Grade: 8 **Content:** Math

Learning Objective:

Greetings 8th graders! We hope you and your families are safe!

We are going to start reviewing how to interpret a proportional relationship and applying proportions to solve percent, discount, and mark up problems.

We've included a video link to review interpreting proportional relationships and solving proportional problems. There's also a link to Khan Academy for the entire topic which includes practice problems if you'd like to try those.

Video Links:

Khan Academy: https://www.khanacademy.org/math/pre-algebra/pre-algebra-ratios-rates/pre-algebrapercent-word-problems/v/solving-percent-problems-2

Khan Academy: https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-ratioproportion/cc-7th-graphs-proportions/v/interpreting-graphs-of-proportional-relationships Khan Academy entire topic practice: https://www.khanacademy.org/commoncore/grade-7-RP

Practice Activities:

Printable Resources:

Skills Practice: see the attached practice pages

Perks of Work

2

Calculating Tips, Commissions, and Simple Interest

WARM UP

Express each percent as a decimal and as a fraction.

1. 47%
2. 3%
3. 12.5%
4. 0.25%
5. 4.99%

LEARNING GOALS

- Use percent equations to solve for unknowns in multistep percent problems.
- Solve multistep percent problems using direct variation.
- Solve multistep percent problems using the constant of proportionality.

KEY TERMS

- percent equation
- simple interest
- commission

You have used proportions to solve percent problems involving markups and markdowns. How can you use proportions and percent equations to solve for the unknown in different types of percent problems?

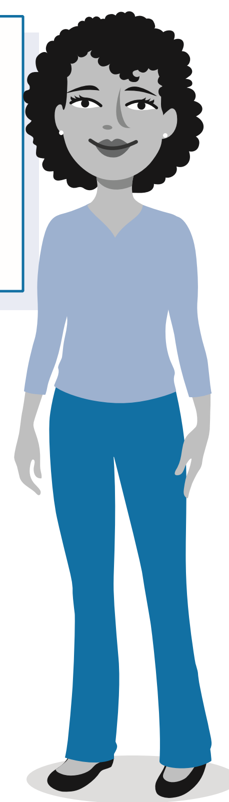
Getting Started

Make Sure to Tip Your Servers

Most restaurant patrons add a tip to the final bill to show their appreciation for the wait staff. Usually, a patron will determine 15% to 20% of the bill, and then add that amount to the total. Many times, patrons will just round off the tip to the nearest dollar.

You can use benchmark percents to estimate the amount of any tip. Common benchmark percents used in calculating tips are 1%, 5%, 10%, and 25%.

A patron is another word for a person who eats at a restaurant.



WORKED EXAMPLE

One strategy to determine a 20% tip for a restaurant bill that is \$38.95 is to first determine 10% of the total and then double that amount. Ten percent of \$38.95 is \$3.90, or approximately \$4. So, a 20% tip should be about \$8.

For each bill amount, use benchmark percents to estimate a 15% and 20% tip.

	15% Tip	**20% Tip**
\$89.45		
\$125.00		
\$12.45		

ACTIVITY 2.1

Introduction to Percent Equations

Now that many people own phones with built-in calculators, some calculate the e
their restaurant bill rather than use rounding and benchmark fractions.

Suppose you want to determine the recommended 15% tip on a restaurant bill of $45.00. You can use a propor
determine the amount of a tip based on the restaurant total.

1. Use a proportion to determine the amount of a tip based on the restaurant total.

You can also use a *percent equation* to determine the tip amount on a restaurant bill. A **percent equation** can b
the form percent 3 whole 5 part, where the percent is often written as a decimal.

WORKED EXAMPLE

percent 5 $\frac{\text{part}}{\text{whole}}$

percent as decimal 3 whole 5 part

(tip percent)	of	(total bill)	5	amount of the tip
↑		↑		↑
$\frac{15}{100}$ or 0.15	3	45	5	t
		6.75	5	t

2. Analyze the worked example.

a. Describe how the percent equation in the form percent 3 whole 5 part is equivalent to a proportion in the form percent 5 $\frac{\text{part}}{\text{whole}}$.

b. Explain how the variable is isolated. Then, describe how the tip amount is calculated using the percent equation.

3. Use proportions and percent equations to calculate tips on the given restaurant bills. For each, isolate the variable first. Then, determine the tip amount. Finally, write your answer in a complete sentence.

Bill	Percent	Use a Proportion	Use a Percent Equation

\$63.89	20%		
Sentence			
\$24.40	15%		
Sentence			

4. **Describe how the strategies you used to solve the proportions and the percent equations are similar.**

ACTIVITY

2.2 Strategies for Calculating Percent

Restaurant servers are not the only people provided with tips for a job well done.

Skylar is a hair stylist at a salon. Her clients pay their bills at the front desk but give her cash for her tip. She wondered what her typical tip percent was, so she calculated the tip percent received from each client on a specific day.

NOTES

WORKED EXAMPLE

Skylar’s first client of the day spent \$150 to have her hair dyed and cut, and gave Skylar a \$30 tip.

Use a Proportion	**Use a Percent Equation**
$\frac{t}{100} = \frac{30}{150}$	$(t)(150) = 30$
	$150t = 30$
$t = \frac{(30)(100)}{150}$	$\frac{150t}{150} = \frac{30}{150}$
	$t = \frac{30}{150}$
$t = 20$	$t = 0.2$

1. **Explain why Skylar's methods result in different values for *t*. What percent tip did Skylar receive from her client?**

2. **Calculate the tip percent for Skylar’s next two clients. Use both proportions and percent equations in the table shown. For each problem, isolate the variable first. Then, calculate the answer. Finally, write your answer in a complete sentence.**

Salon Bill	Tip Amount	Use a Proportion	Use a Percent Equation
$80	$15		
Sentence			
$80	$10		
Sentence			

3. **Describe the strategies you used to solve each proportion and each percent equation.**

ACTIVITY 2.3

Strategies for Calculating the Whole

Tipping for services is not a universal standard, so some business add an automatic gratuity, or tip, onto every bill. Restaurants frequently add an 18% gratuity when the group includes 8 or more people. Some hotels, resorts, and service providers close to tourist areas often add an automatic 18% gratuity to the bill.

An esthetician is someone who is knowledgeable about skin care, particularly the face.

1. The esthetician, manicurist, and massage therapist at the Sun and Sand Resort earn an automatic 18% gratuity on their services. Determine the value of the services each must provide in a day to earn the desired gratuity. Show your work, and then write a sentence to explain your answer.

	Desired Gratuity	Use a Proportion	Use a Percent Equation
Esthetician	$100		
Sentence			
Manicurist	$150		
Sentence			

Massage Therapist	$200		
Sentence			

ACTIVITY 2.4 Tips and Direct Variation

A room service waiter or waitress at a hotel receives an automatic gratuity that varies directly with the amount of the food bill.

1. Suppose a room service waitress at the Sun and Sand Resort receives gratuity represented by the equation $g = 0.15b$, where g represents the gratuity and b represents the food bill.

2. Crystal says that when she solved the proportions in Question 1, she could set up her proportions in any way she wanted because ratios can be written in any way. Do you agree with Crystal's statement? Explain your reasoning.

"What does the form of the equation tell you?"

a. What percent gratuity does the room service waitress receive? How do you know?

b. If the food bill is \$19, how much tip did she receive?

c. If the room service waitress receives a \$2.10 gratuity, how much is the food bill?

2. Write an equation to represent the direct proportional relationship between the amount of gratuity (*g*) received by a room service waitress and the food bill (*b*). Let *k* represent the constant of proportionality. What does the constant of proportionality represent in this equation?

3. Gourmet Eatery has a policy of automatically adding an 18% tip to every restaurant bill.

a. W rite an equation to represent the relationship between the tip (*t*) and the restaurant bill (*b*).

b. How much of a tip is added to a restaurant bill of $54? Use your equation to determine the amount of the tip.

c. Marie receives a tip of $12. How much is the restaurant bill?

d. If a restaurant bill is $12, how much is the tip?

e. How much would a restaurant bill be if it had a tip of $3.20 added to it?

Warm Up Answers

1. 102_____________ cans of food   or an

Complying with Title IX

2

Constant of Proportionality

Warm Up

Washington Middle School collects canned local community food bank. Last year, there were 180 students enrolled at the they collected 102 cans of food.

1. W rite the ratio representing the number of food contributed to the total number of students in the school.

hat is the unit rate of cans contributed per student?

3. T his year, 210 students are enrolled in school. Assume the number of cans of food contributed per student for both years is the same. How many cans of food should the school expect to be contributed this year?

LEARNING GOALS

- Determine if there is a constant ratio food for a between two variables.
- Identify the constant of proportionality in school and proportional relationships.
- Identify the constant of proportionality in of cans equations.
- Represent proportional relationships by equations. 2. W

KEY TERM

- constant of proportionality the

Answers

180 students equivalent ratio

2. 0.567 can per student
3. 119 cans of food

You know how to recognize proportional relationships from tables and graphs. How do you represent proportional relationships with equations?

Answers

Getting Started

Is It Proportional?

Analyze each table to determine if the relationship is proportional. If the table represents a proportional relationship, state the constant ratio that exists between corresponding values of the two quantities.

1. A 30-minute television show has 8 minutes of commercials and 22 minutes of the show. A 120-minute television movie has 32 minutes of commercials and 88 minutes of the movie.

Total Program Length (minutes)	Actual Show Length (minutes)	Commercial Length (minutes)
30	22	8
120	88	32

There are 250 boys in 6th grade, and 75 are in the band. There are 200 girls in 6th grade, and 60 are in the band.

6th Grade Class	Total	Band
Boys	250	75
Girls	200	60

1. This does represent a proportional relationship. Equivalent ratios will vary but can be any combination of part-to-part or part-towhole ratios.
2. This does represent a proportional relationship. Equivalent ratios can be

3________________ students in band or

10 total students

________________10 total students

.

3 students in band

2.

Answers

3. C ommuters in McKnight and Mitenridge either drive to work or take public transportation.

Commuters	Drive to Work	Public Transportation to Work
McKnight	175	120
Mitenridge	525	300

4 Of the 250 middle-school boys who have a subscription to *Boys Noise,* 125 access the magazine through the website. Of the 280 middle-school girls who have a subscription to *Girls Rockstar,* 160 access the magazine through the website.

3. This does not represent a proportional relationship.

4. This does not represent a proportional relationship.

LESSON 2:

Answers

Answers

ACTIVITY

2.1

Defining the Constant of Proportionality

In a proportional relationship, the ratio of all *y*-values, or outputs, to their corresponding *x*-values, or inputs, is constant. This specific ratio, $\frac{y}{x}$, is called the **constant of proportionality**. Generally, the variable *k* is used to represent the constant of proportionality.

Let's revisit the television show scenario. This situation represents a proportional relationship.

Total Program Length (minutes)	Actual Show Length (minutes)	Commercial Length (minutes)
30	22	8
120	88	32

1. The input quantities are 30 and 120, representing the total program length in minutes.

The output quantities are 22 and 88, representing the actual show length in minutes.

Answers

ıppose you want to determine the actual lengths of your favorite television shows, ithout commercials, if you know the total program length.

Identify the input and output quantities in this scenario.

1

› determine the length of a program, without commercials, you will need to ultiply the total program length by a constant of proportionality.

Analyze the different ideas for determining the constant of proportionality.

Jeremiah

We want to know the actual show length and we know the total program length, so

$k = \frac{22 \text{ minutes of show}}{30 \text{ minutes of total length}}$

or $k = \frac{11}{15}$.

Keisha

To determine if a proportional relationship exists the order of the ratio doesn't matter, so the constant of proportionality can be

$k = \frac{15}{11}$

or $k = \frac{11}{15}$.

Susan

I think the constant of proportionality is

$k = \frac{22 \text{ minutes of show}}{8 \text{ minutes of commercials}}$,

or $k = \frac{11}{4}$

Jamie

Jeremiah's correct about which numbers to use but he has them mixed up. The constant of proportionality is

$k = \frac{30 \text{ minutes of total length}}{22 \text{ minutes of show}}$, or $k = \frac{15}{11}$.

2. Explain why Susan's solution is incorrect.

2. The quantities being compared are the total time and the length of the actual show. Susan compared the length of the show and the length of the commercials.

3. Explain why Jeremiah is correct but Jamie and Keisha are incorrect.

3. The constant of proportionality is the ratio of the outputs to inputs, so there can only be one solution. The inputs are the combined lengths of the show and commercials and the outputs are the actual lengths of the shows. So Jeremiah used the correct ratio.

Answers

Answers

1a. There are 250 girls enrolled in the school.

ACTIVITY 2.2

1b. There are 390 boys enrolled in the school.

2. Let b equal the number of boys enrolled in the school, and let g equal the number of girls enrolled in the school.

The term *proportional* is used often in the Title IX document. What does this mean for sports at schools?

Answers

The Meaning of the Constant of Proportionality

·deral government passed Title IX, a law requiring educational ·at receive federal funds to provide equitable athletic for boys and girls. One of the requirements for compliance for institutions to provide opportunities for each gender that *al* to their rates of enrollment or show that they are working ·ch opportunities. Similarly, male and female athletes are to ·c scholarship dollars *proportional* to their participation.

·ut the implications of Title IX at Vista Middle School.

·ls for every 6 boys enrolled in Vista Middle School.

ortions for each question. Then, solve each proportion to the unknown value. Use the information from the ratio given.

are 300 boys , enrolled in the school, how many girls are ·oys are enrolled in the school?

b. If there are 325 girls enrolled in the school, how many boys are enrolled in the school?

1

·ables for the quantities that are changing in

2

this situat

NOTES

3. Set up a proportion using your variables for the quantities to the ratio given for the enrollment of girls to boys enrolled in Vista Middle School.

4. Use your proportion.

a. Write an equation to determine the number of girls enrolled at Vista Middle School if you know the number of boys enrolled.

b. What is the constant of proportionality in this situation? Where is the constant of proportionality in the equation?

c. What does the constant of proportionality mean in this problem situation?

3. $\frac{5 \text{ girls}}{6 \text{ boys}} \, 5 \, \frac{g \text{ girls}}{b \text{ boys}}$

4a. $g \, 5 \, \frac{5}{6}b$

4b. $\frac{5}{6}$

4c. The constant of proportionality is the ratio of the number of girls to the number of boys.

5a. $b \; 5 \; \frac{6}{5}g$

5b. Because the input is number of girls and the output is number of boys, the constant of proportionality is $\frac{6}{5}$. This is the numerical coefficient of the variable *b*, the input, in the equation.

5c. The constant of proportionality means that for every girl enrolled in the school, there is $\frac{5}{6}$ of a boy enrolled.

6. The constants of proportionality are reciprocals of each other.

7. No. The constants of proportionality are fractions of people.

5. **Use your proportion.**

   a. **Write an equation to determine the number of boys enrolled at Vista Middle School if you know the number of girls enrolled.**

   b. **What is the constant of proportionality in this situation? Where do you see the constant of proportionality in the equation?**

   c. **What does the constant of proportionality mean in this problem situation?**

6. **What do you notice about the constant of proportionality in each situation?**

7. **Do you think each constant of proportionality makes sense in terms of the problem situation?**

Answers

8a. g 5 65.833. There should be approximately 66 opportunities for girls to participate in athletics.

Answers

Sometimes, the constant of proportionality is not a whole number. The constant of proportionality can also be a decimal or a fraction. When the constant of proportionality involves whole items, like people, it may seem strange to think about the constant of proportionality in terms of a fraction. Instead, you can think of the constant of proportionality as a way to predict outcomes of a situation.

8. Use your equations and the information about Title IX to answer each question.

a. If there are opportunities for 79 boys to participate in athletics, how many opportunities must be available for girls?

Did you use the constant of proportionality for the girls or for the boys? Does it matter which constant of proportionality you use?

b. If there are opportunities for 119 girls to participate in athletics, how many opportunities must be available for boys?

Answers

8b. b 5 142.8. There should be approximately 143 opportunities for boys to participate in athletics.

ELL Tip

Have students read the paragraph which discusses why the constant of proportionality could be represented as a decimal or fraction. Allow students time to share ideas on how a fractional or decimal constant of proportionality could be used to predict outcomes. Have students use examples from their work or create their own examples to support their ideas.

1a. k 5 $\frac{5}{7}$

1b. k 5 $\frac{7}{5}$

ACTIVITY

2.3

Answers

2. Let p equal the number of girls that play sports, and let t equal the total number of girls enrolled in the school.

Middle Scho

ses the number of athletics opportunities provided to each
e actual participation at schools may differ. Also, the same
ırticipate in multiple sports.

e School, 5 out of every 7 girls play sports. The guidance
. Shanahan, and the athletics director, Coach Culpepper, are
ɔorts about the students at Vista
play spor

ıformation each person knows and use the constant of proportionality to
ıs for each situation.

ce Counselor **Athletics Director**

ı knows the Coach Culpepper knows the
-ls in the school number of girls participating in on
ınd she needs sports during a given season, to be
ate the and she needs to be able to expected number
calculate the expected number of total girls in the

1 **each situ**
ermine the constant of proportionality for

ce Counselor **b. Athletics Director**

2 **these situ**
iables for the quantities that are changing in

ionality

3. Use the constants of proportionality to write equations to determine the information needed by each person.

a. Guidance Counselor

b. Athletics Director

In terms of proportionality, Ms. Shanahan could state that the number of girls who play sports is proportional to the number of total girls in the school at a constant rate equal to the constant of proportionality.

4. Write Coach Culpepper's situation using the language of proportionality and include the value for the constant of proportionality.

5. Consider the given equations, where y represents the dependent or output quantity and x represents the independent or input quantity.

$$\frac{y}{x} 5\ k$$

$$\frac{y}{x} 5 \frac{k}{1}$$

$$y\ 5\ kx$$

a. Describe how the first equation represents the constant of proportionality.

b. Explain how the second equation represents proportional relationships.

c. Describe how the first equation was rewritten to create the third equation.

d. Explain the meaning of the constant of proportionality, k, in the third equation.

Answers

3a. $p\ 5\ \frac{5}{7}t$

3b. $t\ 5\ \frac{7}{5}p$

4. The total number of girls in the school is proportional to the number of girls who play sports at a constant rate of $\frac{7}{5}$.

5a. The constant of proportionality is the ratio of the outputs to the inputs. On a graph, x and y are common variables, with x representing the input and y representing the output. So the constant of proportionality would be $k\ 5\ \frac{y}{x}$.

5b. The output value, y, is proportional to the input value, x, at a rate equal to the constant of proportionality, k.

5c. To write the first equation as the second equation, the constant of proportionality was rewritten as a fraction. Then, the means and extremes method was used to create the third equation.

5d. The y-value will increase by k for every one unit the x-value increases.

Answers

6a. The constant of proportionality is 2. The diameter (y) divided by the radius (x) always equals 2.

6. Identify the constant of proportionality in each equation and describe its meaning.

a. *d* = 2*r*, where *d* represents the diameter of a circle and *r* represents the radius of a circle.

b. *P* = 3*s*, where *P* represents the perimeter and *s* represents the sides of an equilateral triangle.

Answers

6b. The constant of proportionality is 3. The perimeter (y) divided by a side length (x) always equals 3.

Answers

1. Let r equal the number of units of reagent, and let w equal the number of units of water in the solution.
2. The equation is $w = \frac{7}{3} r$. The constant of proportionality is $\frac{7}{3}$.

ACTIVITY
2.4

Using the Constant of Proportionality to Solve Problems

A chemist must use a solution that is 30% of reagent and 70% of water for an experiment. A solution is a mixture of two or more liquids. A reagent is a substance used in a chemical reaction to produce other substances.

1. Define variables for the quantities that are changing in this problem situation.

2. Write an equation for the amount of water needed based on the amount of reagent. What is the constant of proportionality?

3. Use your equation from Question 2 to write an equation for the amount of reagent needed based on the amount of water. Explain your reasoning.

In Question 3, you can say that the amount of reagent varies directly as the amount of water in the solution.

4. Use your equations to answer each question.

   a. If the chemist uses 6 liters of reagent, how many liters of water will she need to make her 30% solution?

   b. If the chemist uses 77 milliliters of water, how many milliliters of reagent will she need to make her 30% solution?

5. Write an equation to show that y is directly proportional to x using the constant of proportionality given. Then solve for the unknown value.

   a. k 5 0.7 and y 5 4

   b. k 5 $\frac{3}{11}$ and x 5 9

   c. k 5 5 and x 5 $1\frac{1}{2}$

   d. k 5 $\frac{1}{6}$ and y 5 $3\frac{1}{3}$

Answers

3. The equation is r 5 $\frac{3}{7}w$. When the input and output quantities are interchanged, the new constant of proportionality is the reciprocal of the original constant of proportionality.

4a. The chemist should use 14 L of water.

4b. The chemist should use 33 mL of reagent.

5a. x 5 $5\frac{5}{7}$

5b. y 5 $2\frac{5}{11}$

5c. y 5 $7\frac{1}{2}$

5d. x 5 20

Answers

1. Answers will vary.
2. Answers will vary.
3. Answers will vary.

NOTES

TALK the TALK

Turning the Tables

Consider the equation $y = kx$. Use the value of the constant of proportionality assigned to you to answer the questions. You will present your work to your class.

1. **Write a scenario for a proportional relationship that would be represented by the equation. Clearly define your variables and identify the direction of the proportional relationship.**

2. **Interpret the constant of proportionality in the context of your scenario.**

3. **Write and solve at least 2 questions that could be solved using your equation.**

Assignment Answers

Write

Answers will vary.

Practice

1a. The second ratio is not the same as the fi rst and third ratios, so this is not a proportional relationship.

1b. The ratios are not the same, so this is not a proportional relationship.

Answers for (a)–(d) will vary based on how students defined variables and set up their equations.

2a. Let h be the time in hours, and let b be the number of bricks.

Assignment

Write

Define *constant of proportionality* in your own words. Provide a specific example with your definition.

Remember

If *y* is directly proportional to *x*, the relationship can be represented by the equation *y* = *kx*, where k is the constant of proportionality.

Practice

1. Analyze each table or problem situation to determine if the relationship is proportional. State a constant of proportionality if possible. Show your work.

a.

Girls	Boys
7	14
9	21
11	22

b. A baby blue whale weighed 5520 pounds at birth. After two days, the baby weighed 5710. After 14 days, the baby weighed 8180 pounds.

2. Maya's construction company builds brick houses. The number of bricks her crew installs varies directly with the number of hours they work.

Hours Worked	Bricks Installed
8	1680
7	1470
6	1260

a. Define variables for the quantities that are changing in this problem situation.
b. Analyze the table to determine the constant of proportionality.
c. What does the constant of proportionality mean in this situation?
d. Write an equation to show the relationship between the number of hours worked, the number of bricks installed, and the constant of proportionality.
e. Use your equation to determine how many bricks Maya's crew can install in 5.5 hours.
f. Use your equation to determine how many hours it will take Maya's crew to install 840 bricks.

3. Given a value for the input variable, *x*, and the output variable, *y*, calculate the constant of proportionality.

a. *x* 5 21 and *y* 5 6

b. *x* 5 60 and *y* 5 18

c. *x* 5 2 $\frac{2}{5}$ and *y* 5 7 $\frac{1}{2}$

d. *x* 5 4 $\frac{8}{11}$ and *y* 5 3 $\frac{6}{11}$

2b. This relationship is proportional, because the ratio of bricks installed to hours worked is the constant 210. The constant of proportionality of bricks installed to hours worked is 210.

2c. The constant of proportionality represents the number of bricks Maya's crew can install in one hour. Each hour, Maya installed 210 bricks.

2d. b 5 210h

2e. Maya's crew can install 1155 bricks in 5.5 hours.

2f. It will take Maya's crew 4 hours to install 840 bricks.

3a. k 5 $\frac{2}{7}$

3b. k 5 $\frac{3}{10}$

3c. k 5 $\frac{25}{8}$ 5 $3\frac{1}{8}$

3d. k 5 $\frac{3}{4}$