiling), tools (standard and non-standard), and formulas (area and perimeter) to determine or estimate measurements.</i>					<i>M.ME-2 Apply appropriate techniques, strategies, and formulas to solve problems involving measurements (including derived measurements and rates).</i>		<i>H.ME-2 Apply and analyze techniques at an appropriate level of precision and use formulas to quantify or interpret abstract events, objects, and situations.</i>	
Grades K-2		Grades 3-4		Grades 5-6	Grades 7-8	Grades 9-12		
Determine measurements ...		Determine measurements ...		Determine measurements ...	Determine measurements ...	Determine measurements ...		
<p>E.ME.2a applying non-standard and common standard units to measure or estimate (length, height, weight, time) 1.MD-2, 3 2.MD-1, 2, 3, 4, 7</p> <p>E.ME.2b selecting tools and using units of measures appropriately and consistently, with no gaps or overlaps in the technique of measuring 1.MD-2, 3 2.MD-1, 3, 7</p> <p>E.ME.2c recognizing situations that require precision and those where an estimation or proportional matching is appropriate 2.MD-1, 3</p> <p>E.ME.2d describing a unit as an amount/quantity (rather than an object or a mark on a scale)</p>		<p>E.ME.2e selecting and applying appropriate customary or metric units and tools to measure or estimate (liquid volume, mass, perimeter, area, time, and angles) 3.MD-1, 2, 4, 6, 7a, 8 4.MD-5a, 6</p> <p>E.ME.2f recognizing relative sizes of units of measure and making simple conversions within systems when solving problems (e.g., 12 in. = 1 ft) 4.MD-1</p> <p>E.ME.2g recognizing situations that require precision (money, time, distances, fractions, decimals) and those where an estimation is appropriate 3.MD-2 4.MD-2</p> <p>E.ME.2h using a variety of strategies (decomposing complex shapes, using counting strategies, arrays, formulas) to estimate or measure area and perimeter (including irregular shapes/objects) 4.MD-3</p> <p>E.ME.2i selecting and using benchmarks to estimate measurements 3.MD-2</p>		<p>M.ME.2a selecting and applying appropriate standard units, tools, and level of precision in real-world measurement problems (e.g., area, surface area, volume, rate) 5.MD-1, 4 6.G-1, 2, 4</p> <p>M.ME.2b using a variety of strategies (decomposing complex shapes, using formulas and models) to measure area (triangles, quadrilaterals, polygons) and volume (rectangular prisms) 5.MD-4, 5 6.G-1, 2, 4</p>	<p>M.ME.2c selecting and applying appropriate standard units and tools to measure to an appropriate level of precision</p> <p>M.ME.2d using various strategies (decomposing complex shapes, using formulas) to measure volume (cones, cylinders, spheres) and area and circumference of circles 7.G-4 8.G-9</p> <p>M.ME.2e solving simple problems involving scale factors, rates, and derived measures 7.G-1; 7.RP-1</p> <p>M.ME.2f applying the Pythagorean theorem to determine lengths/distances in real-world situations 8.G-7, 8</p>	<p>H.ME.2a analyzing levels of precision, accuracy, and approximate error in measurement situations N.Q-3</p> <p>H.ME.2b using techniques of measurement, estimating, or calculating to compare or analyze two- and three-dimensional figures and their parts G.SRT-1, 2 G.C-5 G.MG-1, 2, 3</p>		

Patterns, Relations, and Functions (PRF): Patterns, relations, and functions are used to represent and analyze change in various contexts, make predictions and generalizations, and provide models and explanations for real-world phenomena.

(K-4) Elementary School Learning Targets		(5-8) Middle School Learning Targets		(9-12) High School Learning Targets
<p><i>E.PRF-1 Use concrete, pictorial, and symbolic representations to identify, describe, compare, and model situations that involve change.</i></p>		<p><i>M.PRF-1 Describe and compare situations that involve change and use the information to draw conclusions:</i></p> <ul style="list-style-type: none"> • Model contextual situations using multiple representations; • Calculate rates of change for real-world situations (constant). 		<p><i>H.PRF-1 Approximate, calculate, model, and interpret change:</i></p> <ul style="list-style-type: none"> • Use graphical and numerical data resulting from complex situations; • Model complex real-world phenomena to make predictions and provide explanations.
Grades K-2	Grades 3-4	Grades 5-6	Grades 7-8	Grades 9-12
<p>Demonstrate understanding of change by...</p> <p>E.PRF.1a describing changes qualitatively (e.g., growing taller) and quantitatively (e.g., growing 2 inches in one year) K.CC-6; K.MD-2 1.OA-1; 1.NBT-3; 1.MD-1 2.NBT-4</p> <p>E.PRF.1b exploring and describing how addition or subtraction changes a quantity K.OA-1, 2 1.OA-1, 5</p> <p>E.PRF.1c modeling problem-solving situations that involve addition and subtraction of whole numbers using objects, diagrams, and symbols K.OA-1, 2 1.OA-1, 5 2.OA-1, 3, 4; 2.NBT-7; 2.MD-5</p>	<p>Demonstrate understanding of change by...</p> <p>E.PRF.1d describing and modeling how addition, subtraction, multiplication, or division changes a quantity, including with fractions 3.OA-1, 2, 7 4.OA-1; 4.NBT-1; 4.NF-3, 4</p> <p>E.PRF.1e using representations (tables, graphs, equations) to show how values of one quantity are related to values of another and to draw conclusions 3.OA-9; 3.MD-3, 4 4.OA-2</p> <p>E.PRF.1f representing and explaining equivalence concretely, graphically, and symbolically (equations, rules) 3.MD-1, 7c 4.OA-5; 4.NBT-5, 6; 4.MD-4</p> <p>E.PRF.1g identifying situations with constant or varying rates of change (with two quantities)</p>	<p>Demonstrate understanding of change by...</p> <p>M.PRF.1a describing how multiplication or division changes a quantity, including with fractions or decimals 5.NF-4a, 5, 7b 6.NS-1</p> <p>M.PRF.1b distinguishing linear from nonlinear relationships as represented in graphical and tabular representations 5.OA-3 6.RP-3a</p> <p>M.PRF.1c comparing two rates and evaluating them for a given situation (e.g., best value) 6.RP-1, 2, 3b 6.EE-4, 6</p> <p>M.PRF.1d using symbolic equations to summarize how the quantity of something changes 6.EE-4, 7, 9</p>	<p>Demonstrate understanding of change by...</p> <p>M.PRF.1e representing and computing unit rates associated with ratios of lengths, areas, and other quantities measured in like or different units 7.RP-1, 2, 3 8.EE-5</p> <p>M.PRF.1f identifying essential quantitative relationships in a situation and using symbolic expressions to represent it and draw reasonable conclusions from it 7.RP-1, 2, 3; 7.EE-2, 3, 4 8.EE-5, 7; 8.F-1, 2, 3, 4, 5</p> <p>M.PRF.1g modeling, solving, and explaining contextualized problems using various representations such as graphs, tables, functions, and equations 7.RP-2, 3; 7.EE-2, 4 8.EE-5, 7; 8.F-2, 3, 4, 5</p> <p>M.PRF.1h representing and describing how rates of change can be computed from differences in magnitudes and vice versa</p>	<p>Demonstrate understanding of change by...</p> <p>H.PRF.1a approximating, calculating, and interpreting rates of change using graphical and numerical data S.ID-1, 2, 7 F.LE-1b, 1c, 3</p> <p>H.PRF.1b exploring how the rate of change of something depends on how much there is of something else (e.g., as the rate of change of speed is proportional to the amount of force acting) A.CED-4 S.ID-3 F.LE-1b</p> <p>H.PRF.1c creating mathematical models, using rules and relationships to describe and predict objects and events in the real world A.CED-2 S.IC-2 F.LE-1 F.TF-5</p> <p>H.PRF.1d recognizing that there are limitations in mathematics models A.CED-3 A.REI-2 S.IC-2</p>

Patterns, Relations, and Functions (PRF): Patterns, relations, and functions are used to represent and analyze change in various contexts, make predictions and generalizations, and provide models and explanations for real-world phenomena.

(K-4) Elementary School Learning Targets		(5-8) Middle School Learning Targets		(9-12) High School Learning Targets
<i>E.PRF-2 Give examples, interpret, and analyze repeating and growing patterns and functions involving the four basic operations.</i>		<i>M.PRF-2 Give examples, interpret, and analyze a variety of mathematical patterns, relations, and explicit and recursive functions.</i>		<i>H.PRF-2 Use trends and analyze a variety of mathematical patterns, relations, and explicit and recursive functions.</i>
Grades K-2	Grades 3-4	Grades 5-6	Grades 7-8	Grades 9-12
<p>Demonstrate understanding of patterns by...</p> <p>E.PRF.2a recognizing, describing, and extending simple repeating (ABAB) and growing (A+1, A+2, A+3) patterns (e.g., colors, sounds, words, shapes, numeric – counting, odd-even) K.CC-6; K.OA-1, 4; K.G-2 1.OA-1, 5 1.NBT-2b, 2c, 5, 6 2.OA-3, 4; 2.NBT-1b, 2</p> <p>E.PRF.2b creating and explaining repeating and growing patterns using objects or numbers K.OA-1, 4; K.NBT-1; K.G-2 1.OA-1, 5; 1.NBT-2, 5, 6 2.OA-3, 4; 2.NBT-1, 2</p> <p>E.PRF.2c extending and analyzing simple numeric patterns with rules that involve addition and subtraction K.OA-1, 4; K.NBT-1 1.OA-1, 5; 1.NBT-2, 5, 6 2.OA-3, 4; 2.NBT-1, 2, 8</p>	<p>Demonstrate understanding of patterns by...</p> <p>E.PRF.2d representing and analyzing patterns and rules (e.g. doubling, adding 3) using words, tables, graphs, and models 3.OA-3, 5, 7, 9 4.OA-4, 5</p> <p>E.PRF.2e extending, translating, and analyzing numeric patterns and their rules using addition, subtraction, multiplication, and division 3.OA-3, 5, 7, 9; 3.NBT-3 4.OA-4, 5</p>	<p>Demonstrate understanding of patterns by...</p> <p>M.PRF.2a representing, analyzing, extending, and generalizing a variety of patterns using tables, graphs, words, and symbolic rules 5.OA-3; 5.NBT-2 6.EE-6, 9</p> <p>M.PRF.2b relating and comparing different forms of representation and identifying functions as linear or nonlinear 5.OA-3 6.RP-1, 2, 3 6.EE-6, 7, 9</p>	<p>Demonstrate understanding of patterns by...</p> <p>M.PRF.2c relating and comparing different forms of representation and identifying functions as linear or nonlinear 8.F-3</p> <p>M.PRF.2d solving linear equations and formulating and explaining reasoning about expressions and equations 7.EE-2, 4a 8.EE-6, 7</p> <p>M.PRF.2e using functions to describe quantitative relationships 8.EE-5, 7 8.F-1, 3, 4, 5</p>	<p>Demonstrate understanding of patterns by...</p> <p>H.PRF.2a interpreting and rewriting a variety of expressions or functions to solve problems A.SSE-1, 2, 3, 4 F.BF-1a, 1b, 2</p> <p>H.PRF.2b creating equations and inequalities (in one or two variables) and use them to solve problems and graph solutions A.CED-1, 2 A-REI-3, 4, 6, 7, 10, 12 S.ID-6</p> <p>H.PRF. 2c using trends that follow a pattern and are described mathematically to make generalizations or predictions A-REI-11 F.BF-3, 4a</p> <p>H.PRF. 2d analyzing functions (using technology) by investigating significant characteristics (e.g. intercepts, asymptotes) F.IF-7a, b, e F.BF-3 S.ID-8</p> <p>H.PRF. 2e comparing the properties of classes of functions F.IF-9</p>

Geometry (GM): Visualizations, spatial reasoning, and properties of two- and three-dimensional figures can be used to analyze, represent, and model geometric concepts and relationships.

(K-4) Elementary School Learning Targets		(5-8) Middle School Learning Targets		(9-12) High School Learning Targets
<p>E.GM-1 Recognize that two- and three-dimensional shapes have particular attributes:</p> <ul style="list-style-type: none"> Describe and compare objects and figures based on geometric attributes; Compose, decompose, and draw figures based on spatial reasoning and the properties and attributes of the shapes; Apply concepts of symmetry. 		<p>M.GM-1 Apply reasoning using properties of two- and three-dimensional shapes to analyze, represent, and model geometric relationships:</p> <ul style="list-style-type: none"> Classify objects based on attributes and properties and solve problems using geometric relationships and properties; Decompose figures into new figures and construct figures with given conditions; Apply concepts of parallel and perpendicular. 		<p>H.GM-1 Explain solutions using geometric attributes and relationships in diverse contexts:</p> <ul style="list-style-type: none"> Extend understanding of congruence and similarity working with complex figures and situations; Solve problems involving quadrilaterals and triangles; Perform geometric constructions and use informal proofs to describe relationships and transformations.
Grades K-2	Grades 3-4	Grades 5-6	Grades 7-8	Grades 9-12
<p>Demonstrate understanding of 2- and 3- dimensional shapes and their attributes...</p> <p>E.GM.1a recognizing, describing (using spatial language) and naming shapes regardless of orientation or size and locating shapes in the environment K.G-1, 2</p> <p>E.GM.1b analyzing and comparing 2- (and later) 3- dimensional shapes using informal language (e.g., flat, solid, corners) to describe their differences and similarities, as well as their component parts (number of sides, vertices) and other attributes (e.g., sides of equal length) K.G-3, 4</p> <p>(GM-1 Continued next page)</p>	<p>Demonstrate understanding of 2- and 3- dimensional shapes and their attributes...</p> <p>E.GM.1h describing, analyzing, comparing, and classifying two-dimensional figures (triangles, quadrilaterals) using shared attributes 3.G-1 4.G-2</p> <p>E.GM.1i partitioning shapes into equal parts with equal areas and recognizing that each part is a unit fraction of the whole 3.G-2</p> <p>E.GM.1j recognizing and drawing points, lines, line segments, rays, angles, and perpendicular and parallel lines and identifying these in plane figures 4.G-1</p> <p>E.GM.1k recognizing and drawing lines of symmetry in a variety of figures 4.G-3</p>	<p>Demonstrate understanding of 2- and 3- dimensional shapes and their attributes...</p> <p>M.GM.1a describing and classifying plane figures based on their properties 5.G-3, 4</p> <p>M.GM.1b recognizing and using properties belonging to categories and subcategories of plane figures (e.g., all rectangles have four right angles, so all squares are rectangles and have four right angles) 5.G-3</p> <p>M.GM.1c demonstrating the use of a coordinate system by locating/graphing a given point or polygon using ordered pairs 5.G-1, 2 6.G-3</p> <p>M.GM.1d solving area, surface area, and volume problems by composing and decomposing figures 5.MD-5c 6.G-1, 2, 4</p>	<p>Demonstrate understanding of 2- and 3- dimensional shapes and their attributes...</p> <p>M.GM.1e constructing or drawing geometric shapes from given conditions (e.g., draw triangles given three angle or side measures; change scale) 7.G-1, 2</p> <p>M.GM.1f recognizing and demonstrating rotations, reflections, and translations using multiple contexts (e.g., using coordinates, models, drawings, technology) 8.G-1, 2, 3, 4</p> <p>M.GM.1g demonstrating congruence and similarity using a variety of two-dimensional figures 8.G-2, 4</p> <p>M.GM.1h solving real-world area, surface area, and volume problems using different strategies (formulas and decomposing figures) 7.G-4, 6 8.G-9</p> <p>(GM-1 Continued next page)</p>	<p>Demonstrate understanding of 2- and 3- dimensional shapes and their attributes...</p> <p>H.GM.1a applying the Pythagorean Theorem G.SRT-8</p> <p>H.GM.1b using congruence and similarity relationships to solve problems, including triangle congruence relationships G.CO-7, 8 G.SRT-2, 3, 5</p> <p>H.GM.1c applying understanding of rotations, reflections, and translations to construct figures (e.g., using coordinates, models, drawings, transparencies, dynamic geometry software) G.CO-3, 5</p> <p>H.GM.1d applying scale factors in solving multiple similarity problems, including transformations in the coordinate plane and similarity relationships with right triangles G.CO-2 G.SRT-1b, 2</p> <p>H.GM.1e making various geometric constructions, including use of dynamic geometry software, and creating informal proofs of relationships (lines and angles, circles, polygons) G.CO-12, 13 (GM-1 Continued next page)</p>

Geometry (GM): Visualizations, spatial reasoning, and properties of two- and three-dimensional figures can be used to analyze, represent, and model geometric concepts and relationships.

(K-4) Elementary School Learning Targets		(5-8) Middle School Learning Targets		(9-12) High School Learning Targets
<p>E.GM-1 Recognizing that two- and three-dimensional shapes have particular attributes:</p> <ul style="list-style-type: none"> Describe and compare objects and figures based on geometric attributes; Compose, decompose, and draw figures based on spatial reasoning and the properties and attributes of the shapes; Apply concepts of symmetry. 		<p>M.GM-1 Apply reasoning using properties of two- and three-dimensional shapes to analyze, represent, and model geometric relationships:</p> <ul style="list-style-type: none"> Classify objects based on attributes and properties and solve problems using geometric relationships and properties; Decompose figures into new figures and construct figures with given conditions; Apply concepts of parallel and perpendicular. 		<p>H.GM-1 Explain solutions using geometric attributes and relationships in diverse contexts:</p> <ul style="list-style-type: none"> Extend understanding of congruence and similarity working with complex figures and situations; Solve problems involving quadrilaterals and triangles; Perform geometric constructions and use informal proofs to describe relationships and transformations.
Grades K-2	Grades 3-4	Grades 5-6	Grades 7-8	Grades 9-12
<p>Demonstrate understanding of 2- and 3- dimensional shapes and their attributes... (GM.1 Continued) E.GM.1c composing two-dimensional shapes (rectangles, squares, triangles, half-circles, and quarter-circles) K.G-6 1.G-2</p> <p>E.GM.1d composing three-dimensional shapes, using concrete models/materials (cubes, prisms, cones, and cylinders) 1.G-2</p> <p>E.GM.1e drawing and identifying shapes with specific attributes (e.g., number of sides or equal angles) not determined by direct measuring) 2.G-1</p> <p>E.GM.1f partitioning shapes into 2, 3, or 4 equal parts and describing the parts (halves, quarters, fourths, thirds) 1.G-3; 2.G-2, 3</p> <p>E.GM.1g using spatial language to describe and name more complex or atypical shapes based on their defining characteristics</p>	<p>Demonstrate understanding of 2- and 3- dimensional shapes and their attributes...</p>	<p>Demonstrate understanding of 2- and 3- dimensional shapes and their attributes...</p>	<p>Demonstrate understanding of 2- and 3- dimensional shapes and their attributes... (GM.1 Continued) M.GM.1i exploring and explaining angle relationships (e.g., pairs of parallel lines cut by a transversal, including perpendicular lines) 8.G-5</p> <p>M.GM.1j applying the Pythagorean Theorem 8.G-7, 8</p>	<p>Demonstrate understanding of 2- and 3- dimensional shapes and their attributes... (GM.1 Continued) H.GM.1f solving problems (including proofs) using the relationships among special quadrilaterals (parallelogram, rectangle, rhombus, square, trapezoid, and kite) and describing the characteristics of parallelograms using side, angle, and diagonal properties and relationships G.CO-9, 10, 11</p>

Data Analysis, Probability, and Statistics (DPS): Questions are posed and investigated by collecting data or retrieving existing data, and representing, analyzing, and interpreting data. Investigations, inferences, and predictions are used to make critical and informed decisions.

(K-4) Elementary School Learning Targets		(5-8) Middle School Learning Targets		(9-12) High School Learning Targets
<p>E.DPS-1 Gather and interpret data to answer questions related to a particular/single context.</p> <ul style="list-style-type: none"> Formulate questions, gather data, and build representations; Identify and describe variation in data, and describe and compare shapes of distributions and measures of central tendency. 		<p>M.DPS-1 Design investigations and gather data to answer questions about multiple populations.</p> <ul style="list-style-type: none"> Formulate questions, gather data, and build representations; Compare populations by analyzing distributions in terms of variability and measures of central tendency. 		<p>H.DPS-1 Design and conduct statistical studies:</p> <ul style="list-style-type: none"> Use appropriate statistical measures for analysis; Develop the concepts of statistical inference and statistical significance, especially in relation to probability principles and sampling distributions.
Grades K-2	Grades 3-4	Grades 5-6	Grades 7-8	Grades 9-12
<p>Students gather, organize, and interpret data by...</p> <p>E.DPS.1a posing questions of interest that can be answered by counting or collecting data (e.g., concrete comparisons about students, classroom materials, science topics) <i>with teacher guidance</i> K.CC-5, 6 1.MD-1 2.MD-2, 5, 9</p> <p>E.DPS.1b identifying and sorting data/attributes; identifying rules for classifying data/attributes K.MD-1, 2, 3; K.G-2, 4 1.MD-1, 4; 1.G-1 2.G-1; 2.MD-10</p> <p>E.DPS.1c collecting and organizing/ representing data (e.g., picture graphs, tally charts, bar graphs) K.CC-5; K.MD-1, 2, 3 1.MD-4 2.MD-1, 2, 4, 5, 6, 9, 10</p> <p>E.DPS.1d recognizing that data can take on different values 1.MD-4 2.MD-3, 9, 10</p> <p>E.DPS.1e describing and comparing data and beginning to identify what the data do or do not show (e.g., bar graphs, line plots, picture graphs) 1.MD-4 2.MD-2, 4, 5, 6, 9, 10</p>	<p>Students gather, organize, and interpret data by...</p> <p>E.DPS.1f formulating questions and designing investigations (defining measures and variables) 3.MD-2 4.MD-2</p> <p>E.DPS.1g collecting data and representing data (e.g., bar graphs, frequency tables, line plots) 3. MD-1, 2, 3, 4 4.MD-1, 2, 4</p> <p>E.DPS.1h recognizing and identifying sources of variability in the data (measurement variability and natural variability) 3.MD-2, 3, 4 4.MD-2</p> <p>E.DPS.1i describing data shapes and what the data representations do and do not show (bar graphs, picture graphs, frequency tables, line plots, circle graphs) including the attributes used</p> <p>E.DPS.1j identifying clumps, gaps, trends, or central tendency (mode, median) in the data</p> <p>E.DPS.1k using data to make and support claims and interpretations (e.g., making comparisons among individuals, between individuals and the group, and among groups) 3. MD-1, 3, 4</p>	<p>Students gather, organize, and interpret data by...</p> <p>M.DPS.1a formulating questions about groups larger than classroom groups and comparing different populations or samples 6.SP-1, 2</p> <p>M.DPS.1b distinguishing among populations, censuses, and sampling 6.SP-1, 2</p> <p>M.DPS.1c using representations (e.g., dot plots, scatter plots, line plots) to display data from investigations to describe the shapes of the data 5.MD-2; 5.G-2 6.SP-2, 3, 4, 5</p> <p>M.DPS.1d identifying the range, three common measures of central tendency (mean, median, and mode) and interpreting the mean as a fair share and a center of balance 6.SP-2, 3, 5c, 5d</p> <p>M.DPS.1e making claims about populations from data distributions, supporting interpretations on the basis of mean, median, or mode, and the shape of the distribution 5.G-2 6.SP-3, 5</p>	<p>Students gather, organize, and interpret data by...</p> <p>M.DPS.1f formulating questions about groups larger than classroom groups, comparing different populations or samples, and involving two variables 7.SP-1</p> <p>M.DPS.1g displaying and interpreting univariate data using dot plots, histograms, and circle graphs 7.SP-2, 4</p> <p>M.DPS.1h displaying data in scatter plots and investigating the association between the variables 8.SP-1, 2</p> <p>M.DPS.1i using box plots, interquartile range, mean absolute deviation, range, and the concept of outliers to characterize the distribution (variability) of univariate data 7.SP-2, 3, 4</p> <p>M.DPS.1j comparing two unequal distributions of data using number of data points, measures of central tendency, shape, and variability (numerical data), and two-way tables (categorical variables) 7.SP-3, 4 8.SP-1, 3, 4</p> <p>M.DPS.1k supporting claims about the results of investigations (e.g., coordinating among the measures of central tendency and variability) 7.SP-3, 4 8.SP-1, 3, 4</p>	<p>Students gather, organize, and interpret data by...</p> <p>H.DPS.1a designing and conducting different kinds of studies using categorical and numerical data, explain results, and use data to estimate a population mean or proportion:</p> <ul style="list-style-type: none"> observational studies (e.g., traffic patterns at an intersection near the school); sample surveys (a survey of student nutritional habits); simple comparative experiments (e.g., comparisons of water and fertilizer treatments in a plant growth experiment) <p>S.IC-3, 4, 5</p> <p>H.DPS.1b representing data with plots on the real number line (dot plots, histograms, box plots) S.ID-1</p> <p>H.DPS.1c analyzing and summarizing the data resulting from studies using statistical measures appropriate to shape of the data (median, mean) and spread (interquartile range, standard deviation), and using data to support inferences (population parameters, sample size) or explain possible outliers S.ID-2, 3, 4, 5 S.IC-1</p> <p>H.DPS.1d representing and interpreting data (graphs, scatter plots) to explain how variables are related, or to fit a function to the data S.ID-6</p>

Data Analysis, Probability, and Statistics (DPS): Questions are posed and investigated by collecting data or retrieving existing data, and representing, analyzing, and interpreting data. Investigations, inferences, and predictions are used to make critical and informed decisions.

(K-4) Elementary School Learning Targets		(5-8) Middle School Learning Targets		(9-12) High School Learning Targets
<i>E.DPS-2</i> Conduct simple probability experiments and characterize the outcomes in words, diagrams, or numerically.		<i>M.DPS-2</i> Conduct probability experiments: <ul style="list-style-type: none"> • Generate random samples to characterize variability in estimates and predictions; • Analyze and build models of the association between two variables. 		<i>H.DPS-2</i> Use the rules of probability to interpret data, develop explanations, and address real-world problems.
Grades K-2	Grades 3-4	Grades 5-6	Grades 7-8	Grades 9-12
<p>Students apply probability concepts by...</p> <p><i>E.DPS.2a</i> describing the probability of events as being possible or impossible</p> <p><i>E.DPS.2b</i> describing the probability of events as being certain, likely, unlikely, or impossible</p> <p><i>E.DPS.2c</i> representing all possible outcomes for expectations of varied results (e.g., using words, drawings, tree diagrams to show all different combinations for making sandwiches from a choice of ingredients)</p>	<p>Students apply probability concepts by...</p> <p><i>E.DPS.2d</i> describing the probability of events as being certain, likely, equally likely, unlikely, or impossible.</p> <p><i>E.DPS.2e</i> identifying expectations for varied results in situations involving randomness (e.g., using coin tosses, spinners, dice, playing cards)</p> <p><i>E.DPS.2f</i> representing all possible outcomes for expectations of varied results (e.g., using words, tree diagrams)</p> <p><i>E.DPS.2g</i> conducting repeated trials of simple probability experiments, using display displays (e.g., tables, tree diagrams, histograms) to understand results and explain variations</p>	<p>Students apply probability concepts by...</p> <p><i>M.DPS.2a</i> conducting simple probability experiments and expressing results in terms of relative frequencies or proportions as first estimate of probability</p> <p><i>M.DPS.2b</i> describing and representing (e.g., tree diagrams) all possible outcomes (sample space) and the theoretical probabilities of each outcome (as proportion of a specific outcome relative to all possible outcomes) in simple probability experiments</p> <p><i>M.DPS.2c</i> using two-way tables to characterize distributions of two categorical variables</p>	<p>Students apply probability concepts by...</p> <p><i>M.DPS.2d</i> identifying sample spaces for multi-stage probability experiments (independent events) and determining the theoretical probabilities of specific event combinations 7.SP-5, 6, 7</p> <p><i>M.DPS.2e</i> designing and conducting multi-stage (compound) probability experiments (independent events) and comparing the results with theoretical probabilities 7.SP-8</p> <p><i>M.DPS.2f</i> distinguishing between association of two variables and cause and effect relationship between two variables</p> <p><i>M.DPS.2g</i> using simple lines to model association between two numerical variables in a bivariate data set 8.SP-2, 3, 4</p>	<p>Students apply probability concepts by...</p> <p><i>H.DSP.2a</i> explaining the outcomes of probabilities in words and recognizing equivalent representations of probability, such as one out of ten, 10%, 1/10, 0.10</p> <p><i>H.DSP.2b</i> exploring (framing effects) the degree to which we rate something as “good” or “bad”/ “desirable or “undesirable” when numerical information is presented positively (75% lean) or negatively (25% fat)</p> <p><i>H.DSP.2c</i> designing and conducting multi-stage (compound) probability experiments (independent events) and comparing the results with theoretical probabilities S.CP-2</p> <p><i>H.DSP.2d</i> constructing and interpreting two-way frequency tables when two categories are associated with each object being classified S.CP-4</p> <p><i>H.DSP.2e</i> researching and finding real-world examples and explaining the concept of conditional probability (e.g., compare the chances of having lung cancer if you are a smoker with the chances of being a smoker if you have lung cancer) S.CP-5</p>

NOTE: There is minimal emphasis in the CCSS on Data Analysis, Probability, & Statistics. This is due in part to place greater instructional emphasis on building fluency with basic operations and a deeper understanding of numbers (fractions, decimals, etc.) at the lower grade levels. Because many of the DPS mathematics skills and concepts are essential to science and social studies instruction, progress indicators are included here to guide unit development, especially where organizing and interpreting data is important to overall understanding.