

PRE-CALCULUS

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| Curriculum/Content Area: Mathematics | Course Length: 2 terms |
| Course Title: Pre-Calculus | Date last reviewed: 2020 Link Previous UbD |
| Prerequisites: Algebra 2 and Trigonometry (Trig can be taken concurrently in 1st or 3rd term) | Board approval date: 8/2020 |
| Primary Resource: <ul style="list-style-type: none"> • Glencoe Precalculus - Regular • Foerster - Precalculus with Trigonometry (HONORS) | |

Course description and purpose: Precalculus is a rigorous course encompassing a wide variety of mathematical topics. The content includes mastery of algebraic manipulation of functions (linear, polynomial, rational, exponential, logarithmic, and trigonometric), advanced trigonometry, analytic geometry, logarithms, series and sequences. This course also introduces Calculus outcomes that include limits, asymptotes, continuity and the concept of derivatives (Honors PreCalculus). This course is designed to incorporate theory, process, and application using technology to illustrate concepts whenever appropriate. A graphing calculator is required.

| Enduring Understandings: | Essential Questions: |
|---|---|
| Mathematicians make sense of problems and persevere in solving them. | a. How do we as mathematicians analyze the problem in order to choose the best strategy(ies) or resource to make sense of the problem? b. How do we as mathematicians persevere in solving problems? |
| Mathematicians attend to precision. | How do we as mathematicians know if we fully & accurately answered the problem and does the results make sense in the context of the problem? |
| Mathematicians reason abstractly and quantitatively. | How do we as mathematicians make sense of quantities and situations symbolically? |
| Mathematicians construct viable arguments and critique the reasoning of others. | a. How can we as mathematicians justify our answer(s)? b. How can we as mathematicians evaluate and question whether a mathematical argument is accurate? |
| Mathematicians model with mathematics. | a. What model(s) can we as mathematicians use to solve a problem? b. How can we as mathematicians determine an effective model to use to solve a problem? |

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| Mathematicians use appropriate tools strategically. | What tools are available and efficient for us as mathematicians to use while solving a problem? |
| Mathematicians look for and make use of structure | How can we as mathematicians use and apply patterns and structures to solve problems? |
| Mathematicians look for and express regularity in repeated reasoning. | How can we as mathematicians create and apply generalizations from repeated reasoning? |

Mathematical Practice Standards

The Standards for Mathematical Practice are central to the teaching and learning of mathematics. These practices describe the behaviors and habits of mind that are exhibited by students who are mathematically proficient. Mathematical understanding is the intersection of these practices and mathematics content. It is critical that the Standards for Mathematical Practice are embedded in daily mathematics instruction.

| Mathematical Practice Standards | | Grade Level/Course |
|---------------------------------|---|---|
| Habits of Mind | MP.1 Make sense of problems and persevere in solving them | Understand the meaning of a problem and look for entry points to its conclusion. Analyze information (givens, constraints, relationships, goals). Make conjectures and plan a solution pathway. Monitor and evaluate the progress and change course as necessary Check answers to problems and ask, "Does this make sense?" |
| | MP.6 Attend to precision. | Communicate precisely using clear definitions. State the meaning of symbols, carefully specifying units of measure, and providing accurate labels. State the meaning of symbols, carefully specifying units of measure, and providing accurate labels. Calculate accurately and efficiently, expressing numerical answers with a degree of precision. Provide carefully formulated explanations. Label accurately when measuring and graphing. |
| Reasoning & Explaining | MP.2 Reason abstractly and quantitatively. | Make sense of quantities and relationships in problem situations. Represent abstract situations symbolically and understand the meaning of quantities. Create a coherent representation of the problem at hand. Consider the units involved. Flexibility uses properties of operations. |
| | MP.3 Construct viable arguments and | Use definitions and previously established |

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| | critique the reasoning of others. | causes/effects (results) in constructing arguments. Make conjectures and use counterexamples to build a logical progression of statements to explore and support ideas. Communicate and defend mathematical reasoning using objects, drawings, diagrams, and/or actions. Listen to or read the arguments of others. Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments. |
| Modeling & Using Tools | MP.4 Model with mathematics. | Apply prior knowledge to solve real world problems. Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and/or formulas. Use assumptions and approximations to make a problem simpler. Check to see if an answer makes sense within the context of a situation and change a model when necessary. |
| | MP.5 Use appropriate tools strategically. | Make sound decisions about the use of specific tools (examples might include: calculator, concrete models, digital, technologies, pencil/paper, ruler, compass, protractor) Use technology tools to visualize the results of assumptions, explore consequences, and compare predictions with data. Identify relevant external math resources (digital content on a website) and use them to pose or solve problems. Use technological tools to explore and deepen understanding of concepts. |
| Seeing Structure & Generalizing | MP.7 Look for and make use of structure. | Look for patterns or structure, recognizing that quantities can be represented in different ways. Recognize the significance in concepts and models and use the patterns or structure for solving related problems. View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems. |
| | MP.8 Look for and express regularity in repeated reasoning. | Notice repeated calculations and look for general methods and shortcuts. Continually evaluate the reasonableness of intermediate results (comparing estimates), while attending to details, and make generalizations based on findings. |

Priority Standard Clusters

F-BFA Build a function that models a relationship between two quantities.

- F-BFA.1: Write a function that describes a relationship between two quantities. (C) (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

A-REIC Solve systems of equations. (Honors)

- (A-REIC8): (+) Represent a system of linear equations as a single matrix equation in a vector variable.
- (A-REIC9): (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Supporting Standard Clusters

N-CNA Perform arithmetic operations with complex numbers.

- (N-CNA3): (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

N-CNB Represent complex numbers and their operations on the complex plane.

- (N-CNB4): (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- (N-CNB5): (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120 degrees.
- (N-CNB6): (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

F-BFB Build new functions from existing functions.

- (F-BFB4): Find inverse functions. (B) (+) Verify by composition that one function is the inverse of another. (C) (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (D) (+) Produce an invertible function from a non-invertible function by restricting the domain.
- (F-BFB5): (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

F-TFA Extend the domain of trigonometric functions using the unit circle.

- (F-TFA4): (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F-TFB Model periodic phenomena with trigonometric functions.

- (F-TFB6): (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- (F-TFB7): (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

F-TFC Prove and apply trigonometric identities.

- (F-TFC9): (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

G-SRT Apply trigonometry to general triangles

- (G-SRTD9): (+) Derive the formula $A = \frac{1}{2}ab\sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- (G-SRTD10): (+) Prove the laws of sines and cosines and use them to solve problems.
- (G-SRTD11): (+) Understand and apply the law of sines and the law of cosines to find unknown measurements in right and non-right triangles (for example: surveying problems, resultant forces)

N-VMA Represent and model with vector quantities. - Honors

- Standard 1 (N-VMA1): (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \mathbf{v} , $|\mathbf{v}|$, $\|\mathbf{v}\|$, v).
- (N-VMA2): (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- (N-VMA3): (+) Solve problems involving velocity and other quantities that can be represented by vectors.

N-VMB Perform operations on vectors. - Honors

- (N-VMB5): (+) Multiply a vector by a scalar. (A) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x \mathbf{i} + v_y \mathbf{j}) = (cv_x \mathbf{i} + cv_y \mathbf{j})$. (B) Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|c\mathbf{v}\| = |c|\mathbf{v}\|$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|\mathbf{v}\| \neq 0$, the direction of $c\mathbf{v}$ is either along \mathbf{v} (for $c > 0$) or against \mathbf{v} (for $c < 0$).

N-VMC Perform operations on matrices and use matrices in applications. - Honors

- (N-VMC7): (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

F-TFA Extend the domain of trigonometric functions using the unit circle. - Honors

- (F-TFA3): (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x , where x is any real number.

G-GPEA Translate between the geometric description and the equation for a conic section. - Honors (6)

- (G-GPEA3): (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Unit 1 - Functions from a Calculus Perspective

Essential Questions:

1. How do we as mathematicians make sense of quantities and situations symbolically?
2. What model(s) can we as mathematicians use to solve a problem?

Unit Standards

Priority Standards

F-BFA Build a function that models a relationship between two quantities.

- F-BFA.1: Write a function that describes a relationship between two quantities. (C) (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Supporting Standards

F-BFB Build new functions from existing functions.

- (F-BFB4): Find inverse functions. (B) (+) Verify by composition that one function is the inverse of another. (C) (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (D) (+) Produce an invertible function from a non-invertible function by restricting the domain.
- (F-BFB5): (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Learning Targets

Identify functions and use proper notation

- I can describe sets of numbers using set-builder notation.
- I can write sets of numbers using interval notation.
- I can identify and evaluate functions.
- I can state the domains of functions.
- I can evaluate piecewise functions for specific domain values.

Analyzing graphs of functions and relations

- I can use graphs of functions to estimate function values and find domains, ranges, y-intercepts, and zeros of functions.
- I can algebraically solve for the y-intercepts and zeros of functions.
- I can explore symmetries of graphs and identify even and odd functions.

Determine continuity, end behavior, and limits

- I can use limits to determine the continuity of a function.
- I can apply the Intermediate Value Theorem to continuous functions.
- I can use limits to describe end behavior of functions.

Identify extrema and determine average rates of change

- I can determine the intervals on which functions are increasing, decreasing, or constant.

- I can determine maxima and minima of functions.
- I can use a graphing calculator to approximate extrema.
- I can apply extrema to optimization problems.
- I can determine the average rate of change of a function.

Analyze and graph parent functions and their transformations

- I can identify, graph, and describe parent functions.
- I can identify, graph, and write equations for transformations of parent functions.

Perform operations and find compositions on functions

- I can perform operations with functions.
- I can compose two or more functions.
- I can decompose composite functions

Find inverses of functions and relations

- I can use the horizontal line test to determine if an inverse function exists.
- I can find inverse functions algebraically and graphically.
- I can verify that two functions are inverses.

Assessment Evidence

Performance Assessment Options

May include, but are not limited to the following:

- Feedback & Scoring Rubric based on Priority Standards

Other assessment options

May include, but are not limited to the following:

- Quick Writes
- KWL Chart - (What I Know, Want to know, Learned)
- Marking Text
- Learning Log Reflection - Daily/Weekly
- I-Chart - Gather/Organize Information on a topic
- Focused Note Taking
- CSG - Collaborative Study Groups
- Think/Pair/Share
- Entrance Ticket
- Exit ticket

Digital Tools & Supplementary Resources

- Desmos
- DeltaMath
- TI Calculator
- Excel/Google Sheets

Unit 2 - Power, Polynomial, and Rational Functions

Essential Questions:

1. How do we as mathematicians make sense of quantities and situations symbolically?
2. How can we as mathematicians use and apply patterns and structures to solve problems?

Unit Standards

Priority Standards

F-BFA Build a function that models a relationship between two quantities.

- F-BFA.1: Write a function that describes a relationship between two quantities. (C) (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Supporting Standards

N-CNA Perform arithmetic operations with complex numbers.

- (N-CNA3): (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

F-BFB Build new functions from existing functions. (1, 2, 3, 4, 5, 7)

- (F-BFB4): Find inverse functions. (B) (+) Verify by composition that one function is the inverse of another. (C) (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (D) (+) Produce an invertible function from a non-invertible function by restricting the domain.
- (F-BFB5): (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Learning Targets

Analyze, graph and solve power and radical functions.

- I can describe the domain, the range, intercepts, end behavior, and continuity of a monomial function, and where that function is increasing and decreasing.
- I can describe the domain, the range, intercepts, end behavior, and continuity of a power function (including negative and rational exponents), and where that function is increasing and decreasing.
- I can describe the domain, the range, intercepts, end behavior, and continuity of a radical function, and where that function is increasing and decreasing.
- I can solve radical equations and confirm those solutions or eliminate extraneous ones.

Graph polynomial functions and use them to model real-world data.

- I can analyze, predict and graph transformations of monomial functions.
- I can predict and describe the end behavior of polynomial functions of degree n , using limit properties (including the leading term test and degree of the polynomial).
- I can state the number of possible zeros (including multiplicity) and turning points of polynomials and then determine those zeros by factoring.
- I can apply all aspects of curve graphing to graph a polynomial in detail.
- I can use a graphing calculator to model data with the most appropriate polynomial function.

Divide polynomials using division and factor theorems.

- I can use long division to factor polynomials of different degrees with or without a remainder.
- I can use synthetic division to divide polynomials.
- I can use the Factor Theorem to determine if a binomial is a factor of a polynomial, and use that to factor a polynomial.

Find real and complex zeros using a graphical and algebraic (division/factoring) approach.

- I can list all possible rational zeros of a polynomial function and then determine if those are actual zeros.
- I can use the upper and lower bound test to determine where real zeros of a polynomial function lie, and then find those real zeros.
- I can use Descartes' Rule of Signs to describe the possible real zeros of a polynomial function, using the Fundamental Theorem of Algebra.
- I can write a polynomial function of least degree with real coefficients in standard form, given its zeros.
- I can find the zeros of a polynomial function and factor that function.
- I can find all complex zeros of a polynomial and then write the linear factorization of that polynomial.

Analyze, graph and solve rational functions.

- I can find the domain of a rational function.
- I can find the horizontal and vertical asymptotes of a rational function.
- I can find the x- and y-intercepts of a rational function.
- I can graph a rational function, including all of the features listed above.
- I can graph a rational function, analyzing the degrees of the numerator and denominator.
- I can find holes (removable discontinuities) of rational functions.
- I can solve a rational equation.
- I can confirm solutions of rational equations and discard extraneous solutions.

Solving polynomial and rational inequalities using a sign chart.

- I can solve a polynomial inequality using a sign chart.
- I can solve a polynomial inequality using end behavior.
- I can solve a rational inequality using a sign chart.

Assessment Evidence

Performance Assessment Options

May include, but are not limited to the following:

- Feedback & Scoring Rubric based on Priority Standards

Other assessment options

May include, but are not limited to the following:

- Quick Writes
- KWL Chart - (What I Know, Want to know, Learned)
- Marking Text
- Learning Log Reflection - Daily/Weekly
- I-Chart - Gather/Organize Information on a topic

- Focused Note Taking
- CSG - Collaborative Study Groups
- Think/Pair/Share
- Entrance Ticket
- Exit ticket

Digital Tools & Supplementary Resources

- Desmos
- DeltaMath
- TI Calculator
- Excel/Google Sheets

Unit 3 - Exponential and Logarithmic Functions

Essential Questions:

1. How do we as mathematicians analyze the problem in order to choose the best strategy(ies) or resource to make sense of the problem?
2. What tools are available and efficient for us as mathematicians to use while solving a problem?

Unit Standards

Priority Standards

F-BFA Build a function that models a relationship between two quantities.

- F-BFA.1: Write a function that describes a relationship between two quantities. (C) (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Supporting Standards

F-BFB Build new functions from existing functions. (1, 2, 3, 4, 5, 7)

- (F-BFB4): Find inverse functions. (B) (+) Verify by composition that one function is the inverse of another. (C) (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (D) (+) Produce an invertible function from a non-invertible function by restricting the domain.
- (F-BFB5): (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Learning Targets

Analyze exponential growth and decay functions

- I can evaluate, analyze, and graph exponential functions.
- I can use the Compound Interest Formula and the Continuous Compound Interest Formula to solve exponential growth financial problems.
- I can solve applied exponential growth and decay problems.

Technology integration with Financial Literacy: Exponential Functions

- I can calculate future values of annuities and monthly payments.
- I can use the graphing calculator applications to solve future value problems

Analyzing Logarithmic Functions

- I can evaluate expressions involving logarithms with base b .
- I can apply basic properties of logarithms, common logarithms, and natural logarithms to evaluate logarithmic expressions..
- I can sketch and analyze graphs of logarithmic functions.

Applying Properties of Logarithms

- I can apply properties of logarithms.
- I can expand and condense logarithmic expressions.
- I can apply the change of base formula to evaluate logarithmic expressions.

Solving Exponential and Logarithmic Equations

- I can solve exponential equations using the one-to-one property of exponential functions.
- I can solve logarithmic equations using the one-to-one property of logarithmic functions.
- I can check for extraneous solutions in logarithmic equations.
- I can model exponential growth.

Assessment Evidence

Performance Assessment Options

May include, but are not limited to the following:

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Other assessment options

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- Exit ticket

Digital Tools & Supplementary Resources

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- DeltaMath
- TI Calculator
- Excel/Google Sheets

Unit 4 - Trigonometric Functions

Essential Questions:

1. How do we as mathematicians make sense of quantities and situations symbolically?
2. How can we as mathematicians create and apply generalizations from repeated reasoning?

Unit Standards

Priority Standards

Standard number (space) Priority Standard Cluster that is taught IN THIS UNIT.

- Standard number Grade level standard under the above cluster that is taught IN THIS UNIT

Supporting Standards

F-BFB Build new functions from existing functions.

- (F-BFB4): Find inverse functions. (B) (+) Verify by composition that one function is the inverse of another. (C) (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (D) (+) Produce an invertible function from a non-invertible function by restricting the domain.
- (F-BFB5): (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

F-TFA Extend the domain of trigonometric functions using the unit circle.

- (F-TFA4): (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F-TFB Model periodic phenomena with trigonometric functions.

- (F-TFB6): (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- (F-TFB7): (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

G-SRT Apply trigonometry to general triangles

- (G-SRTD9): (+) Derive the formula $A = \frac{1}{2}ab\sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- (G-SRTD10): (+) Prove the laws of sines and cosines and use them to solve problems.
- (G-SRTD11): (+) Understand and apply the law of sines and the law of cosines to find unknown measurements in right and non-right triangles (for example: surveying problems, resultant forces)

Learning Targets

Solving Right Triangle Trigonometry

- I can find values for the six trigonometric functions of an acute angle in right triangles.
- I can use inverse trigonometry to solve for missing angles.
- I can use trigonometry to solve right triangles for missing side length and angles.
- I can use trigonometry to solve right triangle application problems with angle of elevation and angle of depression.

Using Degrees and Radians in Trigonometry

- I can convert between DMS and decimal degree form.
- I can convert between degree and radian measure and vice versa.
- I can identify and draw co-terminal angles.
- I can calculate the length of an intercepted arc in a circle.

- I can find the linear and angular speed of an object moving at a constant speed along a circular path of radius r .
- I can calculate the area of a sector of a circle.

Evaluating Trigonometric Functions of the Unit Circle

- I can find the exact trigonometric values of an angle in standard position given a point.
- I can evaluate trigonometric functions of quadrantal angles.
- I can find a reference angle and use it to find the exact value of a trigonometric function.
- I can evaluate exact values of trigonometric expressions using the unit circle.
- I can use the concept of periodic functions to evaluate exact values of trigonometric expressions

Graphing Sine and Cosine Functions

- I can graph transformations of sine and cosine functions.
- I can identify dilations, amplitude, frequency, phase shift, and vertical shift of sine and cosine functions.
- I can solve applications of sinusoidal functions.

Graphing Tangent, Cotangent, Secant, and Cosecant

- I can graph tangent and reciprocal trigonometric functions.
- I can graph period transformations of the tangent and reciprocal trigonometric functions.

Evaluating Inverse Trigonometric Functions

- I can restrict the domains of the original trigonometric functions to graph inverse trigonometric functions.
- I can evaluate compositions of trigonometric functions.
- I can evaluate the exact value of an inverse trigonometric expressions.

Solving Using The Law of Sines and Cosines

- I can solve oblique triangles of the law of sines and the law of cosines.
- I can determine if there are zero, one or two possible triangle solutions in the SSA case.
- I can use Heron's Formula to find the area of a triangle given three sides.
- I can find the area of an obtuse triangle given SAS using $\text{Area} = \frac{1}{2} bc\sin A$ or $\text{Area} = \frac{1}{2} ac\sin B$ or $\text{Area} = \frac{1}{2} ab\sin C$

Assessment Evidence

Performance Assessment Options

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Other assessment options

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Digital Tools & Supplementary Resources

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Unit 5 - Trigonometric Identities and Equations

Essential Questions:

1. How can we as mathematicians justify our answer(s)?
2. How can we as mathematicians use and apply patterns and structures to solve problems?
3. What tools are available and efficient for us as mathematicians to use while solving a problem?
4. How can we as mathematicians create and apply generalizations from repeated reasoning?

Unit Standards

Priority Standards

F-BFA Build a function that models a relationship between two quantities.

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Supporting Standards

F-BFB Build new functions from existing functions.

- (F-BFB4): Find inverse functions. (B) (+) Verify by composition that one function is the inverse of another. (C) (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (D) (+) Produce an invertible function from a non-invertible function by restricting the domain.
- (F-BFB5): (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

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- (F-TFB6): (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- (F-TFB7): (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

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- (F-TFC9): (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

G-SRT Apply trigonometry to general triangles

- (G-SRTD9): (+) Derive the formula $A = \frac{1}{2}ab\sin C$ for the area of a triangle by drawing an

- auxiliary line from a vertex perpendicular to the opposite side.
- (G-SRTD10): (+) Prove the laws of sines and cosines and use them to solve problems.
- (G-SRTD11): (+) Understand and apply the law of sines and the law of cosines to find unknown measurements in right and non-right triangles (for example: surveying problems, resultant forces)

Learning Targets

Trigonometric Identities

- I can use reciprocal and quotient identities.
- I can use pythagorean identities.
- I can use co-function and odd-even identities.
- I can simplify identities in terms of only sine and cosine.
- I can simplify by factoring.
- I can simplify by combining (adding and subtracting) fractions.
- I can rewrite to eliminate fractions.

Verifying Trigonometric Identities

- I can verify a trigonometric identity.
- I can verify a trigonometric identity by combining (adding and subtracting) fractions.
- I can verify a trigonometric identity by multiplying by a conjugate.
- I can verify a trigonometric identity by factoring.
- I can verify an identity by working each side separately.
- I can determine whether an equation is an identity.

Solving Trigonometric Equations

- I can solve an equation by isolating trigonometric expressions.
- I can solve by taking the square root of each side.
- I can solve by factoring.
- I can analyze trigonometric functions of multiple angles.
- I can solve by rewriting using a single trigonometric function.
- I can solve by squaring.

Sum and Difference Identities

- I can evaluate a trigonometric expression.
- I can use a sum or difference identity.
- I can rewrite a single trigonometric expression.
- I can write as an algebraic expression.
- I can verify cofunction identities.
- I can verify reduction identities.
- I can solve a trigonometric equation.

Multiple/Half Angle

- I can evaluate expressions involving double angles.
- I can solve an equation using a double-angle identity.
- I can evaluate an expression involving a half angle.
- I can solve an equation using a half-angle identity.

Product-to-Sum/Power Reducing Identities

- I can use an identity to reduce a power.
- I can solve an equation using a power-reducing identity.
- I can use an identity to write a product as a sum or difference.
- I can use a product-to-sum identity.
- I can solve an equation using a sum-to-product identity.

Assessment Evidence

Performance Assessment Options

May include, but are not limited to the following:

- Feedback & Scoring Rubric based on Priority Standards

Other assessment options

May include, but are not limited to the following:

- Quick Writes
- KWL Chart - (What I Know, Want to know, Learned)
- Marking Text
- Learning Log Reflection - Daily/Weekly
- I-Chart - Gather/Organize Information on a topic
- Focused Note Taking
- CSG - Collaborative Study Groups
- Think/Pair/Share
- Entrance Ticket
- Exit ticket

Digital Tools & Supplementary Resources

- Desmos
- DeltaMath
- TI Calculator
- Excel/Google Sheets

Unit 6 - System of Equations and Matrices - **HONORS ONLY**

Essential Questions:

1. What tools are available and efficient for us as mathematicians to use while solving a problem?
2. How do we as mathematicians know if we fully & accurately answered the problem and does the results make sense in the context of the problem?

Unit Standards

Priority Standards

A-REIC Solve systems of equations. (Honors)

- (A-REIC8): (+) Represent a system of linear equations as a single matrix equation in a vector variable.
- (A-REIC9): (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Supporting Standards

N-VMA Represent and model with vector quantities. - Honors

- Standard 1 (N-VMA1): (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \mathbf{v} , $|\mathbf{v}|$, $\|\mathbf{v}\|$, v).
- (N-VMA2): (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- (N-VMA3): (+) Solve problems involving velocity and other quantities that can be represented by vectors.

N-VMB Perform operations on vectors. - Honors

- (N-VMB5): (+) Multiply a vector by a scalar. (A) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x \mathbf{i} + v_y \mathbf{j}) = (cv_x \mathbf{i} + cv_y \mathbf{j})$. (B) Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|c\mathbf{v}\| = |c|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|\mathbf{v}$ does not = 0, the direction of $c\mathbf{v}$ is either along \mathbf{v} (for $c > 0$) or against \mathbf{v} (for $c < 0$).

N-VMC Perform operations on matrices and use matrices in applications. - Honors

- (N-VMC7): (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

F-TFA Extend the domain of trigonometric functions using the unit circle. - Honors

- (F-TFA3): (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x , where x is any real number.

G-GPEA Translate between the geometric description and the equation for a conic section. - Honors (6)

- (G-GPEA3): (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Learning Targets

Solving Systems of Equations Using Matrices

- I can solve a system of equations algebraically.
- I can solve a system of equations using the matrix function on a graphing calculator.
- I can solve a system of equations using the RREF feature on the graphing calculator.

Partial Fractions

- I can find a partial fraction decomposition of a rational function.

Assessment Evidence

Performance Assessment Options

May include, but are not limited to the following:

- Feedback & Scoring Rubric based on Priority Standards

Other assessment options

May include, but are not limited to the following:

- Quick Writes
- KWL Chart - (What I Know, Want to know, Learned)
- Marking Text
- Learning Log Reflection - Daily/Weekly
- I-Chart - Gather/Organize Information on a topic
- Focused Note Taking
- CSG - Collaborative Study Groups

- Think/Pair/Share
- Entrance Ticket
- Exit ticket

Digital Tools & Supplementary Resources

- Desmos
- DeltaMath
- TI Calculator
- Excel/Google Sheets

Unit 7 - Parametric Equations, Polar Coordinates and Complex Numbers

Essential Questions:

1. How do we as mathematicians make sense of quantities and situations symbolically?
2. How can we as mathematicians determine an effective model to use to solve a problem?
3. What tools are available and efficient for us as mathematicians to use while solving a problem?

Unit Standards

Priority Standards

F-BFA Build a function that models a relationship between two quantities.

- F-BFA.1: Write a function that describes a relationship between two quantities. (C) (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

A-REIC Solve systems of equations. (Honors)

- (A-REIC8): (+) Represent a system of linear equations as a single matrix equation in a vector variable.
- (A-REIC9): (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Supporting Standards

N-CNA Perform arithmetic operations with complex numbers.

- (N-CNA3): (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

N-CNB Represent complex numbers and their operations on the complex plane.

- (N-CNB4): (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- (N-CNB5): (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120 degrees.
- (N-CNB6): (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

F-BFB Build new functions from existing functions.

- (F-BFB4): Find inverse functions. (B) (+) Verify by composition that one function is the inverse of another. (C) (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (D) (+) Produce an invertible function from a non-invertible function by restricting the domain.
- (F-BFB5): (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

F-TFA Extend the domain of trigonometric functions using the unit circle.

- (F-TFA4): (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F-TFB Model periodic phenomena with trigonometric functions.

- (F-TFB6): (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- (F-TFB7): (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

F-TFC Prove and apply trigonometric identities.

- (F-TFC9): (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Learning Targets

Manipulating Parametric Equations

- I can graph parametric equations.
- I can eliminate the parameter.
- I can write parametric equations in rectangular form.
- I can solve problems related to the motion of projectiles.

Exploring Polar Coordinates

- I can graph points with polar coordinates.
- I can find multiple representations of single polar coordinates.
- I can graph polar equations.
- I can find the distance between two polar coordinates.

Graphing Polar Equations

- I can graph polar equations.
- I can use symmetry to graph certain polar equations.
- I can identify and graph classic polar curves.

Polar and Rectangular Forms of Equations

- I can convert from polar and to rectangular coordinates and vice versa.
- I can convert from rectangular to polar equations.
- I can convert from polar to rectangular equations.

Operations with Complex Numbers

- I can perform operations with pure imaginary numbers and complex numbers
- I can use complex conjugates to write quotients of complex numbers in standard form.

Operations with Complex Numbers and DeMoivre's Theorem

- I can graph a complex number in the complex plane.
- I can find the absolute value of a complex number.
- I can convert complex numbers from rectangular form to polar form and vice versa.
- I can find the product and quotient of a complex number in polar form.
- I can apply DeMoivre's Theorem to find a power of a complex number.

- I can find n distinct roots of a complex number using the distinct root formula.

HONORS ONLY

Assessment Evidence

Performance Assessment Options

May include, but are not limited to the following:

- Feedback & Scoring Rubric based on Priority Standards

Other assessment options

May include, but are not limited to the following:

- Quick Writes
- KWL Chart - (What I Know, Want to know, Learned)
- Marking Text
- Learning Log Reflection - Daily/Weekly
- I-Chart - Gather/Organize Information on a topic
- Focused Note Taking
- CSG - Collaborative Study Groups
- Think/Pair/Share
- Entrance Ticket
- Exit ticket

Digital Tools & Supplementary Resources

- Desmos
- DeltaMath
- TI Calculator
- Excel/Google Sheets

Unit 8 - Sequences and Series

Essential Questions:

1. How can we as mathematicians create and apply generalizations from repeated reasoning?
2. How do we as mathematicians make sense of quantities and situations symbolically?
3. How can we as mathematicians use and apply patterns and structures to solve problems?

Unit Standards

Beyond the scope of Common Core Standards

Learning Targets

Sequences, Series, and Sigma Notation

- I can find the next terms of a given sequence.
- I can determine if a sequence is convergent or divergent.
- I can find a partial sum of an infinite series.
- I can use sigma notation to represent and calculate the sums of series.

Arithmetic Sequences and Series

- I can find nth terms and arithmetic means of arithmetic sequences.
- I can find a quadratic model for a sequence using second differences.
- I can find the sum of a finite arithmetic series.

Geometric Sequences and Series

- I can find nth terms and geometric means of geometric sequences.
- I can find the sum of a finite geometric series.
- I can find the sum of an infinite geometric series.

Spreadsheets to Detect Patterns in Data

- I can use spreadsheets to organize and display data.
- I can detect patterns and departures from patterns from data in spreadsheets.

Assessment Evidence

Performance Assessment Options

May include, but are not limited to the following:

- Feedback & Scoring Rubric based on Priority Standards

Other assessment options

May include, but are not limited to the following:

- Quick Writes
- KWL Chart - (What I Know, Want to know, Learned)
- Marking Text
- Learning Log Reflection - Daily/Weekly
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- Exit ticket

Digital Tools & Supplementary Resources

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- TI Calculator
- Excel/Google Sheets

Unit 9 - Calculus: Limits and Derivatives

Essential Questions:

1. What model(s) can we as mathematicians use to solve a problem?
2. How can we as mathematicians use and apply patterns and structures to solve problems?

Unit Standards

Learning Targets

Estimating Limits Graphically

- I can estimate limits of functions at a point using a graph.
- I can evaluate one-sided limits
- I can determine if a limit exists at a point.
- I can identify limits that do not exist.
- I can evaluate the limits of functions at infinity.

Evaluating Limits Algebraically

- I can apply the properties of limits to evaluate limits of polynomial and rational functions at a point.
- I can use direct substitution to evaluate limits.
- I can identify indeterminate forms and use factoring or rationalizing techniques to evaluate the limits of these functions at a point.
- I can evaluate limits of polynomial and rational functions at infinity.

Tangent Lines and Velocity

- I can find instantaneous rates of change by using the difference quotient to calculate the slope of tangent lines.
- I can find the slope of a graph at a point.
- I can find an equation for the slope of an equation at any point.
- I can find the average and instantaneous velocity and relate these to slopes of secant lines and tangent lines, respectively.
- I can understand the relationship between position and velocity functions.

Derivatives

- I can recognize various notations for derivative functions.
- I can relate the derivative to the difference quotient, slope and instantaneous rates of change.
- I can use the power rule and other derivative rules to find derivatives.
- I can solve for instantaneous rates of change by using derivatives.
- I can understand that critical points occur when the derivative of a function is 0 or undefined, and that relative extrema occur only at critical points.
- I can use the product and quotient rules to calculate derivatives.

Assessment Evidence

Performance Assessment Options

May include, but are not limited to the following:

- Feedback & Scoring Rubric based on Priority Standards

Other assessment options

May include, but are not limited to the following:

- Quick Writes
- KWL Chart - (What I Know, Want to know, Learned)
- Marking Text
- Learning Log Reflection - Daily/Weekly
- I-Chart - Gather/Organize Information on a topic
- Focused Note Taking
- CSG - Collaborative Study Groups
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- Entrance Ticket

- Exit ticket

Digital Tools & Supplementary Resources

- Desmos
- DeltaMath
- TI Calculator
- Excel/Google Sheets

Unit 10 - Vectors - HONORS ONLY

Essential Questions:

1. What model(s) can we as mathematicians use to solve a problem?
2. How can we as mathematicians use and apply patterns and structures to solve problems?

Unit Standards

Supporting Standards

N-VMA Represent and model with vector quantities. - Honors

- Standard 1 (N-VMA1): (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \mathbf{v} , $|\mathbf{v}|$, $\|\mathbf{v}\|$, v).
- (N-VMA2): (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- (N-VMA3): (+) Solve problems involving velocity and other quantities that can be represented by vectors.

N-VMB Perform operations on vectors. - Honors

- (N-VMB5): (+) Multiply a vector by a scalar. (A) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x \mathbf{i} + v_y \mathbf{j}) = (cv_x \mathbf{i} + cv_y \mathbf{j})$. (B) Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|c\mathbf{v}\| = |c|v$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|v$ does not = 0, the direction of $c\mathbf{v}$ is either along \mathbf{v} (for $c > 0$) or against \mathbf{v} (for $c < 0$).

Learning Targets

Introduction to Vectors

- I can find the component form and magnitude of a vector.
- I can perform the basic vector operations -- vector addition and scalar multiplication.
- I can find a unit vector in the same direction as a given vector.
- I can find the direction angle of a vector.
- I can write a vector in the linear combination form of the standard unit vectors.
- I can apply my knowledge of vector addition to solve wind and plane problems.

Dot Products and Vector Projections

- I can find the dot product of two vectors.
- I can find the angle between two vectors.
- I can determine whether two vectors are orthogonal, parallel, or neither.
- I can use a projection vector to decompose a vector into its orthogonal components.
- I can apply my knowledge of dot products and vector projections to solve ramp problems and work problems.

Assessment Evidence

Performance Assessment Options

May include, but are not limited to the following:

- Feedback & Scoring Rubric based on Priority Standards

Other assessment options

May include, but are not limited to the following:

- Quick Writes
- KWL Chart - (What I Know, Want to know, Learned)
- Marking Text
- Learning Log Reflection - Daily/Weekly
- I-Chart - Gather/Organize Information on a topic
- Focused Note Taking
- CSG - Collaborative Study Groups
- Think/Pair/Share
- Entrance Ticket
- Exit ticket

Digital Tools & Supplementary Resources

- Desmos
- DeltaMath
- TI Calculator
- Excel/Google Sheets

Unit 11 - Conics - **HONORS ONLY**

Essential Questions:

1. What model(s) can we as mathematicians use to solve a problem?
2. How can we as mathematicians use and apply patterns and structures to solve problems?

Unit Standards

Supporting Standards

G-GPEA Translate between the geometric description and the equation for a conic section. - Honors (6)

- (G-GPEA3): (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Learning Targets

- I can transform a Cartesian equation for a conic section to other equivalent forms and sketch or plot the graph..
- I can write a Cartesian equation given the graph of a conic section
- I can find the foci, the directrix, and the eccentricity, and vice versa, given the equation for a conic section,
- I can plot a conic section rotated by a specified angle to the coordinate axes.
- I can identify a rotated conic from its Cartesian equation.
- I can plot a rotated conic using its Cartesian equation of parametric equations.

Assessment Evidence

Performance Assessment Options

May include, but are not limited to the following:

- Feedback & Scoring Rubric based on Priority Standards

Other assessment options

May include, but are not limited to the following:

- Quick Writes
- KWL Chart - (What I Know, Want to know, Learned)
- Marking Text
- Learning Log Reflection - Daily/Weekly
- I-Chart - Gather/Organize Information on a topic
- Focused Note Taking
- CSG - Collaborative Study Groups
- Think/Pair/Share
- Entrance Ticket
- Exit ticket

Digital Tools & Supplementary Resources

- Desmos
- DeltaMath
- TI Calculator
- Excel/Google Sheets