PUBLIC SCHOOLS OF EDISON TOWNSHIP

OFFICE OF CURRICULUM AND INSTRUCTION

Calculus III (Multivariable Calculus)

Length of Course:	Term
Elective/Required:	Required
Schools:	High School
Eligibility:	Grade 12
Credit Value:	5 Credits
Date Approved:	August 17, 2021

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STATEMENT OF PURPOSE

Calculus III is a course designed for those students who have completed the AP Calculus AB/BC. This course carries dual enrollment credits and students are able to use it to satisfy the Edison Township School District's four year requirement in mathematics.

The course covers the topics that are typically covered in a college Multivariable Calculus course that follows two full semesters worth of study of Calculus of a Single Variable. Students will study the three-dimensional coordinate system, vector calculus, vector-valued functions, multivariable functions, partial derivatives, and multiple integrals.

COURSE OBJECTIVES

Students will be able to:

- Compute arithmetical operations with vectors, as well as vector operations
- Develop models and equations for figures in the three-dimensional plane as well as parametric surfaces
- Use calculus to graph and solve vector-valued functions
- Apply calculus to physics problems
- Use calculus to determine curvature as well as tangent and normal vectors
- Evaluate and graph functions of several variables
- Compute partial derivatives
- Use partial derivatives to find equations to tangent planes and linear approximations
- Use the Chain Rule for partial derivatives
- Compute gradients and directional derivatives
- Find the extrema of several variables and optimize functions using Lagrange multipliers
- Compute double integrals and use them to find area, volume, and center of mass
- Work with double integrals in polar coordinates
- Compute triple integrals and work with cylindrical and spherical coordinates
- Compute multiple integrals using change of variables
- Sketch vector fields and conservative vector fields
- Compute line integrals and utilize Green's Theorem as it pertains to them
- Determine curl and use partial derivatives to find divergence
- Compute surface integrals
- Apply the Divergence Theorem and Stokes' Theorem

Pacing Guide for Units Of Study

Unit / Section / Topic	# of Days	Notes
Review of AP Calculus Concepts: Chapters 2, 3, 4, 6, and 7	**23 days**	
(Various sections) Conceptual Derivative Concepts **2.3, 2.4, 2.5, 2.6:** Review of Differentiation Rules **2.7** Related Rates **3.7** Optimization **Review and Quiz #1** **4.2, 4.3, 4.4**: Conceptual Integration Concepts **4.5** Integration by U-Substitution **Review and Quiz #2** **7.1** Integration by Parts **7.2** Trig Integrals **7.4** Integration by Partial Fractions **Practice/Review/Test : AP Concepts**	2 2 1 1 **2** 3 2 **2** 2 2 2 **2**	
Chapter 12: Vectors and the Geometry of Space	**26 days**	

12.1 3D Coordinates **12.2** Vectors in the Plane, Vectors in Space, applications **Review and Quiz #1** **12.3** Dot Product and Applications **12.4** Cross Product and Applications **Review and Quiz #2** **Practice / Test (12.1-12.4)** **12.5** Lines and Planes in Space **Review and Quiz #3** **12.6** Surfaces in Space **Practice/Review/Test (Ch 12)**	2 3 **2** 2 2 **2** **2*** 4 **2** 3 **2**	*Estimated End MP1 after Test #1 (4 Quiz / 2 Test)
Chapter 13: Vector Functions	**21 days**	
(**10.1-10.2** Parametric Equations in 2D) **13.1** Vector-Valued Functions & Parametric Surfaces **Review and Quiz** **13.2** The Calculus of Vector-Valued Functions **13.3** (Arc Length) Curvature, Tangent and Normal Vectors **Review and Quiz** **13.4** (?)Motion in Space (not sure what this refers to?) **Practice/ Review / Test (Ch 13)**	2 4 **2** 3 3 **2** 3 **2**	
Chapter 14: Functions of Several Variables & Partial Derivatives	**32 days**	
14.1 Functions of Several Variables **14.2** Limits and Continuity **14. 3** Partial Differentiation **Review and Quiz** **14.4** Tangent Planes and Linear Approximations **14.5** The Chain Rule / Implicit Differentiation **Practice / Review / Test (14.1-14.5)** **14.6** Gradient and Directional Derivatives **14.7** Extrema of Functions of Several Variables **Review and Quiz** **14.8** Lagrange Multipliers **Practice / Review / Test (14.6-14.8)**	3 2 3 **2*** 3 3 **2** 4 3 **2** 3 2	*Estimated End MP2 After 1st Quiz (4 quiz / 2 test)
Chapter 15: Multiple Integrals	**31 days**	

15.1 Double Integrals over Rectangular Regions **15.2** Double Integrals over General Regions **15.4** Applications of Integration: Center of Mass **Review and Quiz** (**10.3-10.4** Polar Coordinates in 2D) **15.3** Double Integrals in Polar Coordinates **Review and Quiz** **15.5** Surface Area **15.6** Triple Integrals **15.7** Cylindrical Coordinates **15.8** Spherical Coordinates **15.9** Change of Variables in Multiple Integrals **Practice / Review / Test**	3 3 2 **2** 3 2 **2*** 3 3 3 3 **2**	*Estimated End MP 3 after 2nd Quiz (3 Quiz / 2 test)
Chapter 16: Vector Calculus	**30 days**	
16.1 Vector Fields **16.2** Line Integrals **16.3** Independence of Path & Conservative Vector Fields **Review and Quiz** **16.4** Green's Theorem **16.5** Curl and Divergence **16.6** Parametric Surfaces and their Areas **16.7** Surface Integrals **Review and Quiz** **16.8** Stokes Theorem **16.9** The Divergence Theorem **Practice / Review / Test**	3 3 2 **2** 3 4 3 **2** 3 3 2*	*Estimated End MP 4 after Test Quiz 2-3quiz / 2 Test
TOTAL DAYS OF INSTRUCTION	**163 days**	

Unit 1: Chapters 2, 4, 6, 7 - Fundamental Concepts of AP Calculus

Unit Objectives/Conceptual Understandings: *Students will be able to:* ● *Understand the overall conceptual connections between functions and their derivatives, integration and the area under a curve* ● *Use and apply all basic rules of differentiation* ● *Compute integrals using various techniques*
Unit Assessment: *Teacher generated assessments (formative and summative), short graded classworks, webassign online practice*

Core Content Objectives		Instructional Action	
Concepts Student will know:	**Skills** Students will be able to:	**Activities/Strategies** Technology Implementation Interdisciplinary Connections	**Assessment Check Points**
● The derivatives for the following functions: ○ All 6 basic trigonometric ○ Exponential ○ Logarithmic ● The following derivatives rules: ○ Power ○ Product ○ Quotient ○ Chain ○ Implicit ○ Logarithmic ● The following integration techniques: ○ U-substitution ○ By Parts ○ Partial Fractions ○ Trigonometric Integration ● The following theorems: ○ Intermediate Value Theorem ○ Mean-Value Theorem ○ Fundamental Theorem of Calculus (Parts 1 and 2) ● The following concepts: ○ Numerical, analytical, and graphical relationship between f(x), f'(x), f''(x) ○ 1st and 2nd derivative tests ○ Related Rates	● Apply the power, product, quotient, and chain rules to calculate the derivatives of polynomial, rational, trigonometric, exponential, and logarithmic functions ● Find derivatives using implicit differentiation ● Find derivatives using logarithmic differentiation ● Apply the methods of u-substitution, by parts, & partial fractions to integrate functions ● Apply methods of trigonometric integration to integrate functions involving higher powers of sine and cosine or tangent and secant ● Apply the Intermediate Value theorem ● Apply the Mean Value theorem ● Apply both parts of the Fundamental Theorem of Calculus ● Use the connections between f, f', and f'' to analyze a function's	● Students match an f(x) graph handed to them with an f'(x) graph to be found on desk to find seats day 1 ● Use WS "Discussion Questions" to review conceptual connections in differential calculus ● Use "Questions for Thought (QFT)" to engage in more conceptual review. ● Utilize the "Text" questions in the Instructor's Manual to formatively assess student understanding of fundamentals after reading an assigned section ● Establish student groups early on to build class collaboration ● Use questions that focus on a more comprehensive/ summative conceptual review as opposed to section by section skills ● Use Desmos Card sort	● "Graded classworks" for quick assessment of topics ● HW assignments: ○ Teacher generated problem sets ○ "Webassign" problem sets ○ Textbook assignments: (odds for solutions available/evens for no solutions available) ● Assessment Schedule: ○ Graded CW's for various topics ○ Quiz on derivative rules ○ Quiz after U-substitution ○ Test on Advanced Integration Techniques (Chapter 7)

<table>
<tr>
<td>○ Optimization
○ Integrals as area under curve
○ Integrals as accumulation function
○ Properties of definite integrals
○ Connection between derivatives and integrals</td>
<td>slope and concavity
● Solve related rates problems
● Solve optimization problems
● Use definite integrals to calculate the area under a curve
● Analyze an accumulation function as it relates to graphs and real-world examples
● Apply the properties of definite integrals</td>
<td>activities</td>
<td></td>
</tr>
<tr>
<td colspan="2">Resources
● Textbook: Calculus, 9th Edition, Stewart, Clegg, and Watson
● cengage.com (Publisher Site for Instructor resources)
● webassign.net (For problems and exercises- included as part of textbook adoption)
● https://assessment.cengage.com/Instructor/TestGenerator.aspx
● geogebra.org/3d (to create 3 dimensional surfaces, etc)
● desmos.com</td>
<td colspan="2">Instructional Adjustments
● Underlying algebraic misunderstandings can lead to incorrectly calculated derivatives. Be sure to point out when a mistake is based on incorrect application of a derivative rule versus algebra: ie, believing $\frac{x}{x+5} = 1 + \frac{x}{5}$ or that $(a+2)^2 = a^2 + b^2$
● Related Rates and Optimizations problems are notoriously difficult for students but will both come back during the multivariable functions unit and so their overall underlying concepts should be reviewed
● Encourage problem solving strategies that will help them to organize the given information as well as what they need to find
● Students often confuse the Extreme, Intermediate, and Mean Value Theorems and what in fact they are saying: be sure to review and emphasize both the algebraic as well as graphical significance of all three and the conditions that need to be met for them to hold
● Students coming directly from AB Calculus will not have seen integration by parts, partial fractions, or trig integrals, so they will need to be instructed as if for the first time</td>
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Unit 2: Chapter 12 - Vectors and the Geometry of Space

Unit Objectives/Conceptual Understandings: *Students will be able to:*

- *Understand how vectors can be used and interact in two and three dimensional space*
- *Visualize functions of two variables as surfaces in space*

Unit Assessment: *Teacher generated assessments (formative and summative), short graded classworks, webassign online practice*

Core Content Objectives		Instructional Action	
Concepts Student will know:	**Skills** Students will be able to:	**Activities/Strategies** Technology Implementation Interdisciplinary Connections	**Assessment Check Points**
• All critical notation involved with vectors in 2 and 3 dimensional space • The graphical representations of simple planes and spheres • The algebraic and geometric formulations of the dot product • The relationship between the sign of a•b and the angle between the two vectors • The meaning and significance of orthogonal vectors, as well as vector and scalar projections • The algebraic and geometric formulations of the cross product • How to use vector and parametric equations to represent points, lines, and planes in space given various pieces of information	• Plot points in 3 dimensional space • Identify the equation of a plane parallel to one of the axes and of a sphere • Find the length of and a unit vector for any vector • Write vectors in component form of a vector in ai+bj+ck form. • Compute vector addition and scalar multiplication and interpret these computations geometrically • Calculate the dot product of two vectors and apply the properties of the dot product • Find the angle between two vectors using dot product • Find the direction cosines and the direction angles of a vector • Calculate the projection of one vector onto another and represent it geometrically • Find the cross product and triple scalar product of three vectors in space. • Apply the cross product to find volume, torque, and area of a parallelogram • Determine if two vectors are	• Have students read parts of sections ahead of lessons and use the “question” feature on google classroom or a short google form to quickly assess their understanding • Use 12.1 Group Work 1 as a matching activity to have students visualize 3D surfaces • Use geogebra 3D graphing technology to help generate surfaces and manipulate them to show functions from several viewpoints in space • Use desmos card sorting activities to match equations with surfaces and two dimensional traces	• “Graded classworks” for quick assessment of topics • HW assignments: ○ Teacher generated problem sets ○ “Webassign” problem sets ○ Textbook assignments: (odds for solutions available/evens for no solutions available) • Assessment Schedule: ○ Quiz after 12.1-12.2 ○ Quiz after 12.3-12.4 ○ Test 12.1-12.4 ○ Quiz after 12.5 ○ Test entire Unit

	orthogonal • Apply vectors to topics such as circles, force, and velocity • Write parametric and vector equations for lines in space • Write a linear equation to represent a plane in space • Find the distance between points, planes, and lines using vectors		

Resources

- **Textbook:** Calculus, 9th Edition, Stewart, Clegg, and Watson
- cengage.com (Publisher Site for Instructor resources)
- webassign.net (For problems and exercises- included as part of textbook adoption)
- https://assessment.cengage.com/Instructor/TestGenerator.aspx
- geogebra.org/3d (to create 3 dimensional surfaces, etc)
- desmos.com

Instructional Adjustments

- Emphasize the importance that the basic calculations with vectors will have in their study of functions of several variables
- Make the connection that this chapter is very much like Algebra II but for three-dimensions. Connect the equations/shapes of surfaces like paraboloids and spheres to those of two dimensional parabolas and circles
- Focus mostly on students' ability to recognize and match computer generated surfaces with equations as opposed to them drawing them by hand
- Students will need to be shown HOW each of the equations for lines and planes are derived. They will need to be reminded which answers will be unique and which will have multiple ways to represent based on method of solving so that they are not discouraged if their answers check out differently than written solutions
- Most students have not seen algebraic systems that have not enough or an extra variable for a unique solution. Be sure to address how to handle a linear system with 3 equations and only 2 unknowns
- As you discuss surfaces in space for the first time, and the students visualize cross sections, be sure to emphasize that these ideas will come back in their study of level curves

Unit 3: Chapter 13 - Vector Functions

Unit Objectives/Conceptual Understandings: *Students will be able to:*
● *Understand vector functions and how they are used to describe curves and surfaces in space and the motion of objects through space*
Unit Assessment: *Teacher generated assessments (formative and summative), short graded classworks, webassign online practice*

Core Content Objectives		**Instructional Action**	
Concepts Student will know:	**Skills** Students will be able to:	**Activities/Strategies** Technology Implementation Interdisciplinary Connections	**Assessment Check Points**
● The connection between space curves and ranges of vector functions ● How to match vector equations with their curves ● Parametrizations of curves in space are not unique ● How to visualize curves in three dimensions ● The vector derivative and the unit tangent vector ● The definition of the line tangent to a space curve ● The geometric interpretation of the tangent vector and smooth curves ● Integrals of vector functions ● The arc length and curvature formulas ● The independence of arc length and parametrization ● The geometric definition of curvature ● The TNB frame ● Definitions of velocity and acceleration as vector functions ● How to derive velocity from acceleration and position from velocity ● Tangential and normal components of acceleration	● Sketch and find the domain and limit of vector functions ● Draw projections of curves onto coordinate planes ● Find vector and parametric equations ● Match parametric equations with their graphs ● Find equations of planes containing given curves and vector equations ● Graph curves with given vector and parametric equations ● Find vector functions that represent the intersection of two surfaces ● Sketch plane curves with given vector equations ● Sketch position vectors and tangent vectors ● Find derivatives of vector functions ● Find unit tangent vectors ● Find parametric equations for tangent lines to curves with given parametric equations ● Evaluate definite and indefinite integrals of vector functions ● Find the length of a curve	● Desmos activities ● Applied/discovery projects from TXBK: Kepler's Laws ● Chapter Review and Problems Plus question sets	● "Graded classworks" for quick assessment of topics ● HW assignments: ○ Teacher generated problem sets ○ "Webassign" problem sets ○ Textbook assignments: (odds for solutions available/evens for no solutions available) ● Assessment Schedule: ○ Graded CW's for various topics ○ Quiz 13.1 ○ Quiz 13.2-13.3 ○ Test 13.1-13.4

	• Find torsion of a curve • Find the unit tangent and unit normal vectors • Find curvature of functions • Find equations of the normal and osculating planes of curves • Find velocity, speed, and acceleration of a particle • Sketch the path of a particle • Draw velocity and acceleration vectors • Analyze and solve projectile word problems • Find tangential and normal components of an acceleration vector		

Resources • **Textbook:** Calculus, 9th Edition, Stewart, Clegg, and Watson • cengage.com (Publisher Site for Instructor resources) • webassign.net (For problems and exercises- included as part of textbook adoption) • https://assessment.cengage.com/Instructor/TestGenerator.aspx • geogebra.org/3d (to create 3 dimensional surfaces, etc) • desmos.com	**Instructional Adjustments** • Differentiated instruction/activities • Error analysis • Word problem practice • Workshop/discussion exercises suggested in the Instructor's Guide

Unit 4: Chapter 14 - Functions of Several Variables and Partial Derivatives

Unit Objectives/Conceptual Understandings: *Students will be able to:*

- Understand how functions of more than one variable can be represented verbally, numerically, algebraically, and visually
- Extend the concepts of differential calculus to multivariable functions by finding partial derivatives and applying them to find the extrema of a multivariable function, with and without restrictions

Unit Assessment: *Teacher generated assessments (formative and summative), short graded classworks, webassign online practice*

Core Content Objectives		Instructional Action	
Concepts Student will know:	**Skills** Students will be able to:	**Activities/Strategies** Technology Implementation Interdisciplinary Connections	**Assessment Check Points**
● That most concepts including domain and range are an extension of functions of a single variable ● The properties of limits and the test for continuity of multivariable functions ● The meanings of all notations for partial derivatives (1st and 2nd order) ● How to find partial derivatives of first and second order ● What the partial derivatives and "mixed partials" represent graphically ● How to extend the chain rule and implicit differentiation formulas to functions containing 2,3,4,or more independent variables ● The following conceptual and graphical meaning behind the following topics: ○ Tangent vectors & linear approximations ○ Differentials ○ directional derivatives, gradient vectors, and tangent planes ● How to apply the 2nd partials test to find extrema of a multivariable function ● Lagrange's method for maximizing or minimizing a general function f(x,y,z) with a constraint of the form g(x,y,z)=k	● Find and describe the domain and range of a function of two variables ● Visualize the graph of a function of two variables ● Identify and create level curves for a function of two variables ● Match a function, its level curves, and its graph ● Find the limit of a function f(x,y) as (x,y) approaches (a,b) ● Show that a limit does not exist ● test a function of two variables for continuity ● Calculate the partial derivative for a function of two or more variables ● Calculate higher order partial derivatives for a function of two or more variables ● Find an equation of the tangent plane to a given surface ● Explain why a function is differentiable at a given point ● Find the linearization of a function at a given point ● Find dz/dt as well as $\partial z/\partial x$ and	● Use geogebra to show how certain functions look from different perspectives ● Also use geogebra to create functions and level curves and create multiple matching type practice exercises for students to match function to level curve, function to surface, and surface to level curve ● Use computer generated graphs to highlight the graphical significance of f_x and f_y	● "Graded classworks" for quick assessment of topics ● HW assignments: ○ Teacher generated problem sets ○ "Webassign" problem sets ○ Textbook assignments: (odds for solutions available/evens for no solutions available) ● Assessment Schedule: ○ Quiz after 14.1-14.3 ○ Test 14.1-14.5 ○ Quiz after 14.6-14.7 ○ Test entire Unit

<table>
<tr>
<td></td>
<td>$\partial z/\partial y$ using the chain rule for functions of more than one variable
• Apply the chain rule to solve related rate problems
• Find the gradient vector of a function at a point
• Find the directional derivative for a function at a point
• Find the equations of the tangent plane and normal lines to a given surface at a specified point
• Find the all critical points of a function and classify them as local maximums, minimums, or saddle points
• Identify all absolute extrema for a function over a closed domain
• Use Lagrange multipliers to find the extreme values of a function subject to a given constraint</td>
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<tr>
<td colspan="2">Resources
• Textbook: Calculus, 9th Edition, Stewart, Clegg, and Watson
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• webassign.net (For problems and exercises- included as part of textbook adoption)
• https://assessment.cengage.com/Instructor/TestGenerator.aspx
• geogebra.org/3d (to create 3 dimensional surfaces, etc)
• desmos.com</td>
<td colspan="2">Instructional Adjustments
• When looking at the graphical representation of a function of two variables, students can often forget to take the perspective from which the graph is being shown into account, so they should be reminded to always do this: use geogebra to show how certain functions look from different perspectives
• Students often underestimate the distinction between the symbol dz/dt and $\partial z/\partial x$, which can cause issues with the chain rule and implicit differentiation
• Students can often get stuck on the very open-ended algebraic approaches to solving the systems involved in maximization problems. Explore enough examples to prepare students for multiple approaches</td>
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Unit 5: Chapter 15 - Multiple Integrals

Unit Objectives/Conceptual Understandings: *Students will be able to:*
- *Expand the idea of a definite integral to double and triple integrals of 2 or 3 variables*
- *Compute volumes, masses, and centroids of general regions*
- *Use double integrals to calculate probabilities involving 2 random variables*
- *Use polar coordinates to compute double integrals*
- *Apply cylindrical and spherical coordinates to simplify computations of triple integrals over commonly occurring solid regions*

Unit Assessment: *Teacher generated assessments (formative and summative), short graded classworks, webassign online practice*

Core Content Objectives		Instructional Action	
Concepts Student will know:	**Skills** Students will be able to:	**Activities/Strategies** Technology Implementation Interdisciplinary Connections	**Assessment Check Points**
• The definition and properties of the double integral, including the analogy between single and double integration • Notation of a double integral for a positive function f(x,y) over a rectangle [a,b] x [c,d] • Volume and average value applications of double integrals • The statement of Fubini's Theorem and how it makes computations easier • The geometric meaning of Fubini's Theorem: slicing the area in two different ways • The geometric interpretation of double integrals • How to set up the limits of double integrals, given a region over which to integrate • How to change the order of integration • The definition of a polar rectangle: what it looks like, and its differential area *r dr dθ* • The idea that some integrals are simpler to compute in polar coordinates • Integration over general polar regions	• Sketch and find the volume of a solid • Use Riemann sums • Analyze contour maps • Evaluate double integrals • Set up and evaluate iterated integrals • Express double integrals as iterated integrals • Change the order of integration • Decide whether to use polar or rectangular coordinates to write a double integral as an iterated integral • Change to polar coordinates to evaluate an integral • Use a double integral to find the area of a given region • Use polar coordinates to find the volume of a solid • Express a double integral as a single integral • Analyze and solve word problems	• Desmos activities • Applied/Discovery projects from TXBK: Volume of Hyperspheres, The Intersection of Three Cylinders, Roller Derby • Chapter Review and Problems Plus question sets	• "Graded classworks" for quick assessment of topics • HW assignments: ○ Teacher generated problem sets ○ "Webassign" problem sets ○ Textbook assignments: (odds for solutions available/evens for no solutions available) • Assessment Schedule: ○ Graded CW's for various topics ○ Quiz after 15.1-15.2, 15.4 ○ Quiz after 15.3 ○ Test 15.6-15.9

• Density, mass, and centers of mass • The derivation of the formula for the area used to approximate the surface area over a small region • Various techniques and difficulties associated with surface area integrals • The basic definition of a triple integral • The various types of volume domain, and how to set up the volume integral based on each of them • How to change the order of integration in triple integrals • The cylindrical coordinate system as an extension of polar coordinates in R^2 • The basic shapes of cylindrical solids • The idea that the cylindrical coordinate system can be used to simplify equations and volume integrals of certain three--dimensional surfaces and solids • The geometry of the spherical coordinate system • The basic shapes of solids in spherical coordinates • The idea that the spherical coordinate system can be used to simplify equations and volume integrals of certain three--dimensional surfaces and solids • Reason for change of variables: to reduce a complicated multiple integration problem to a simpler integral or an integral over a simpler region in the new variables • What happens to area over a change in variables: The role of the Jacobian • Various methods to construct a change of variables	involving applications of double integrals • Find surface area • Evaluate triple integrals • Evaluate iterated triple integrals • Express a triple integral as an iterated integral • Use a triple integral to find the volume of a solid • Plot points with cylindrical or spherical coordinates • Convert between cylindrical or spherical and rectangular coordinates • Identify and describe surfaces given in polar, spherical, and rectangular forms • Write equations in cylindrical or spherical coordinates • Apply knowledge of cylindrical or spherical coordinates to evaluating triple integrals and finding the volume of a solid • Evaluate triple integrals by changing to cylindrical or spherical coordinates • Determine which situations are appropriate to use cylindrical or spherical coordinates • Match transformations with given images • Find the image of a set under a given transformation • Find equations of transformations • Find the Jacobian of a transformation • Use transformations to evaluate integrals • Evaluate integrals by appropriately changing variables		

Resources	Instructional Adjustments
• **Textbook:** Calculus, 9th Edition, Stewart, Clegg, and Watson • cengage.com (Publisher Site for Instructor resources) • webassign.net (For problems and exercises- included as part of textbook adoption) • https://assessment.cengage.com/Instructor/TestGenerator.aspx • geogebra.org/3d (to create 3 dimensional surfaces, etc) • desmos.com	• Differentiated instruction/activities • Error analysis • Word problem practice • Workshop/discussion exercises suggested in the Instructor's Guide

Unit 6: Chapter 16 - Vector Calculus

Unit Objectives/Conceptual Understandings: *Students will be able to:* ● *Understand the Calculus of vector fields* ● *Define line and surface integrals* ● *Relate line and surface integrals to single, double, and triple integrals using Green's Theorem, Stokes' Theorem, and the Divergence Theorem*
Unit Assessment: *Teacher generated assessments (formative and summative), short graded classworks, webassign online practice*

Core Content Objectives		**Instructional Action**	
Concepts Student will know:	**Skills** Students will be able to:	**Activities/Strategies** Technology Implementation Interdisciplinary Connections	**Assessment Check Points**
● Two-- and three--dimensional vector fields ● Vector fields can either be drawn "scaled," so that the lengths of the vectors are proportional to their magnitudes and the longest vectors in the field have a specified length, or "unscaled," so that the vectors appear at their true lengths ● Gradient fields in R^2 and R^3, and their relationships to level curves and surfaces ● The meaning of the line integral of a scalar function f(x,y) along a curve C ● Vector fields and work ● Path independence under suitable conditions ● The Law of Conservation of Energy ● The statement of Green's Theorem over a region with a boundary curve ● The extension of Green's Theorem to domains with holes ● The importance of Green's Theorem, in that it allows us to replace a difficult line integration by an easier area integration, or a difficult area integration by an easier	● Sketch vector fields ● Match vector fields with given plots ● Find and sketch the gradient vector field of a function ● Match functions with given plots of their gradient vector fields ● Plot a gradient vector field together with a contour map of a function and explain how they are related ● Evaluate a line integral given a plane curve, space curve, or vector function ● Analyze line integrals using vector fields ● Determine if a vector function is a conservative vector field ● Apply the Fundamental Theorem for line integrals ● Evaluate line integrals by using Green's Theorem ● Find the curl and divergence of vector fields ● Determine whether points lie on a given surface	● Desmos activities ● Chapter Review and Problems Plus question sets	● "Graded classworks" for quick assessment of topics ● HW assignments: ○ Teacher generated problem sets ○ "Webassign" problem sets ○ Textbook assignments: (odds for solutions available/evens for no solutions available) ● Assessment Schedule: ○ Graded CW's for various topics ○ Quiz after 16.1-16.3 ○ Quiz after 16.4-16.7 ○ Test 16.8-16.9

line integration ● The definition of curl ● If F has continuous partial derivatives, F is conservative if and only if the curl of F = 0 ● Physical interpretations of curl and divergence ● Parametric surfaces and the role of gridlines in studying these surfaces ● How the form and/or symmetry of a surface helps one in choosing a parametrization ● Differentiability and tangent planes to parametric surfaces ● The definition of the surface integral of a scalar function f(x,y,z) viewed as an extension of the surface area integral ● The intuitive idea of an oriented surface with orientation given by a unit normal vector ● The concept of positive orientation ● The surface integral of a vector field over an oriented surface ● The statement of Stokes' Theorem ● The connection between the curl and the circulation of a velocity field ● The meaning of a simple closed solid region R and its boundary surface ● A careful statement of the Divergence Theorem	● Identify a surface with a given vector equation ● Match equations with given graphs ● Find a parametric representation/equations for given surfaces ● Find an equation of the tangent plane to a given parametric surface ● Find the area of a surface ● Evaluate surface integrals ● Find the flux of a vector field ● Use Stokes' Theorem ● Use the Divergence Theorem		

Resources ● **Textbook:** Calculus, 9th Edition, Stewart, Clegg, and Watson ● cengage.com (Publisher Site for Instructor resources) ● webassign.net (For problems and exercises- included as part of textbook adoption) ● https://assessment.cengage.com/Instructor/TestGenerator.aspx ● geogebra.org/3d (to create 3 dimensional surfaces, etc) ● desmos.com	**Instructional Adjustments** ● Differentiated instruction/activities ● Error analysis ● Word problem practice ● Workshop/discussion exercises suggested in the Instructor's Guide