LAUNCH -- Mathematics for Data Science

Curriculum/Content Area: Mathematics	**Course Length:** 2 Terms
Course Title: Mathematics for Data Science	**Date last reviewed:** 2021
Prerequisites: Honors Pre-Calculus or Pre-Calculus & Trigonometry	**Board approval date:** February 2021

Secondary Resources and Teacher Reference Materials:

- Lial, Greenwell and Ritchey (2016). Finite Mathematics and Calculus with Applications, 10th Ed. New York, NY: Pearson [ISBN-139780133981070].
- 3Blue1Brown YouTube Channel / Coursera Mathematics for Machine Learning Specialization
- Free Software Tools and Learning Platforms (e.g. Python, Dataquest, DataCamp, etc.)

Desired Results

Course Description and Purpose Mathematics is the underlying basis of many aspects of data science. In this course, students will experience authentic and effective ways to work with data through cross cutting concepts that can be applied to multiple career fields. This course will provide students with the mathematical foundation needed to understand and deploy the algorithms that drive many of the important data science tools and techniques related to tasks that require optimization, approximation, prediction, classification, and recommendation. The Python programming language will be used to assist with the computationally intensive techniques that will be explored in the course. The learning in this course may be an alternate or extension to Advanced Placement Calculus. By the end of this course, students will have acquired the prerequisite mathematical knowledge to take more advanced courses in data science.

Enduring Understandings:	Essential Questions:
Mathematicians make sense of problems and persevere in solving them.	a. How do we as mathematicians analyze the problem in order to choose the best strategy(ies) or resource to make sense of the problem? b. How do we as mathematicians persevere in solving problems?
Mathematicians attend to precision.	How do we as mathematicians know if we fully & accurately answered the problem and does the results make sense in the context of the problem?
Mathematicians reason abstractly and quantitatively.	How do we as mathematicians make sense of quantities and situations symbolically?
Mathematicians construct viable arguments and critique the reasoning of others.	a. How can we as mathematicians justify our answer(s)? b. How can we as mathematicians evaluate and question whether a mathematical argument is accurate?
Mathematicians model with mathematics.	a. What model(s) can we as mathematicians use to solve a problem? b. How can we as mathematicians determine an effective model to use to solve a problem?
Mathematicians use appropriate tools	What tools are available and efficient for us as mathematicians to use

strategically.	while solving a problem?
Mathematicians look for and make use of structure	How can we as mathematicians use and apply patterns and structures to solve problems?
Mathematicians look for and express regularity in repeated reasoning.	How can we as mathematicians create and apply generalizations from repeated reasoning?

PRIORITY STANDARDS	**MEANING-** *(The Priority Standards help students construct understanding of…)*
The Mathematics for Data Science Course Skills Priority Standards are distinct skills that are integrated throughout the course and derived from Elmbrook Mathematical Priority Standards & Progressions, Advanced Placement Statistics (APS), Advanced Placement Calculus (APC) and Advanced Placement Computer Science Principles (APCS). These standards ensure our Elmbrook Scholars learn to think and act like data science modelers and problem solvers, and are authentically integrated in each unit through the instructional approach of problem-based, experiential learning.	
➔ APS1.A & APC1.A- SELECTING STATISTICAL METHODS & IMPLEMENTING MATHEMATICAL PROCESSES: Identify the question to be answered or the problem to be solved. ➔ APS1.B & APC1.B- SELECTING STATISTICAL METHODS & IMPLEMENTING MATHEMATICAL PROCESSES: Identify key and relevant information to answer a question or solve a problem	1. Defining the Data Science/Mathematical Problem and Identifying What Type of Problem It Is: *Identifying the problem type (e.g. Prediction, Classification, Optimization, Approximation, Recommendation, etc.) is critical in determining the proper algorithms to use to solve the problem.*
➔ APCS 1B- COMPUTATIONAL SOLUTION DESIGN: Determine and design an appropriate method or approach to achieve a purpose. ➔ APCS2.B- ALGORITHMS AND PROGRAM DEVELOPMENT: Implement and apply an algorithm ➔ APC1.D-IMPLEMENTING MATHEMATICAL PROCESSES: Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.	2. Selecting the Proper Data Science/Mathematical Methods and Algorithms: *The choice of method/algorithm is determined by the type of problem you are trying to solve.* 3. Building the Mathematical Foundation Needed to Obtain Data Driven Solutions: *The effective deployment of the algorithms that drive data science tools and techniques requires an understanding of mathematics in the areas of Linear Algebra, Calculus, Probability and Statistics.*
➔ APCS5.A- COMPUTING INNOVATIONS: Explain how computing systems work. ➔ APCS5.B- COMPUTING INNOVATIONS: Explain how knowledge can be generated from data ➔ APCS5.C- COMPUTING INNOVATIONS:	4. Using State of the Art Data Science Software: *By using the open-source free Python Programming language students will be able to work with large data sets and perform computationally intensive tasks.*

Describe the impact of computing innovation. ➔ APCS5.D- COMPUTING INNOVATIONS: Describe the impact of gathering data	
➔ APCS5.E- COMPUTING INNOVATIONS: Evaluate the use of computing based on legal and ethical factors ➔ APC3.E- JUSTIFICATION: Provide reasons or rationales for solutions and conclusions. ➔ APC3.F- JUSTIFICATION: Explain the meaning of mathematical solutions in context ➔ APC3.G- JUSTIFICATION: Confirm that solutions are accurate and appropriate ➔ APS4.A: STATISTICAL ARGUMENTATION: Interpret statistical calculations and findings to assign meaning or assess a claim	5. Explaining Results: *By interpreting the results of a mathematical procedure or data science model the data driven analysis can be used to inform decisions and guide action.* 6. Incorporating a Process Approach to Data Science Problems: *In addition to building models and deploying algorithms, necessary assumptions will be validated, proper terminology and notation will be used, and the accuracy of results and goodness of fit will be evaluated.*

#1 LINEAR ALGEBRA

Essential Unit Questions

1. How do we as mathematicians analyze the problem in order to choose the best strategy(ies) or resource to make sense of the problem?
2. How do we as mathematicians make sense of quantities and situations symbolically?

Guiding Content Questions

1. How can higher-order linear systems be solved?
2. How can vectors be used to represent objects that move around in space?
3. How can matrices be used to represent objects that operate on vectors?
4. How can matrices be used to make linear mappings?
5. How can eigenvalues and eigenvectors be applied to data problems?

Learning Targets:

- I can solve systems of linear equations usingthe Row-Echelon and Gauss-Jordan methods.
- I can manipulate matrices using addition, subtractionand multiplication.
- I can calculate and apply the inverse of matrices.
- I can perform operations with vectors.
- I can calculateinner products and dot products.
- I can solve application problems by changing the basis.
- I can solve simultaneous equation problems.
- I can perform matrix transformations and composite matrix transformations.
- I can use Gaussian elimination to find the inverse of a matrix.

- I can use determinants to find inverses.
- I can use matrices to make transformations.
- I can solve linear equations using the inverse matrix.
- I can do a transformation in a changed basis.
- I can perform a non-square matrix multiplication.
- I can use non-square matrices to do a projection.
- I can calculate eigenvectors and eigenvalues.
- I can apply eigenvectors and eigenvalues to data problems.

Assessment Evidence:

Performance Assessment Options *May include, but are not limited to the following:*	**Other assessment options** *May include, but are not limited to the following:*
• Problem Sets • Project-based/Problem-based activities • Unit Assessment • Coding Tasks • Feedback on Success/Professional Skills	• Project reflection

Digital Tools & Supplementary Resources:

Graphing Calculator, Desmos Web-Based Calculator,Python software, Dataquest, DataCamp, Coursera, Amazon Machine Learning and Data Science Certificate Program, etc.

#2 LINEAR PROGRAMMING

Essential Unit Questions

1. How do we as mathematicians analyze the problem in order to choose the best strategy(ies) or resource to make sense of the problem?
2. What tools are available and efficient for us as mathematicians to use while solving a problem?

Guiding Content Questions

1. How can a graphical approach be used to achieve the best outcome (such as maximum profit or minimum cost) in a mathematical problem involving two variables that is subject to linear inequality constraints?
2. How can the simplex method be used to achieve the best outcome (such as maximum profit or minimum cost) in a mathematical problem involving more than two variables that is subject to linear inequality constraints?

Learning Targets:

- I can apply the graphical method to solve linearprogramming to real-world optimization problems.
- I can define and understand the meaning and applicationsof slack variables and the pivot.
- I can solve maximization and minimization problemsusing the simplex tableau and method.

Assessment Evidence:

Performance Assessment Options *May include, but are not limited to the following:* ● Problem Sets ● Project-based/Problem-based activities ● Unit Assessment ● Coding Tasks ● Feedback on Success/Professional Skills	Other assessment options *May include, but are not limited to the following:* ● Project reflection

Digital Tools & Supplementary Resources:

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#3 LIMITS

Essential Unit Questions

1. How do we as mathematicians make sense of quantities and situations symbolically?
2. What tools are available and efficient for us as mathematicians to use while solving a problem?

Guiding Content Questions

1. How can limits be calculated algebraically, graphically, and numerically?
2. How can limits be used to prove continuity?
3. How are infinite limits calculated?

Learning Targets:

- I can define limits and use correct limit notation.
- I can estimate limit values from a graph.
- I can estimate limit values from a table.
- I can determine limits using algebraic propertiesof limits.
- I can determine limits using algebraic manipulation.
- I can select different procedures for determininglimits.
- I can determine limits using the squeeze theorem.
- I can connect multiple representations of limits.
- I can explore types of discontinuities.
- I can confirm continuity over an interval.
- I can determine removable discontinuities.
- I can work with the Intermediate Value Theorem(IVT).
- I can connect infinite limits and vertical asymptotes.
- I can connect limits at infinity and horizontalasymptotes.

Assessment Evidence:

Performance Assessment Options *May include, but are not limited to the following:* ● Problem Sets ● Project-based/Problem-based activities ● Unit Assessment	Other assessment options *May include, but are not limited to the following:* ● Project reflection

• Coding Tasks • Feedback on Success/Professional Skills	

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#4 DERIVATIVES/DIFFERENTIATION

Essential Unit Questions

1. How do we as mathematicians make sense of quantities and situations symbolically?
2. What tools are available and efficient for us as mathematicians to use while solving a problem?

Guiding Content Questions

1. How can the limit definition be used to find the derivative of a function?
2. What are the basic rules of differentiation?
3. What is the difference between average and instantaneous rates of change?

Learning Targets:

- I can define average and instantaneous ratesof change at a point.
- I can define the derivative of a function anduse derivative notation.
- I can estimate derivatives of a function at apoint.
- I can connect differentiability and continuity.
- I can determine when derivatives do and do notexist.
- I can apply the power rule.
- I can use the constant, sum, difference, andconstant multiple derivative rules.
- I can find derivatives of e^x and $\ln(x)$.
- I can find derivatives using the product rule,quotient rule, and chain rule.
- I can use implicit differentiation.
- I can select procedures for calculating derivatives.
- I can calculate higher-order derivatives.

Assessment Evidence:

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#5 GRAPHS AND THE DERIVATIVE

Essential Unit Questions

1. How can we as mathematicians use and apply patterns and structures to solve problems?
2. What tools are available and efficient for us as mathematicians to use while solving a problem?

Guiding Content Questions

1. What are the procedures for determining critical points and whether they are extrema?
2. What must a function have if there is a change in concavity?

Learning Targets:

- I can use the Mean Value Theorem (MVT).
- I can use the Extreme Value Theorem (EVT), distinguish between global and local extrema, and find critical points.
- I can determine intervals on which a function is increasing or decreasing.
- I can use the first derivative test to determine relative (local) extrema.
- I can use the candidates test to determine absolute (global) extrema.
- I can determine concavity of functions over their domains.
- I can use the second derivative test to determine extrema.
- I can sketch graphs of functions and their derivatives.
- I can connect a function, its first derivative, and its second derivative.

Assessment Evidence:

Performance Assessment Options *May include, but are not limited to the following:*	Other assessment options *May include, but are not limited to the following:*
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#6 APPLICATIONS OF THE DERIVATIVE

Essential Unit Questions

1. What tools are available and efficient for us as mathematicians to use while solving a problem?
2. How can we as mathematicians use and apply patterns and structures to solve problems?

Guiding Content Questions

1. What are the guidelines for solving applied minimum and maximum problems?
2. What is an indeterminate form and how can a limit be found when dealing with these types of problems?

3. How can implicit differentiation be used to solve related rates problems?
4. How can differentials be used for linear approximation?

Learning Targets:

- I can interpret the meaning of the derivative in context.
- I can use L'Hospital's Rule for determining limits of indeterminate forms.
- I can solve max-min optimization and other business/economics based application problems.
- I can solve related rates problems.
- I can approximate values of a function using local linearity and linearization.

Assessment Evidence:

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#7 INTEGRATION

Essential Unit Questions

1. How do we as mathematicians make sense of quantities and situations symbolically?
2. How can we as mathematicians use and apply patterns and structures to solve problems?

Guiding Content Questions

1. How can the area under a curve be found geometrically and by using the Fundamental Theorem of Calculus?
2. How can the area under a curve be approximated using left endpoints, right endpoints, or midpoints?
3. What are some different interpretations of the Fundamental Theorem of Calculus?
4. How can limits be used to find the area under a curve?
5. How can integration be used to find the area between two curves?
6. How can numerical integration be used to evaluate integrals that could not otherwise be evaluated?

Learning Targets:

- I can find accumulations of change
- I can approximate areas with Riemann sums.
- I can relate Riemann sums, summation notation, and definite integral notation.
- I can use the Fundamental Theorem of Calculus (FTC) and accumulation functions.
- I can interpret the behavior of accumulating functions involving area.

- I can apply properties of definite integrals.
- I can relate the FTC and definite integrals.
- I can relate antiderivatives and indefinite integralsusing basic rules and notation.
- I can integrate using substitution.
- I can integrate functions using long divisionand completing the square.
- I can integrate using integration by parts.
- I can integrate using linear partial fractions.
- I can evaluate improper integrals.
- I can determine which technique is most appropriatefor antidifferentiation.
- I can apply integration techniques to solve business/economicsrelated application questions.

Assessment Evidence:

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#8 INFINITE SEQUENCES AND SERIES

Essential Unit Questions

1. How do we as mathematicians make sense of quantities and situations symbolically?
2. How can we as mathematicians use and apply patterns and structures to solve problems?

Guiding Content Questions

1. What is the difference between a sequence and a series?
2. What test for convergence is most appropriate for a given series?
3. How can the accuracy of a Taylor polynomial approximation be determined from the actual value?
4. What is the general procedure for finding the form that every convergent power series must take?

Learning Targets:

- I can define convergent and divergent infiniteseries.
- I can work with geometric series.
- I can test for divergence using the nth termtest.
- I can use the integral test for convergence.
- I can recognize the harmonic series and p-series.
- I can use the comparison tests for convergence.

Assessment Evidence:	
Performance Assessment Options *May include, but are not limited to the following:* ● Problem Sets ● Project-based/Problem-based activities ● Unit Assessment ● Coding Tasks ● Feedback on Success/Professional Skills	**Other assessment options** *May include, but are not limited to the following:* ● Project reflection
Digital Tools & Supplementary Resources:	
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#9 MULTIVARIATE CALCULUS

Essential Unit Questions

1. How do we as mathematicians make sense of quantities and situations symbolically?
2. How can we as mathematicians use and apply patterns and structures to solve problems?

Guiding Content Questions

1. What are the informal and formal definitions of a partial derivative.
2. How can extrema of multivariate functions be found?
3. What are the guidelines for solving applied minimum and maximum problems involving multivariate functions?
4. How can functions subject to constraints be optimized?
5. How can the concept of a differential for functions of one variable be extended to the concept of a total differential for functions of more than one variable?
6. How can multiple (e.g. double) integrals be used to find antiderivatives of functions of two or more variables?
7. How can partial differentiation be used to calculate the Jacobian and the Hessian?
8. How can the Taylor Series be used to find the linear approximation to a function at a given point?
9. How can the Gradient Descent method be used to finda local minimum for a differentiable function?

Learning Targets:

- I can evaluate functions of several variables.
- I can evaluate the change in the dependent variableof a multivariable function when one of the independent variables is changed while the other independent variables are held constant.
- I can solve application problems involving partialderivatives.
- I can identify relative extrema and saddle points.
- I can use Lagrange multipliers to optimize functionssubject to constraints.
- I can analyze results of total differentials.
- I can calculate the Jacobian and the Hessian.
- I can perform the multivariate chain rule.
- I can apply the Taylor Series to solve problems.

- I can use the Gradient Descent Method to optimize a fitting function.

Assessment Evidence:

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#10 GRAPH THEORY (POSSIBLE EXTENSION TOPIC)

Essential Unit Questions

1. How do we as mathematicians analyze the problem in order to choose the best strategy(ies) or resource to make sense of the problem?
2. What tools are available and efficient for us as mathematicians to use while solving a problem?

Guiding Content Questions

1. How can I use graph theory to find the shortest path in a given graph or network.

Learning Targets:

- I can create graphs to model real world problems.
- I can represent graphs using incidence matrices.
- I can construct Euler and Hamilton paths and circuits.
- I can solve shortest-path problems.
- I can recognize the properties of trees.

Assessment Evidence:

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#11 PROBABILITY (POSSIBLE EXTENSION TOPIC)

Essential Unit Questions

1. How do we as mathematicians make sense of quantities and situations symbolically?
2. How can we as mathematicians use and apply patterns and structures to solve problems?

Guiding Content Questions

1. How does probability allow us to quantify uncertainty?
2. How can probability be used to calculate the likelihood of an event happening?
3. What is the difference between permutations and combinations?
4. How can the total number of possible outcomes in a given situation be determined?
5. What is a random variable and how is the distribution of a random variable determined?

Learning Targets:

- I can define basic probability vocabulary,including experiments, outcome, sample space, event, complement of the event, simple events, theoretical probability, experimental probability and subjective probability.
- I understand the Law of Large Numbers and the range of probabilities.
- I can calculate probabilities and conditionalprobabilities of different events.
- I can calculate the probability and the complementof a given event.
- I understand the difference between independentand dependent events.
- I can properly notate probabilities.
- I can calculate probabilities of events happeningin sequence.
- I can determine whether two events are mutuallyexclusive.
- I can use the addition rule to calculate probabilities.
- I can use the Fundamental Counting Principleto calculate probabilities.
- I can identify if an arrangement is a permutationor combination and calculate the number of arrangements.
- I can calculate the number of distinguishablepermutations.
- I can use permutations and combinations tofind probabilities of events.
- I can apply the basic concepts of probabilityto real-world situations.
- I can apply Bayes' Theorem to find probabilities.
- I can apply permutations and combinations toreal-world problems.
- I can calculate expected values using probabilitydistributions.

Assessment Evidence:

Performance Assessment Options *May include, but are not limited to the following:*	Other assessment options *May include, but are not limited to the following:*
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Digital Tools & Supplementary Resources:

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#12 REGRESSION ANALYSIS: THE ORDINARY LEAST SQUARES REGRESSION LINE AND MULTIPLE REGRESSION (POSSIBLE EXTENSION TOPIC)

Essential Unit Questions

1. What model(s) can we as mathematicians use to solve a problem?
2. What tools are available and efficient for us as mathematicians to use while solving a problem?

Guiding Content Questions

1. What is a regression analysis and what are the steps in a regression analysis
2. How do you perform an exploratory data analysis and what is its purpose?
3. How do you find and interpret the correlation coefficient and the multiple correlation coefficient?
4. What is the form of a simple linear regression model and of a multiple regression model?
5. How are parameters/coefficients of regression models found and interpreted?
6. How can regression models be used for prediction?
7. What criteria can be used to evaluate regression models?
8. What feature/variable selection procedures can be used to build optimal regression models?
9. What are the standard regression assumptions and what diagnostic tools can be used to check these assumptions?
10. How can outliers be analyzed in a regression model?
11. How can indicator/dummy variables and interaction variables be incorporated into a regression model?
12. How can transformations of the original data be used to achieve linearity, stabilize variance, and remove heteroscedasticity?
13. How can collinearity be detected and addressed?
14. How can matrix algebra be used to represent a multiple regression problem?

Learning Targets:

- I can define what a regression analysis is.
- I can give examples of where regression analysiscan be applied to solve real-world problems.
- I can identify, describe, and execute the stepsin the dynamic iterative regression process.
- I can identify and perform the steps requiredto do a comprehensive exploratory data analysis.
- I can compute and interpret the covariance andthe correlation coefficient.
- I can estimate the parameters in a simple linearregression model.
- I can make predictions using a simple linearregression model.
- I can estimate the parameters in a multiplelinear regression model.
- I can interpret the regression coefficientsfor continuous variables in a multiple linear regression model.
- I can interpret the regression coefficientsfor discrete/indicator variables in a multiple linear regression model.
- I can compute the multiple correlation coefficient R^2 and the adjusted R-squared and use

these measures to evaluate the goodness-of-fit of a regression model.

- I can describe the different uses of a regression model and how these specific uses affect the variable selection process.
- I can use the residual mean square, R-squared, and Adjusted R-squared, Mallows C_p, and information criteria (AIC/BIC) to evaluate competing regression equations.
- I can use a variety of selection procedures (e.g. forward selection, backward elimination, stepwise method, etc.) to select the optimal subset of potential predictor variables to incorporate into the regression model.
- I can use specialized procedures for selecting variables when analyzing collinear data.
- I can classify and describe the standard regression assumptions and name the issues that arise when these assumptions are violated.
- I can use the appropriate residual and graphical analyses to validate the standard regression assumptions.
- I can describe the implications of outliers in the response variable and outliers in the predictor variables.
- I can use the following measures – Cook's Distance, Welsch and Kuh Measure, Hadi's Influence Measure – to determine whether there are influential observations in the data set.
- I can explain the purpose and interpretation of an indicator variable.
- I can explain the purpose and interpretation of an interaction variable.
- I can select and execute an appropriate transformation to linearize a non-linear data set.
- I can select and execute an appropriate transformation to stabilize the error variance.
- I can detect heteroscedastic errors and then remove the heteroscedasticity by performing an appropriate transformation or by using the weighted least squares technique.
- I can recognize when to use a logarithmic or power transformation and I also understand how to properly perform either of those transformations.
- I can describe the effects that multicollinearity issues have on the forecasting that is done when using a regression model.
- I can use simple signs of collinearity, variance inflation factors, and condition indices to detect collinearity issues.
- I can describe the different approaches that can be used to address collinearity issues.
- I can use matrix algebra to represent a multiple regression problem.

Assessment Evidence:

Performance Assessment Options *May include, but are not limited to the following:*	**Other assessment options** *May include, but are not limited to the following:*
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Digital Tools & Supplementary Resources:

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#13 BINARY MULTIPLE LOGISTIC REGRESSION (POSSIBLE EXTENSION TOPIC)

Essential Unit Questions

1. What model(s) can we as mathematicians use to solve a problem?
2. What tools are available and efficient for us as mathematicians to use while solving a problem?

Guiding Content Questions

1. How can logistic regression be used to build classification models involving dichotomous dependent variables?
2. What issues arise when trying to use ordinary least squares regression to model situations that involve dichotomous dependent variables?
3. How is Maximum Likelihood Estimation used to estimate the parameters of a logistic regression model?
4. How are the coefficients of a logistic regression model interpreted?
5. How do you evaluate a binary multiple logistic regression model?

Learning Targets:

- I can define what is meant by a dichotomous (binary) dependent variable and recognize situations where it is appropriate to use this type of dependent variable.
- I can describe why there are problems using ordinary least squares regression for modeling situations that involve a binary response variable.
- I can compute (or recognize from computer output) and interpret the odds and odds ratios in a logistic regression model.
- I can compute (or recognize from computer output) and interpret the logit and log-likelihood of a logistic regression model.
- I can explain how the parameter estimation process is different for the logistic regression process than it is for the multiple regression process.
- I can use computer software to perform a maximum likelihood estimate of the parameters of a logistic regression model.
- I can interpret the coefficients found in a logistic regression model for both continuous and class/discrete predictor variables.
- I can use the Likelihood Ratio, Score, and Wald global chi-square tests to determine if the regression model is better than no model.
- I can use the AIC, SC, and -2 Log L model fit statistics to compare the fit of models with different sets of predictors.
- I can use the deviance, Pearson chi-square and HL tests to evaluate whether a model can be improved by including interactions and non-linearities.
- I can use several generalized versions of R^2 to evaluate the predictive power of the logistic regression model.
- I can use several ordinal measures of association to get model-free measures of the predictive power of the logistic regression model.
- I can use ROC curves to evaluate the predictive power of models for binary outcomes.
- I can split a data set into a training set and a test set to gauge model effectiveness.
- I can use lift charts to analyze model effectiveness.

Assessment Evidence:

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• Feedback on Success/Professional Skills	

Digital Tools & Supplementary Resources:

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#14 DECISION TREES (POSSIBLE EXTENSION TOPIC)

Essential Unit Questions

1. What model(s) can we as mathematicians use to solve a problem?
2. What tools are available and efficient for us as mathematicians to use while solving a problem?

Guiding Content Questions

1. How can decision trees be used for classification problems?
2. What are the requirements for using decision trees?
3. How can decision rules in the form of "if antecedent, then consequent" statements be used to interpret the results of a decision tree analysis?
4. What is meant by the support of a decision tree and by the confidence of a decision tree?

Learning Targets:

- I can identify problems where performing a decisiontree analysis is appropriate.
- I understand the requirements that must be satisfiedin order to use a decision tree.
- I can use the CART algorithm and the C4.5 algorithmto perform a decision tree analysis.
- I can determine the classification error ratefor a decision tree.
- I can create decision tree rules in the formof "if antecedent, then consequent" statements that can be used to interpret the results of a decision tree analysis.
- I can calculate the support and confidence ofa decision tree.

Assessment Evidence:

Performance Assessment Options *May include, but are not limited to the following:*	Other assessment options *May include, but are not limited to the following:*
• Problem Sets • Project-based/Problem-based activities • Unit Assessment • Coding Tasks • Feedback on Success/Professional Skills	• Project reflection

Digital Tools & Supplementary Resources:

Graphing Calculator, Desmos Web-Based Calculator,Python software, Dataquest, DataCamp, Coursera, Amazon Machine Learning and Data Science Certificate Program, etc.