Functions and Trigonometry

Curriculum/Content Area: Mathematics	**Course Length: 1 Term**
Course Title: Functions and Trigonometry	**Date last reviewed: 2019** **Previous UbD 2014**
Prerequisites: Algebra 2	**Board approval date: 11/2019**
Primary Resource: Trigonometry, Larson/Hostetler, 6th edition	

Desired Results

Course description and purpose:
Functions and Trigonometry is designed for the student who wishes to expand the concepts in Algebra 2. The course integrates the ideas of functions and trigonometry. This course will build an understanding with real world problems and establish a firm foundation in future work in mathematics courses. A graphing calculator is required

Enduring Understandings: *We will utilize the math practice standards as our Enduring understandings.*	**Essential Questions**: *big picture questions, aligned with enduring understandings* *Examples found here.*
Mathematicians make sense of problems and persevere in solving them.	a. How do we as mathematicians analyze the problem in order to choose the best strategy(ies) or resource to make sense of the problem? b. How do we as mathematicians persevere in solving problems?
Mathematicians attend to precision.	How do we as mathematicians know if we fully & accurately answered the problem and does the results make sense in the context of the problem?
Mathematicians reason abstractly and quantitatively.	How do we as mathematicians make sense of quantities and situations symbolically?
Mathematicians construct viable arguments and critique the reasoning of others.	a. How can we as mathematicians justify our answer(s)? b. How can we as mathematicians evaluate and question whether a mathematical argument is accurate?
Mathematicians model with mathematics.	a. What model(s) can we as mathematicians use to solve a problem? 5b. How can we as mathematicians determine an effective model to use to solve a problem?
Mathematicians use appropriate tools	6. What tools are available and efficient for us as mathematicians to use while solving a problem?

strategically.	
Mathematicians look for and make use of structure	How can we as mathematicians use and apply patterns and structures to solve problems?
Mathematicians look for and express regularity in repeated reasoning.	How can we as mathematicians create and apply generalizations from repeated reasoning?

Unit 1 - Foundations of Trigonometry

A. Graphing angles on the unit circle
B. Trigonometric ratios and values with the unit circle
C. Inverse trigonometric functions and the unit circle
D. Graphing periodic functions
E. Applications

Essential Unit Questions:

1. How can the unit circle be used to represent trigonometric ratios?
2. How can right triangle trigonometry be used to solve problems?
3. How can transformations be applied to the graphs of the six trigonometric functions?

Unit Standards

Priority Standards

Cluster Number F-IFC Analyze functions using different representations.

- **(F-IFC7)**: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (D) (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior

Cluster Number F-BFA Build a function that models a relationship between two quantities.

- **(F-BFA1)**: Write a function that describes a relationship between two quantities. (C) (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Supporting Standards

Cluster Number F-TFA Extend the domain of trigonometric functions using the unit circle.

- **(F-TFA3)**: (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for pi/3, pi/4 and pi/6, and use the unit circle to express the values of sine, cosine, and tangent for pi-x, pi+x, and 2pi-x in terms of their values for x, where x is any real number.
- **(F-TFA4)**: (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Learning Targets- I can...

- I can describe an angle and convert between radian and degree measure.
- I can identify a unit circle and its relationship to real numbers.

- I can evaluate the trigonometric functions of any angle
- I can evaluate the inverse trigonometric functions.
- I can evaluate the compositions of trigonometric functions and inverse trigonometric functions.
- I can sketch the graph of the six trigonometric functions and their transformations.
- I can solve applications using right triangle trigonometry.

Assessment Evidence

Performance Assessment Options *May include, but are not limited to the following:* ● Formative and Summative Assessment ● Feedback & Scoring Rubric based on Priority Standards	**Other assessment options** *May include, but are not limited to the following:* ● Projects

Digital Tools & Supplementary Resources

TI-83/84 Calculator /Interactive software

Unit 2 - Analytic Trigonometry

A. Simplifying and verifying trigonometric expressions.
B. Solving trigonometric equations

Essential Unit Questions:

1. How can the unit circle be used to represent trigonometric ratios?
2. How can right triangle trigonometry be used to solve problems?
3. How can transformations be applied to the graphs of the six trigonometric functions?
4. How can trigonometric identities be used to solve problems?

Unit Standards

Priority Standards

Cluster Number F-IFC Analyze functions using different representations.

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Cluster Number F-BFB Build new functions from existing functions.

- **(F-BFB4)**: Find inverse functions. (B) (+) Verify by composition that one function is the inverse of another. (C) (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. (D) (+) Produce an invertible function from a non-invertible function by restricting the domain.

Cluster Number F-TFB Model periodic phenomena with trigonometric functions.

- **(F-TFB7):** (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Cluster Number F-TFC Prove and apply trigonometric identities.

- **(F-TFC9)**: (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Learning Targets- I can...

- I can use fundamental trigonometric identities to evaluate trigonometric functions and simplify expressions.
- I can verify trigonometric identities.
- I can use standard algebraic techniques and inverse trigonometric functions to solve trigonometric equations.
- I can use the sum and difference and double angle formulas to rewrite and solve trigonometric functions.

Assessment Evidence

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Unit 3 - Non Right triangle Trigonometry and Vectors

A. Law of Sines & Cosines
B. Vectors in the plane

Essential Unit Questions:

1. How can the unit circle be used to represent trigonometric ratios?
2. How can right triangle trigonometry be used to solve problems?
3. How can transformations be applied to the graphs of the six trigonometric functions?

4. How can trigonometric identities be used to solve problems?
5. How can trigonometry be applied to non right triangle problems?

Unit Standards

Priority Standards

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- **(F-TFC9)**: (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

***Cluster Number N-VMA* Represent and model with vector quantities.**

- **(N-VMA1)**: (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, v).

Learning Targets- I can...

- I can use the Law of Sines and Cosines to solve oblique triangles.
- I can find the areas of oblique triangles.
- I can solve applications using non right triangle trigonometric formulas.
- I can write the component form of vectors and perform basic vector operations.
- I can find the direction angles of vectors and the angle between two vectors.
- I can solve applications using vectors.

Assessment Evidence

Performance Assessment Options	**Other assessment options**

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Digital Tools & Supplementary Resources

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Unit 4 - Complex Numbers

A. Complex Numbers
B. Trigonometric Form of a Complex Number

Essential Unit Questions:

1. How can the unit circle be used to represent trigonometric ratios?
2. How can right triangle trigonometry be used to solve problems?
3. How can transformations be applied to the graphs of the six trigonometric functions?
4. How can trigonometric identities be used to solve problems?
5. How can trigonometry be applied to non right triangle problems?
6. How does trigonometry connect the real number system to the complex number system?

Unit Standards

Priority Standards

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Cluster Number F-TFC Prove and apply trigonometric identities.

- **(F-TFC9)**: (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Cluster Number N-VMA **Represent and model with vector quantities.**

- **(N-VMA1)**: (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, v).
- **(N-VMA2):** (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- **(N-VMA3)**: (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Cluster Number N-CNA Perform arithmetic operations with complex numbers. (N-CNA)

- **(N-CNA3):** (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers

Cluster Number N-CNB Represent complex numbers and their operations on the complex plane.

- **(N-CNB4):** (+) Represent complex numbers on the complex plane in rectangular and (polar form) (including real and imaginary numbers), and explain why the rectangular and (polar forms) of a given complex number represent the same number. (Note: Polar form is an extension topic)
- **(N-CNB5)**: (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, (-1 + square root of 3 i)^3 = 8 because (-1 + the square root of 3 i) has modulus 2 and argument 120 degrees.

Cluster Number N-VMB Perform operations on vectors.

- **(N-VMB4): (+)** Add and subtract vectors. (A) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. (B) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. (C) Understand vector subtraction v - w as v + (-w), where -w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- **(N-VMB5)**: (+) Multiply a vector by a scalar. (A) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(v x^3 v y) = (cv x^3 cv y). (B) Compute the magnitude of a scalar multiple c**v** using ||c**v**|| = |c|v. Compute the direction of c**v** knowing that when |c|v does not = 0, the direction of c**v** is either along **v** (for c > 0) or against **v** (for c < 0).

Learning Targets- I can...

- I can perform operations on complex numbers.
- I can find complex solutions to quadratic equations.
- I can write a complex number in trigonometric form.
- I can graph a complex number.

Assessment Evidence

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