

AP Calculus BC

Curriculum/Content Area: Mathematics	Course Length: 2 terms
Course Title: AP Calculus BC	Date last reviewed: 2014 Previous UbD 2014
Prerequisites: Honors Precalculus or AP Calculus AB	Board approval date: 11/2019
Primary Resource: Calculus: Early Transcendental Functions by Cengage Learning	

Desired Results

Course description and purpose:

AP Calculus BC is an introductory college-level calculus course. Students cultivate their understanding of differential and integral calculus through engaging with real-world problems represented graphically, numerically, analytically, and verbally and using definitions and theorems to build arguments and justify conclusions as they explore concepts like change, limits, and the analysis of functions. Technology will be used by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

AP Mathematical Practice Standards	
The AP Mathematical Practice Standards are distinct skills that students should practice throughout the year-- skills that will help them learn to think and act like mathematicians. These skills span the entire course.	
AP Calculus Mathematical Practices	Essential Questions:
1. Students will be able to implement mathematical processes.	1. How can mathematical procedures and rules be used to determine expressions and values?
2. Students will be able to connect different representations of mathematical problems.	2a. How can mathematical ideas be represented verbally, algebraically, numerically, and graphically? 2b. How can mathematical information from a single representation be translated across multiple representations?
3. Students will be able to justify their reasoning and solutions.	3. How can supportive data acquired from sign charts, graphs, and knowledge from other curricular areas be used to justify answers?
4. Students will be able to	4. What are the correct notation, language, and mathematical

communicate effectively using proper notation.	conventions that should be used to communicate results or solutions?
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Big Ideas

The big ideas serve as the foundation of the course and allow students to create meaningful connections among concepts. They are often abstract concepts or themes that become thread that runs throughout the course. Revisiting the big ideas and applying them in a variety of contexts allows students to develop deeper conceptual understanding. Below

BIG IDEA 1: CHANGE (CHA)

Using derivatives to describe rates of change of one variable with respect to another or using definite integrals to describe the net change in one variable over an interval of another allows students to understand change in a variety of contexts. It is critical that students grasp the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus—a central idea in AP Calculus.

BIG IDEA 2: LIMITS (LIM)

Beginning with a discrete model and then considering the consequences of a limiting case allows us to model real-world behavior and to discover and understand important ideas, definitions, formulas, and theorems in calculus..

BIG IDEA 3: ANALYSIS OF FUNCTIONS (FUN)

Calculus allows us to analyze the behaviors of functions by relating limits to differentiation, integration, and infinite series and relating each of these concepts to the others.

Unit 1 - Limits and Continuity

Essential Unit Questions:

1. How can limits be calculated algebraically, graphically, and numerically?
2. How can limits be used to prove continuity?
3. How are infinite limits calculated?

AP Calculus Mathematical Practices

Priority Standards

1. **Implementing Mathematical Practices:** Determine expressions and values using mathematical procedures and rules.
2. **Connecting Representations:** Translate mathematical information from a single representation or across multiple representations.
3. **Justification:** Justify reasoning and solutions.
4. **Communication and Notation:** Use correct notation, language, and mathematical conventions to communicate results or solutions.

Learning Targets

- I can define limits and use correct limit notation.
- I can estimate limit values from a graph.
- I can estimate limit values from a table.
- I can determine limits using algebraic properties of limits.

- I can determine limits using algebraic manipulation.
- I can select different procedures for determining limits.
- I can determine limits using the squeeze theorem.
- I can connect multiple representations of limits.
- I can explore types of discontinuities.
- I can confirm continuity over an interval.
- I can determine removable discontinuities.
- I can work with the Intermediate Value Theorem (IVT).
- I can connect infinite limits and vertical asymptotes.
- I can connect limits at infinity and horizontal asymptotes.

Assessment Evidence

Performance Assessment Options

May include, but are not limited to the following:

- Formative and summative assessments.
- Feedback & Scoring Rubric based on Priority Standards

Other assessment options

May include, but are not limited to the following:

- Unit Personal Progress Checks

Digital Tools & Supplementary Resources

A graphing calculator is required for this unit.
AP Classroom Resources

Unit 2 - Differentiation: Definition and Basic Derivative Rules

Essential Unit Questions:

1. How can the limit definition be used to find the derivative of a function?
2. What are the basic rules of differentiation?
3. What is the difference between average and instantaneous rates of change?

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Learning Targets

- I can define average and instantaneous rates of change at a point.

- I can define the derivative of a function and use derivative notation.
- I can estimate derivatives of a function at a point.
- I can connect differentiability and continuity.
- I can determine when derivatives do and do not exist.
- I can apply the power rule.
- I can use the constant, sum, difference, and constant multiple derivative rules.
- I can find derivatives of $\cos(x)$, $\sin(x)$, e^x , and $\ln(x)$.
- I can find derivatives using the product rule and quotient rule.
- I can find the derivative of tangent, cotangent, secant, and/or cosecant functions.

Assessment Evidence

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Digital Tools & Supplementary Resources

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AP Classroom Resources

Unit 3 - Differentiation: Composite, Implicit, and Inverse Functions

Essential Unit Questions:

1. In what instances is it necessary to use implicit differentiation?
2. How can it be determined which method of differentiation should be eased for certain problems?

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Learning Targets

- I can use the chain rule to find a derivative.
- I can use implicit differentiation.

- I can differentiate inverse functions.
- I can differentiate inverse trigonometric functions.
- I can select procedures for calculating derivatives.
- I can calculate higher-order derivatives.

Assessment Evidence

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Other assessment options

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Digital Tools & Supplementary Resources

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AP Classroom Resources

Unit 4 - Contextual Applications of Differentiation

Essential Unit Questions:

1. How can velocity and acceleration be found when given a position function?
2. What is an indeterminate form and how can a limit be found when dealing with these types of problems?
3. What are the guidelines for solving a related rates problem?

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Learning Targets

- I can interpret the meaning of the derivative in context.
- I can connect position, velocity, and acceleration in straight-line motion.
- I can find and apply rates of change in contexts other than motion.
- I can solve related rates problems.
- I can approximate values of a function using local linearity and linearization.
- I can use L'Hospital's Rule for determining limits of indeterminate forms.

Assessment Evidence

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Other assessment options

May include, but are not limited to the following:

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Digital Tools & Supplementary Resources

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AP Classroom Resources

Unit 5 - Analytical Applications of Differentiation

Essential Unit Questions:

1. What are the procedures for determining critical points and whether they are extrema?
2. What must a function have if there is a change in concavity?
3. What are the guidelines for solving applied minimum and maximum problems?

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Learning Targets

- I can use the Mean Value Theorem (MVT).
- I can use the Extreme Value Theorem (EVT), distinguish between global and local extrema, and find critical points.
- I can determine intervals on which a function is increasing or decreasing.
- I can use the first derivative test to determine relative (local) extrema.
- I can use the candidates test to determine absolute (global) extrema.
- I can determine concavity of functions over their domains.
- I can use the second derivative test to determine extrema.
- I can sketch graphs of functions and their derivatives.
- I can connect a functions, its first derivative, and its second derivative.
- I can solve optimization problems.
- I can explore behaviors of implicit relations.

Assessment Evidence	
<p>Performance Assessment Options <i>May include, but are not limited to the following:</i></p> <ul style="list-style-type: none"> ● Formative and summative assessments. ● Feedback & Scoring Rubric based on Priority Standards 	<p>Other assessment options <i>May include, but are not limited to the following:</i></p> <ul style="list-style-type: none"> ● Unit Personal Progress Checks
Digital Tools & Supplementary Resources	
<p>A graphing calculator is required for this unit. AP Classroom Resources</p>	

Unit 6 - Integration and Accumulation of Change
<p>Essential Unit Questions:</p> <ol style="list-style-type: none"> 1. How can the area under a curve be found geometrically and by using the Fundamental Theorem of Calculus? 2. How can the area under a curve be approximated using left endpoints, right endpoints, or midpoints? 3. How can limits be used to find the area under a curve?
AP Calculus Mathematical Practices
<p>Priority Standards</p> <ol style="list-style-type: none"> 1. Implementing Mathematical Practices: Determine expressions and values using mathematical procedures and rules. 2. Connecting Representations: Translate mathematical information from a single representation or across multiple representations. 3. Justification: Justify reasoning and solutions. 4. Communication and Notation: Use correct notation, language, and mathematical conventions to communicate results or solutions.
Learning Targets
<ul style="list-style-type: none"> ● I can find accumulations of change ● I can approximate areas with Riemann sums. ● I can relate Riemann sums, summation notation, and definite integral notation. ● I can use the Fundamental Theorem of Calculus (FTC) and accumulation functions. ● I can interpret the behavior of accumulating functions involving area. ● I can apply properties of definite integrals. ● I can use relate the FTC and definite integrals. ● I can relate antiderivatives and indefinite integrals using basic rules and notation. ● I can integrate using substitution. ● I can integrate functions using long division and completing the square.

- I can integrate using integration by parts.
- I can integrate using linear partial fractions.
- I can evaluate improper integrals.
- I can determine which technique is most appropriate for antidifferentiation.

Assessment Evidence

Performance Assessment Options

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Other assessment options

May include, but are not limited to the following:

- Unit Personal Progress Checks

Digital Tools & Supplementary Resources

A graphing calculator is required for this unit.

AP Classroom Resources

Unit 7 - Differential Equations

Essential Unit Questions:

1. What is a differential equation?
2. How are slope fields and differential equations related?
3. What are some examples of exponential growth and decay models?

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Learning Targets

- I can model situations with differential equations.
- I can verify solutions for differential equations.
- I can sketch slope fields.
- I can approximate solutions using Euler's Method.
- I can find general solutions using separation of variables.
- I can find particular solutions using initial conditions and separation of variables.
- I can use exponential models with differential equations.

- I can use logistic models with differential equations.

Assessment Evidence

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Other assessment options

May include, but are not limited to the following:

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Digital Tools & Supplementary Resources

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AP Classroom Resources

Unit 8 - Applications of Integration

Essential Unit Questions:

1. What is the difference between the disc and washer methods?
2. How can total distance traveled and displacement be calculated using integration?
3. How is it determined whether to use curves expressed as functions of x or functions of y when finding area?

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Learning Targets

- I can find the average value of a function on an interval.
- I can connect position, velocity, and acceleration of functions using integrals.
- I can use accumulation functions and definite integrals in applied contexts.
- I can find the area between curves expressed as functions of x .
- I can find the area between curves expressed as functions of y .
- I can find the area between curves that intersect at more than two points.
- I can find volumes with cross sections that are squares, rectangles, triangles, and semicircles.
- I can find the volume by revolving around the x - or y -axis using the disc method.
- I can find the volume using disc method and revolving around other axes.

- I can find the volume by revolving around the x- or y-axis using the washer method.
- I can find the volume using washer method and revolving around other axes.
- I can find the arc length of a smooth, planar curve and distance traveled.

Assessment Evidence

Performance Assessment Options

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Other assessment options

May include, but are not limited to the following:

- Unit Personal Progress Checks

Digital Tools & Supplementary Resources

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AP Classroom Resources

Unit 9 - Parametric Equations, Polar Coordinates, and Vector-Valued Functions

Essential Unit Questions:

1. How is slope and concavity determined from parametric equations?
2. What is the procedure for converting polar coordinates to rectangular coordinates?
3. What are the major differences in calculating area between two curves given as rectangular equations versus two curves given as polar equations?

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Learning Targets

- I can define and differentiate parametric equations.
- I can find second derivatives of parametric equations.
- I can find arc lengths or curves given by parametric equations.
- I can define and differentiate vector-valued functions.
- I can integrate vector-valued functions.
- I can solve motion problems using parametric and vector-valued functions.
- I can define polar coordinates and differentiate in polar form.

- I can find the area of a polar region or the area bounded by a single polar curve.
- I can find the area of the region bounded by two polar curves.

Assessment Evidence

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Other assessment options

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Digital Tools & Supplementary Resources

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AP Classroom Resources

Unit 10 - Infinite Sequences and Series

Essential Unit Questions:

1. What is the difference between a sequence and a series?
2. What test for convergence is most appropriate for a given series?
3. How can the accuracy of a Taylor polynomial approximation be determined from the actual value?
4. What is the general procedure for finding the form that every convergent power series must take?

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Learning Targets

- I can define convergent and divergent infinite series.
- I can work with geometric series.
- I can test for divergence using the nth term test.
- I can use the integral test for convergence.
- I can recognize the harmonic series and p-series.
- I can use the comparison tests for convergence.

- I can use the alternating series test for convergence.
- I can use the ratio test for convergence.
- I can determine absolute or conditional convergence.
- I can find the alternating series error bound.
- I can find Taylor polynomial approximations of functions.
- I can find the Lagrange error bound.
- I can find the radius and interval of convergence of a power series.
- I can find Taylor or Maclaurin series for a function.
- I can represent functions as a power series.

Assessment Evidence

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Other assessment options

May include, but are not limited to the following:

- Unit Personal Progress Checks

Digital Tools & Supplementary Resources

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AP Classroom Resources