Name _____

Model Integer Addition

Essential Question How can you use a number line to model addition of integers?
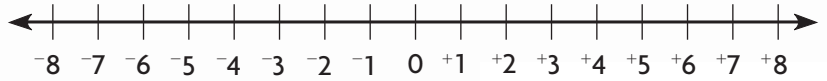
🔑 Unlock the Problem (Real World)

In the first round of a game, Laura lost 5 points. Then she won 9 points in the second round. What is her score after the second round?

- How can you represent a loss of 5 points with an integer? _____
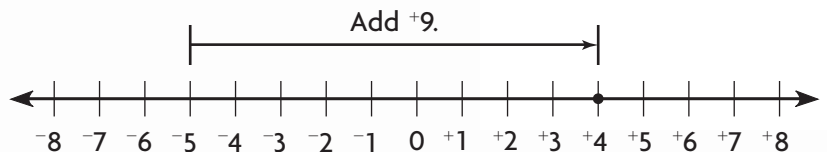- How can you represent a gain of 9 points with an integer? _____

🔑 **Find** $^-5 + {}^+9$.

STEP 1 Draw a number line.

$^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 $^+1$ $^+2$ $^+3$ $^+4$ $^+5$ $^+6$ $^+7$ $^+8$

STEP 2 Start at 0. Move 5 units to the _____ to show $^-5$.

Show $^-5$.

$^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 $^+1$ $^+2$ $^+3$ $^+4$ $^+5$ $^+6$ $^+7$ $^+8$

STEP 3 From $^-5$, move 9 units to the _____ to add $^+9$.

Add $^+9$.

$^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 $^+1$ $^+2$ $^+3$ $^+4$ $^+5$ $^+6$ $^+7$ $^+8$

$^-5 + {}^+9 =$ _____

So, Laura's score after the second round is _____.

Math Talk **Mathematical Practices**

Will $^-5 + {}^+8$ be the same as $^+8 + {}^-5$? **Explain.**

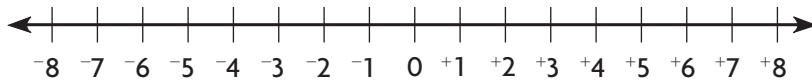**Try This!** **Tell how to find the sum using a number line.**

A. $^+6 + {}^-8$

_____

_____

_____

_____

B. $^-2 + {}^-6$

_____

_____

_____

_____

Share and Show

MATH BOARD

1. Use the number line to find $^+4 + {}^-7$.

$$\leftarrow\!\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!+\!\!\rightarrow$$

$^-8 \quad ^-7 \quad ^-6 \quad ^-5 \quad ^-4 \quad ^-3 \quad ^-2 \quad ^-1 \quad 0 \quad ^+1 \quad ^+2 \quad ^+3 \quad ^+4 \quad ^+5 \quad ^+6 \quad ^+7 \quad ^+8$

Start at _____.

Move 4 units to the _____. Then move 7 units to the _____.

The sum is _____.

Draw a number line to find the sum.

2. $^-3 + {}^+1$ _____

3. $^-5 + {}^-1$ _____

4. $^+6 + {}^-6$ _____

On Your Own

Draw a number line to find the sum.

5. $^-8 + {}^+4$ _____

6. $^-3 + {}^-3$ _____

7. $^+7 + {}^-9$ _____

8. $^+5 + {}^-4$ _____

9. $^-4 + {}^-3$ _____

10. $^-2 + {}^+10$ _____

Problem Solving Real World

11. In a football game, Jim's team gained 7 yards on the first play, lost 2 yards on the second play, and lost 10 yards on the third play. How many total yards did Jim's team gain or lose after three plays?

_____

12. In the morning the temperature was $^-3$°F. By noon it had risen by 10°F. What was the temperature at noon?

_____

Name

Model Integer Subtraction

Essential Question How can you use a number line to model subtraction of integers?

Unlock the Problem (Real World)

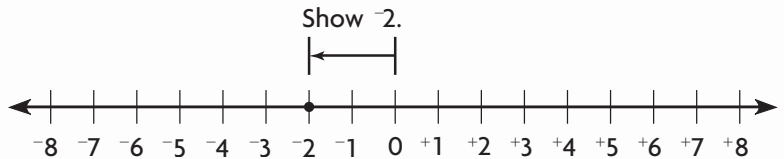At 6:00 P.M., the temperature was ⁻2˚F. By midnight, it had dropped 5˚F. What was the temperature at midnight?

> **Math Idea**
> * Move left on a number line to subtract a positive integer.
> * Move right on a number line to subtract a negative integer.

🔑 **Find ⁻2 − ⁺5.**

STEP 1 Draw a number line.

Start at 0, move 2 units to the

_____ to show ⁻2.

Show ⁻2.

⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8

STEP 2 From ⁻2, move 5 units to the

_____ to subtract ⁺5.

⁻2 − ⁺5 = _____

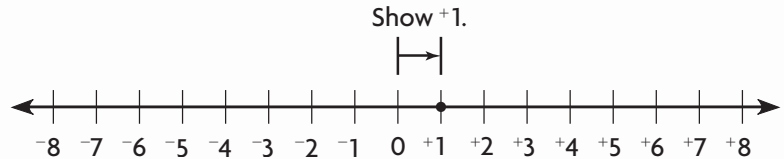⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8

So, the temperature at midnight was ⁻7˚F.

EXAMPLE Find ⁺1 − ⁻4.

STEP 1 Draw a number line. Start at 0,

move 1 unit to the _____ to show ⁺1.

Show ⁺1.

⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8

STEP 2 From ⁺1, move 4 units to the right to subtract ⁻4.

So, ⁺1 − ⁻4 = _____.

⁻8 ⁻7 ⁻6 ⁻5 ⁻4 ⁻3 ⁻2 ⁻1 0 ⁺1 ⁺2 ⁺3 ⁺4 ⁺5 ⁺6 ⁺7 ⁺8
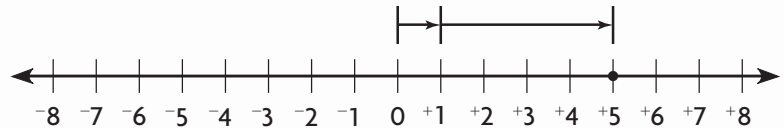
Math Talk **Mathematical Practices**

Will ⁺1 − ⁻4 be the same as ⁻4 − ⁺1? **Explain.**

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1. Use the number line to find $^-3 - ^-3$.

Start at _____.

Move 3 units to the _____.

Move _____ on the number line to subtract $^-3$.

The difference is _____.

Draw a number line to find the difference.

2. $^-3 - ^+4$ _____

3. $^+5 - ^+9$ _____

4. $^+2 - ^-4$ _____

On Your Own

Draw a number line to find the difference.

5. $^-2 - ^+2$ _____

6. $^+1 - ^-6$ _____

7. $^-7 - ^-7$ _____

8. $^+4 - ^-4$ _____

9. $^+3 - ^+6$ _____

10. $^-8 - ^-3$ _____

Problem Solving · Real World

11. In a golf tournament, Tim got a score of $^+2$ in the first round and a score of $^-3$ in the second round. What was the difference in his scores between the first round and the second round?

_____

12. The high temperature one day was $^-3°$F. The low temperature was $^-7°$F. What was the difference between the high and low temperatures that day?

_____

Name _____

Model Integer Multiplication

Essential Question How can you use a number line to model multiplication of integers?
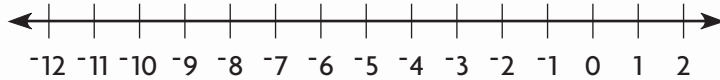
🔑 Unlock the Problem (Real World)

Kayla is scuba diving to explore coral reefs. She makes 5 equal descents of 2 meters each. What is Kayla's elevation at the end of her descent?
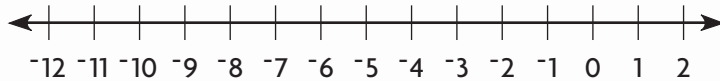
🔑 **Find $^-2 \times {}^+5$.**

By the Commutative Property, $^-2 \times 5 = 5 \times {}^-2$.

STEP 1 Draw a number line.

$$\longleftarrow \underset{^-12\ \ ^-11\ \ ^-10\ \ ^-9\ \ ^-8\ \ ^-7\ \ ^-6\ \ ^-5\ \ ^-4\ \ ^-3\ \ ^-2\ \ ^-1\ \ 0\ \ 1\ \ 2}{|\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |}\longrightarrow$$

STEP 2 Start at 0. Show five groups of $^-2$.

$$\longleftarrow \underset{^-12\ \ ^-11\ \ ^-10\ \ ^-9\ \ ^-8\ \ ^-7\ \ ^-6\ \ ^-5\ \ ^-4\ \ ^-3\ \ ^-2\ \ ^-1\ \ 0\ \ 1\ \ 2}{|\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |\ \ |}\longrightarrow$$

$^+5 \times {}^-2 = {}^-10$

So, Kayla's elevation at the end of her descent is _____ feet.

Math Idea

$5 \times {}^-2$ means 5 groups of $^-2$.

Math Talk — **Mathematical Practices**

What do you notice about the sign of the product when you multiply a positive integer and a negative integer?

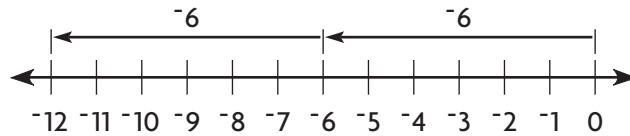**Try This!** Tell how to find the product using a number line.

A. $^+3 \times (^-2)$

_____

_____

_____

_____

B. $^-4 \times {}^+6$

_____

_____

_____

_____

Share and Show

MATH BOARD

1. Use the number line to find $^+2 \times (^-6)$.

Start at _____.

Show _____ groups of _____.

The product is _____.

Draw a number line to find the product.

2. $^+4 \times (^-1)$

_____

3. $^-5 \times {}^+3$

_____

4. $^+7 \times (^-2)$

_____

On Your Own

Draw a number line to find the product.

5. $^-4 \times {}^+3$

_____

6. $^+3 \times (^-7)$

_____

7. $^-2 \times {}^+4$

_____

8. $^+5 \times (^-6)$

_____

9. $^-9 \times {}^+2$

_____

10. $^+6 \times (^-8)$

_____

Problem Solving Real World

11. The Milam glacier is changing size at a rate of $^-3$ feet per month. Write a number sentence to show the change in size of the glacier after 3 months.

_____

12. An elevator starts at the lobby of a building and descends into the basement. The elevator's height changes by $^-6$ meters each second. What is the change in the elevator's height after 3 seconds?

_____

Name _____

✓ Checkpoint

Concepts and Skills

Draw a number line to find the sum, difference, or product.

1. $^+4 + (^-2)$

2. $^+5 - (^-1)$

3. $^+6 \times (^-2)$

_____

4. $^-2 - {}^+7$

5. $^+7 + (^-2)$

6. $^-3 \times {}^+1$

_____

7. $0 - {}^+4$

8. $^+3 \times (^-2)$

9. $^-1 + {}^+5$

_____

10. $^-3 \times {}^+7$

11. $^-6 + {}^+8$

12. $^-2 - {}^+4$

_____

13. $^+10 + (^-12)$

14. $^-1 - {}^+5$

15. $^-6 \times {}^+3$

_____

Problem Solving (Real World)

16. The price of a stock increased by $6, then decreased by $8. What integer represents the overall change in the price?

_____

17. The captain of a boat is 1 meter above sea level. A diver is 8 meters below sea level. What is the difference in elevation between the captain and the diver?

_____

18. A researcher lowers the temperature of a specimen by 3° each hour for four hours. What integer shows the total change in temperature?

_____

Choose the letter of the correct answer.

19. When Carlos went to bed, the temperature was $^-2°$F. While he slept, the temperature rose by $5°$F. What was the temperature when Carlos woke up?

    (A) $^-7°$F

    (B) $^-3°$F

    (C) $3°$F

    (D) $7°$F

20. To prepare for landing, a jet makes 5 equal descents of 200 meters. Which integer represents the total change in elevation?

    (A) $^-200$

    (B) $^-500$

    (C) $^-1,000$

    (D) $^-2,000$

21. The temperature at the base of a mountain is $16°$F. The temperature at the summit is $^-4°$F. What is the difference in temperatures between the base of the mountain and the summit?

    (A) $20°$F

    (B) $12°$F

    (C) $^-12°$F

    (D) $^-20°$F

22. In a carnival game, Juan won 11 tickets on his first turn, lost 5 tickets on his second turn, and lost 4 tickets on his third turn. What integer represents the number of tickets Juan has after three turns?

    (A) 20

    (B) $^-2$

    (C) 2

    (D) $^-20$

23. A submarine sailing 30 meters below sea level rises 20 meters. What integer represents the submarine's new elevation?

    (A) $^-50$

    (B) $^-10$

    (C) 10

    (D) 20

24. Which of these expressions has the greatest value?

    (A) $^-1 + {}^+7$

    (B) $^+2 \times ({}^-4)$

    (C) $^+8 - ({}^-3)$

    (D) $^-4 + {}^+5$

GR8

Name _____

Simplify Complex Fractions

Essential Question How can you simplify complex fractions?

A **complex fraction** is a fraction in which the numerator, denominator, or both contain fractions.

⚷ Unlock the Problem Real World

Jerrod swam $\frac{1}{2}$ mile in $\frac{1}{4}$ hour. The complex fraction $\dfrac{\frac{1}{2}}{\frac{1}{4}}$ is a ratio that represents Jerrod's speed in miles per hour.

🔑 **Simplify the complex fraction that shows Jerrod's speed to find the unit rate.**

STEP 1 Write the complex fraction as division.

$$\dfrac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \div \frac{1}{4}$$

STEP 2 Use the reciprocal of the divisor to write a multiplication problem. Multiply. Write your answer in simplest form.

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{1} = 2$$

So, Jerrod's speed was 2 miles per hour.

- What operation does the bar between the numerator and denominator of a fraction represent?

_____

Math Talk **Mathematical Practices**

Explain how you can use division to simplify a complex fraction.

🔑 Example Simplify $\dfrac{\frac{2}{3}}{\frac{4}{5}}$.

STEP 1 Write the complex fraction as division.

$$\dfrac{\frac{2}{3}}{\frac{4}{5}} = \frac{2}{3} \div \frac{4}{5}$$

STEP 2 Use the reciprocal of the divisor to write a multiplication problem. Multiply. Write your answer in simplest form.

$$\frac{2}{3} \div \frac{4}{5} = \dfrac{\overset{1}{\cancel{2}}}{3} \times \dfrac{5}{\underset{2}{\cancel{4}}} = \text{_____}$$

So, $\dfrac{\frac{2}{3}}{\frac{4}{5}} = \text{_____}$.

MATH
BOARD

Simplify $\dfrac{\frac{1}{4}}{\frac{3}{5}}$.

1. Write the complex fraction using division: $\dfrac{\frac{1}{4}}{\frac{3}{5}} =$ _____

2. Use the reciprocal of the divisor to write a multiplication problem. Multiply. Write your answer in simplest form.

$\dfrac{1}{4} \times$ _____ $=$ _____

Simplify the complex fraction. Write your answer in simplest form.

3. $\dfrac{\frac{1}{4}}{\frac{2}{3}}$

_____

4. $\dfrac{\frac{4}{5}}{\frac{1}{5}}$

_____

5. $\dfrac{\frac{1}{2}}{\frac{3}{4}}$

_____

Simplify the complex fraction. Write your answer in simplest form.

6. $\dfrac{\frac{1}{4}}{\frac{2}{5}}$

_____

7. $\dfrac{\frac{1}{6}}{\frac{2}{5}}$

_____

8. $\dfrac{\frac{1}{8}}{\frac{7}{8}}$

_____

9. $\dfrac{\frac{3}{8}}{\frac{3}{4}}$

_____

10. $\dfrac{\frac{2}{9}}{\frac{2}{3}}$

_____

11. $\dfrac{\frac{1}{5}}{\frac{3}{8}}$

_____

Problem Solving Real World

12. Meg ran $\frac{7}{8}$ mile in $\frac{1}{8}$ hour. What was her running speed in miles per hour?

_____

13. Kareem needs $\frac{3}{4}$ cup of flour to bake a batch of cupcakes. He has $\frac{1}{2}$ cup. What fraction of a batch can Kareem bake?

_____

Name _____

Identify Proportional Relationships

Essential Question How can you identify a proportional relationship?

A **proportional relationship** is a relationship between two quantities in which the ratio of one quantity to the other quantity is constant.

🔑 Unlock the Problem (Real World)

Kudzu is a fast-growing plant that is found in the southeastern United States. In summer, kudzu grows 12 inches per day. Is the relationship between the length of a kudzu plant and the number of days it has been growing a proportional relationship?

• What operation can you use to find the length of a kudzu plant after a certain number of days?

_____

🔑 **Find and compare the ratios of the length of a kudzu plant to the number of days it has been growing.**

STEP 1 Make a table of values.

Number of days	1	2	3	4	5
Length (in.)	12	24	36	48	60

STEP 2 Find and compare ratios.

$$\frac{\text{length (in.)}}{\text{number of days}} = \frac{12}{1} = \frac{24}{2} = \frac{36}{3} = \frac{48}{4} = \frac{60}{5} = 12$$

The ratios are constant.
So, the relationship is a proportional relationship.

🔑 Example

Judy drives 150 miles in 3 hours, 250 miles in 5 hours, and 400 miles in 8 hours. Is the relationship between distance and time a proportional relationship? If so, what is the unit rate?

Find and compare ratios: $\frac{\text{distance}}{\text{time}} = \frac{150}{3} = \frac{250}{5} = \frac{400}{8} = $ _____

The ratios are constant.

So, the relationship is a proportional relationship.

The unit rate is the ratio that gives the distance traveled in

one hour. The unit rate is _____ miles per hour.

Math Talk **Mathematical Practices**

Describe the connection between proportional relationships and unit rates.

Share and Show

MATH BOARD

There are 4 mg of vitamin C in every cup of blueberries. Is the relationship between the amount of vitamin C and the number of cups a proportional relationship?

1. Make a table of values.

Number of cups	1	2	3	4	5
Vitamin C (mg)	4				

2. Find the ratios of the amount of vitamin C to the number of cups of blueberries.

_____

3. Is the relationship a proportional relationship?

_____

On Your Own

4. Each pound of dried cranberries costs $3.50. Is the relationship between cost and the number of pounds a proportional relationship?

_____

5. The equation $y = 2x$ represents the cost y of buying x pounds of cheese. Complete the table and plot the ordered pairs. Tell whether the relationship between y and x is a proportional relationship and describe what you notice about the points you plotted.

Pounds, x	1	2	3	4
Dollars, y				

_____

_____

_____

Cost of Cheese

Problem Solving (Real World)
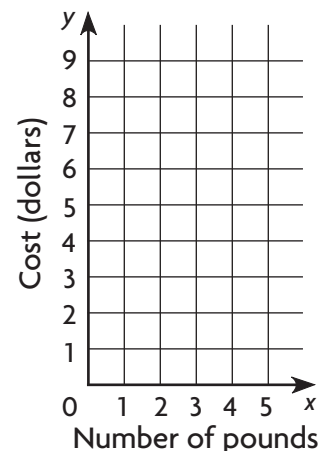
6. An elevator rises 40 feet in 2 seconds, 100 feet in 5 seconds, and 180 feet in 9 seconds. Is the relationship between distance and time a proportional relationship? If so, what is the unit rate?

_____

7. Drew types 45 words in 1 minute, 120 words in 3 minutes, and 184 words in 4 minutes. Is the relationship between the number of words and time a proportional relationship? If so, what is the unit rate?

_____

GR12

Analyze Proportional Relationships

Essential Question How can you identify the constant of proportionality in proportional relationships?

A proportional relationship is a relationship between two variables, x and y, that can be written in the form $y = kx$, or $\frac{y}{x} = k$, where k is a nonzero number called the *constant of proportionality*. The graph of a proportional relationship is a straight line through the origin.

Unlock the Problem *Real World*

Potato salad costs $3 per pound at a local deli. Write and graph an equation for the proportional relationship. Give the constant of proportionality.

- What operation will you use in your equation for this relationship?

  _____

Analyze the relationship.

STEP 1 Write an equation for the relationship. Let x represent the number of pounds of potato salad. Let y represent the cost of buying x pounds.

Cost = $3 times the number of pounds
$$y = 3 \cdot x$$
$$y = 3x$$

STEP 2 Make a table of values. Then graph the relationship by plotting several points and drawing a line through the points and through the origin.

x	y
1	3
2	6
3	9

STEP 3 Identify the constant of proportionality.
The constant of proportionality in $y = 3x$ is 3.

Potato Salad Cost

Math Talk **Mathematical Practices**

Explain why the equation $y = x$ shows a proportional relationship. What is the constant of proportionality?
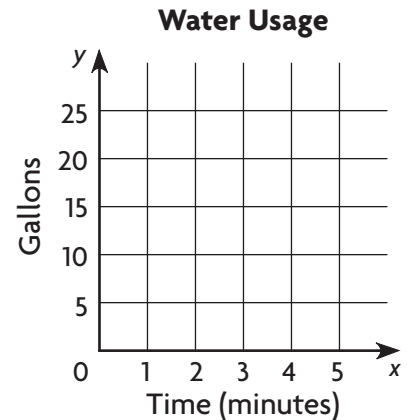
Share and Show

MATH BOARD

A shower uses 5 gallons of water per minute. Use this information for 1–3.

1. Let x represent the number of minutes. Let y represent the number of gallons of water used. Write an equation that relates x and y.

2. Give the constant of proportionality.
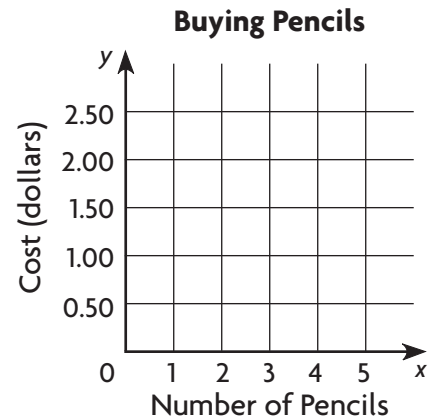
3. Graph the equation you wrote in Exercise 1.

Water Usage

On Your Own

Pencils cost $0.50 each. Use this information for 4–6.

4. Let x represent the number of pencils and let y represent the cost. Write an equation that relates x and y.

5. Give the constant of proportionality. _____

6. Graph the equation you wrote in Exercise 4.

Buying Pencils

Problem Solving **Real World**

The graph shows the data about a typical whale's heartbeats. Use the graph for 7–9.

7. Complete the table.

x	1	2	4		
y				140	160

8. Use the table to find the ratio $\frac{y}{x}$. _____

9. Write an equation that relate x and y.

Whale Heartbeat Data
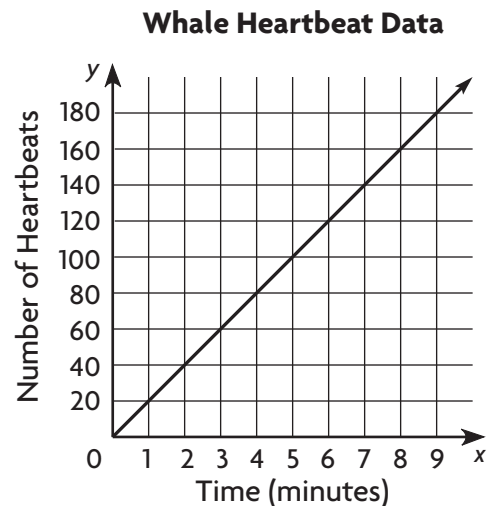
Name _____

Apply Percents

Essential Question How can you solve percent problems involving discounts and sales tax?

A **discount** is a percent of a price that is subtracted from the price. **Sales tax** is a tax that is a percent of a price that is added to the price of an item.

Unlock the Problem

The regular price of a pair of sneakers is $40. A store is offering a 15% discount on the shoes. What is the sale price?

- How can you write 15% as a decimal?

  _____

Find the discount and sale price of the sneakers.

STEP 1 Find 15% of 40.

$$15\% \text{ of } 40 = \frac{15}{100} \times 40 = 6$$

The discount is $6.

STEP 2 Subtract to find the sale price.

regular price − discount = sale price

$40 − $6 = $34

So, the sale price is $34.

Example

A pair of jeans costs $23 plus tax. The sales tax rate is 8%. What is the sales tax? What is the final cost of the jeans?

STEP 1 Find 8% of 23.

$$8\% \text{ of } 23 = \frac{8}{100} \times 23 = \underline{\hspace{2cm}}$$

The sales tax is $1.84.

STEP 2 Add to find the total cost.

price + sales tax = final cost

$23 + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}

So, the final cost is \underline{\hspace{3cm}}.

Math Talk **Mathematical Practices**

How can you use estimation to show that your answer is reasonable?

A video game is on sale for 10% off. The regular price is $29.00.

1. Find the discount.

   10% of 29.00 = _____ × 29 = _____

   The discount is _____.

2. Find the sale price of the game.

   Regular price – discount = sale price

   _____ – _____ = _____

   The sale price is _____.

On Your Own

Find the discount and the sale price.

3. regular price: $50

   discount: 20%

   discount: $ _____

   sale price: $ _____

4. regular price: $56

   discount: 25%

   discount: $ _____

   sale price: $ _____

Find the sales tax and the final cost.

5. price: $75

   sales tax: 6%

   sales tax: $ _____

   final cost: $ _____

6. price: $25

   sales tax: 5%

   sales tax: $ _____

   final cost: $ _____

Problem Solving Real World

7. A sweater that is regularly sold for $35 is on sale for 20% off. What is the sale price of the sweater?

   _____

8. Eileen has a $15 gift card to a music store. She uses the card to pay for a CD that costs $12 + tax. If the tax rate is 5%, how much will be left on the gift card after the purchase?

   _____

Name _____

Percent of Change

Essential Question How can you find a percent of change?

A **percent of change** is an amount, stated as a percent, that a number goes up or down. If the number goes up, it is a **percent of increase**. If the number goes down, it is a **percent of decrease**. To find a percent of change, use the following formula:

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

Unlock the Problem (Real World)

The manager of a store raises the price of a pair of shoes from $40 to $42. What is the percent of change in the price?

- What clue word tells you that this problem involves a price increase?

  _____

Use the formula to find the percent of change.

STEP 1 The change is an increase. Find the amount of increase: $42 - 40 = 2$.

STEP 2 Find the percent of increase.

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}} \quad \text{Write the formula.}$$

$$= \frac{2}{40} \quad \text{Substitute.}$$

$$= 0.05 = 5\% \quad \text{Divide. Write the quotient as a percent.}$$

So, the percent of change is a 5% increase.

Example Find the percent of change when the amount of water in a storage tank drops from 640 gallons to 512 gallons.

STEP 1 The change is a decrease. Find the amount of decrease: $640 - 512 = 128$.

STEP 2 Find the percent of change.

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}} \quad \text{Write the formula.}$$

$$= \frac{128}{640} \quad \text{Substitute.}$$

$$= \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \quad \text{Divide. Write the quotient as a percent.}$$

So, the percent of change is a 20% decrease.

Math Talk **Mathematical Practices**

Explain what it means when a price increases by 100%.

Share and Show

Use these steps to find the percent of change for the prices in the advertisement at right.

> **Model Train Set**
> **Original Price: $50**
> **Now reduced to $29!**

1. Tell whether the change is an increase or decrease. Then find the amount of change.

2. Substitute values in the formula and divide.

3. Write the quotient as a percent.

Find the percent of change. Label the change as increase or decrease.

4. 60 is increased to 75.

5. 1,200 is decreased to 1,176.

On Your Own

Find the percent of change. Label the change as increase or decrease.

6. 85 is increased to 119.

7. 5 is decreased to 4.

8. 35 is decreased to 21.

9. 22 is increased to 44.

10. 18 is increased to 26.1.

11. 700 is increased to 777.

Problem Solving · Real World

12. The owner of a boutique buys necklaces from a jewelry maker for $25 each. Then the boutique owner sells the necklaces for $40 each. What is the percent of change in the price?

13. On Saturday, 400 people attended a school festival. On Sunday, 366 people attended the festival. What is the percent of change in the attendance for the festival from Saturday to Sunday?

Name _____

✓ Checkpoint

Concepts and Skills

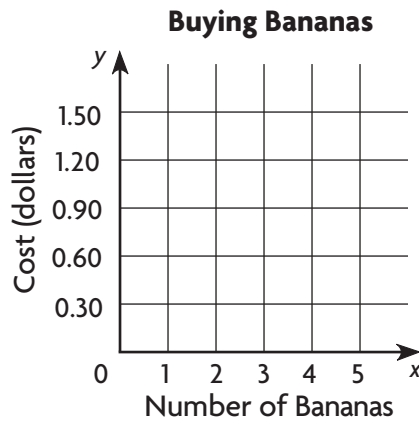**Simplify the complex fraction. Write your answer in simplest form.**

1. $\dfrac{\frac{1}{4}}{\frac{2}{3}}$

2. $\dfrac{\frac{1}{6}}{\frac{2}{3}}$

3. $\dfrac{\frac{3}{5}}{\frac{7}{10}}$

_____   _____   _____

Bananas cost $0.30 each. Use this information for 4-6.

4. Let x represent the number of bananas. Let y represent the cost. Write an equation that relates x and y.

_____

5. Give the constant of proportionality.

_____

6. Graph the equation you wrote in Exercise 4.

Buying Bananas

Problem Solving Real World

7. Joelle made 3 bracelets in 15 minutes, 4 bracelets in 20 minutes, and 6 bracelets in 30 minutes. Is the relationship between the number of bracelets and time a proportional relationship? If so, what is the unit rate?

_____

8. A calculator that normally sells for $45 is on sale for 20% off. What is the amount of the discount?

_____

9. The number of members in the Service Club rose from 18 to 27. What is the percent of change in the membership?

_____

Choose the letter of the correct answer.

10. A box contains $\frac{5}{6}$ pound of cereal. Each serving of cereal is $\frac{1}{12}$ pound. How many servings are in the cereal box?

(A) $\frac{5}{72}$ serving

(B) $\frac{5}{2}$ servings

(C) 6 servings

(D) 10 servings

11. Kaylin reads 96 pages in 3 hours, 128 pages in 4 hours, and 192 pages in 6 hours. Which statement is correct?

(A) The relationship is a proportional relationship. The unit rate is 32 pages per hour.

(B) The relationship is a proportional relationship. The unit rate is 96 pages per hour.

(C) The relationship is a proportional relationship. The unit rate is 192 pages per hour.

(D) The relationship is not a proportional relationship.

12. To attend a school Movie Night, students are asked to donate canned goods. The table below shows data for the relationship between the number of people and the number of cans collected.

People	10	20	40	100
Cans	20	40	80	200

Let x represent the number of people and y represent the number of cans. What is the constant of proportionality for the relationship?

(A) 2

(B) 10

(C) 20

(D) 100

13. What is the total cost of a cell phone if the price of the cell phone is $60 and the sales tax rate is 5%?

(A) $60

(B) $63

(C) $65

(D) $66

14. After exercising, Ellen's heart rate went from 120 beats per minute to 84 beats per minute. What is the percent of change in her heart rate?

(A) 36% decrease

(B) 30% decrease

(C) 20% decrease

(D) 16% decrease

Name _____

Add Algebraic Expressions

Essential Question How can you add algebraic expressions?

Unlock the Problem (Real World)

During track practice, Steven runs laps and then runs cross-country. The expression $3x + 2$ represents the distance in miles he ran on Monday, where x is the length in miles of each lap. The expression $4x + 3$ represents the distance in miles he ran on Tuesday. Write and simplify an expression to show the total distance in miles Steven ran on both days.

Math Idea

Is $3x + 2 = 5x$? Explain.

_____

_____

_____

Day	Laps	Cross-Country Distance (mi)
Monday	3	2
Tuesday	4	3

Write and simplify an algebraic expression for the total distance.

STEP 1 Write the algebraic expression for the total distance.

Monday   Tuesday

$(3x + 2) + (4x + 3)$

STEP 2 Use the Associative Property of Addition to remove the parentheses.

$3x + 2 + 4x + 3$

STEP 3 Simplify the expression by combining like terms.

Use the Commutative Property of Addition to reorder the terms.

$3x + 4x + 2 + 3$

Use parentheses to group like terms.

$(3x + 4x) + (2 + 3)$

Combine like terms.

$7x + 5$

So, the simplified expression $7x + 5$ represents the total distance in miles. Steven ran $7x + 5$ miles on Monday and Tuesday.

Math Talk

Mathematical Practices

Explain how you can identify like terms in an algebraic expression.

Share and Show

Find the sum of the expressions $(5y + 29)$ and $(8 + 6y)$.

1. Write the algebraic expression for the total.

_____

2. Use the Associative Property of Addition to remove the parentheses.

_____

3. Use the Commutative Property of Addition to reorder the terms.

_____

4. Use parentheses to group like terms.

_____

5. Combine like terms.

_____

Find the sum of the expressions.

6. $(x + 12) + (11 + 2x)$

_____

7. $(4s + 1) + (8s + 14)$

_____

8. $(1 + 41y) + (6y + 2)$

_____

On Your Own

Find the sum of the expressions.

9. $(10t + 7) + (8 + 3t)$

_____

10. $(8 + 3x) + (11 + 17x)$

_____

11. $(9 + 19c) + (18c + 4)$

_____

12. $(16y + 0) + (y + 23)$

_____

13. $(45t + 27) + (23t + 32)$

_____

14. $(35 + 6x) + (6 + x)$

_____

Problem Solving

15. Zoe orders 3 books for her friend Amelia and 4 books for her friend Edward. She pays an additional shipping charge of $5 for Amelia's order and $7 for Edward's order. Simplify the expression $(3c + 5) + (4c + 7)$, where c represents the cost of each book, to find the total Zoe spent on both orders.

_____

16. Molly works 4 hours on Saturday and earns an additional $22 in tips. On Sunday, she works 6 hours and earns an additional $15 in tips. Simplify the expression $(4h + 22) + (6h + 15)$, where h represents the amount she is paid per hour, to find the total she earned over the weekend.

_____

Name _____

Solve Two-Step Equations

Essential Question How do you solve two-step equations?

Unlock the Problem (Real World)

Olivia orders 5 sets of beads. She pays $7 for shipping, and the total cost of the order is $52.
Solve the equation $5p + 7 = 52$ to find the price p in dollars of each set of beads.

> **Math Idea**
>
> Suppose Olivia had a coupon. What operation would this indicate?
>
> _____

Solve the equation to find the price of each set of beads.

STEP 1 Write the equation.

$$5p + 7 = 52$$

STEP 2 Use the Properties of Equality and inverse operations to get the variable by itself on one side. First undo addition or subtraction, and then undo multiplication or division.

Undo the addition. Subtract 7 from both sides.

$$5p + 7 - 7 = 52 - 7$$
$$5p = 45$$

Undo the multiplication. Divide both sides by 5.

$$\frac{5p}{5} = \frac{45}{5}$$
$$p = 9$$

So, the price of each set of beads is $9.

Math Talk **Mathematical Practices**

Explain how you know that your answer is correct.

Try This! Tell how to solve the equation for *x*.

A. $6x - 9 = 15$

_____

_____

_____

_____

1. Solve the equation $\frac{1}{4}c + 6 = 18$.

First undo the _____ by using _____.

Then undo the _____ by using _____.

$c =$ _____

Solve the equation.

2. $12x + 2 = 38$

_____

3. $\frac{1}{3}y - 5 = 3$

_____

4. $3 + 7p = 52$

_____

On Your Own

Solve the equation.

5. $23 + 4t = 59$

_____

6. $2x - 8 = 64$

_____

7. $5r + 30 = 105$

_____

8. $\frac{1}{2}p + 15 = 29$

_____

9. $3c + 58 = 97$

_____

10. $6y - 37 = 29$

_____

Problem Solving Real World

11. Lee started a round on a game show with 65 points. He answered all 5 questions during the round correctly. Lee's score at the end of the round was 105 points. Solve the equation $65 + 5p = 105$ to find the number of points p that Lee earned for each correct answer.

_____

12. To repair a bike, a shop charges a fee of $11, plus $13 for each hour that the mechanic works on the bike. Minh paid $63 to have his bike fixed. Solve the equation $11 + 13h = 63$ to find the number of hours h the mechanic worked on Minh's bike.

_____

Name _____

Solve Inequalities

Essential Question How can you solve inequalities?

Solving inequalities is much like solving equations. To solve an
inequality, get the variable on one side by itself using the Properties of
Inequality and inverse operations.

Addition and subtraction properties of inequality

You can add or subtract the same
number on both sides of an inequality,
and the inequality will still be true.

$$3 + 2 < 8$$
$$3 + 2 - 2 < 8 - 2$$
$$3 + 0 < 6$$
$$3 < 6$$

Multiplication and division properties of inequality

You can multiply or divide both sides of an
inequality by the same positive number,
and the inequality will still be true.

$$2 \times 4 > 6$$
$$\frac{2 \times 4}{2} > \frac{6}{2}$$
$$1 \times 4 > 3$$
$$4 > 3$$

Unlock the Problem (Real World)

A person must be at least 50 inches tall to be allowed to
ride a roller coaster. Belinda is 38 inches tall. The inequality
$38 + n \geq 50$ can be used to find the number of inches n
Belinda must grow to be able to ride the roller coaster.
Solve the inequality. Explain what the solution means.

> ### Math Idea
> Inequalities may have more than one
> solution. *Any* value of n that when
> added to 38 totals more than 50 is a
> solution for the inequality
>
> $38 + n \geq 50$.

Solve the inequality.

STEP 1 Write the inequality.

$$38 + n \geq 50$$

STEP 2 Use the Properties of Inequality and
inverse operations to get the variable by
itself on one side.

$$38 - 38 + n \geq 50 - 38$$
$$n \geq 12$$

Undo the addition. Subtract 38 from both sides.

So, the solution of the inequality is $n \geq 12$.

This means that Belinda must grow 12 or more inches before she is
able to ride the roller coaster. Any amount of growth she experiences
that is 12 inches or more will allow her to ride the roller coaster.

Math Talk

Mathematical Practices

Describe the inverse
operation you would use to
solve $3x < 18$.

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1. Solve the inequality $2s \leq 6$.

   Use the Properties of Inequality and inverse operations to get the variable by itself on one side.

   Undo the multiplication by _____.

   The solution is _____.

Solve the inequality.

2. $x + 3 < 4$

3. $n - 12 > 10$

4. $\frac{p}{3} \geq 9$

On Your Own

Solve the inequality.

5. $n + 5 < 9$

6. $x - 1 \leq 0$

7. $7c > 7$

8. $\frac{m}{2} \geq 2$

9. $a + 16 > 26$

10. $y - 5 \geq 19$

Problem Solving Real World

11. An elephant weighs more than 30 times what a tiger weighs. An average elephant weighs 12,000 pounds. The inequality $30w < 12{,}000$ can be used to find the possible weight w in pounds of the tiger. Solve the inequality and explain what the solution means.

12. The inequality $m + 12 \leq 20$ can be used to find the amount of money m in dollars that Nolan can spend at a circus. Solve the inequality and explain what the solution means.

Vertical Angles

Essential Question How can you find the unknown measure of vertical angles?

🔑 Unlock the Problem (Real World)

Vertical angles are formed when two lines or line segments intersect. Vertical angles are opposite congruent angles.

E   B
35°
A
D   C

- What is true about congruent angles?

  _____

- What is the measure of a straight angle?

  _____

- ∠EAB and ∠DAC are vertical angles.
- ∠BAC and ∠EAD are vertical angles.

🔑 **Find the measure of ∠EAD.**

THINK: Vertical angles are congruent.

- ∠BAC and ∠EAD are vertical angles, so the measure of ∠BAC = the measure of ∠EAD.
- The measure of ∠BAC is 35°.

E   B
?   35°
A
D   C

So, the measure of ∠EAD, written m∠EAD, is _____.

🔑 **Find the measure of ∠EAB and ∠DAC.**

THINK: A straight angle is 180°.

- Together, ∠EAB and ∠BAC make up a straight angle, ∠EAC.

- Subtract the measure of ∠BAC from _____ to find the

  measure of ∠EAB. 180° − _____ = _____

E   B
?   35°
A
D   C

Since ∠EAB and ∠DAC are vertical angles and the measure of

∠EAB is _____, the measure of ∠DAC is _____.

Try This!

Materials ■ protractor

- Draw two intersecting lines. Use a protractor to measure one angle.
- Find and label the measure of the other three angles using what you know about vertical angles and straight angles.

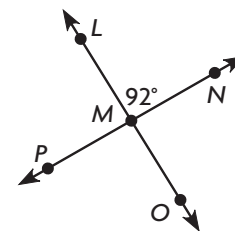**For 1–3, use the drawing to find the measure of the angle.**

1. $m\angle SQR = 180° - 51° = $ _____

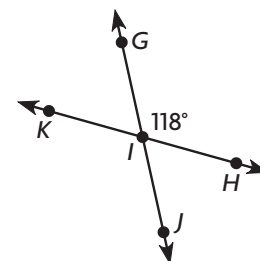2. $m\angle UQS = $ _____

3. $m\angle UQT = $ _____

On Your Own

For 4–6, use the drawing to find the measure of the angle.

4. $m\angle PMO = $

5. $m\angle LMP = $

6. $m\angle NMO = $

_____   _____   _____

For 7–9, use the drawing to find the measure of the angle.

7. $m\angle HIJ = $

8. $m\angle JIK = $

9. $m\angle KIG = $

_____   _____   _____

Problem Solving Real World

10. Two intersecting lines form vertical angles that are not obtuse or acute. Describe the angles that are formed and make a sketch of the lines.

_____

11. The figure at the right shows three lines intersecting to form an isosceles right triangle. How many acute angles are formed? **Explain** why they all are congruent.

_____

_____

_____

_____
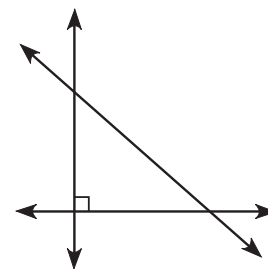
Name _____

# Complementary and
Supplementary Angles

Essential Question How can you find the unknown measure of complementary or supplementary angles?

When the sum of the measures of two angles equals 90°, the angles are **complementary angles**. The angles do not need to be adjacent in order to be complementary. For example, $\angle MNO$ and $\angle PQR$ are complementary, and each angle is the complement of the other.

$$35° + 55° = 90°$$

When the sum of the measures of two angles is 180°, the angles are called **supplementary angles**. You can show that $\angle ABC$ and $\angle DEF$ are supplementary by adding their measures.
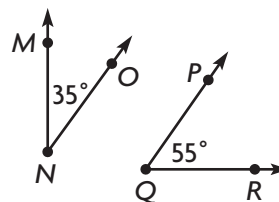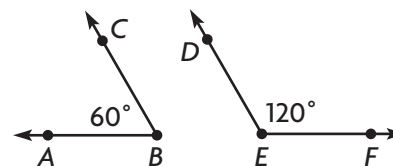
$$60° + 120° = 180°$$

Unlock the Problem — Real World

The figure shows the support beams of a half-pipe skateboard ramp. Find $m\angle TQG$.

$\angle TQY$ is a right angle, so it measures _____.

$\angle TQG$ and $\angle GQY$ together form $\angle TQY$, so they are _____ angles.
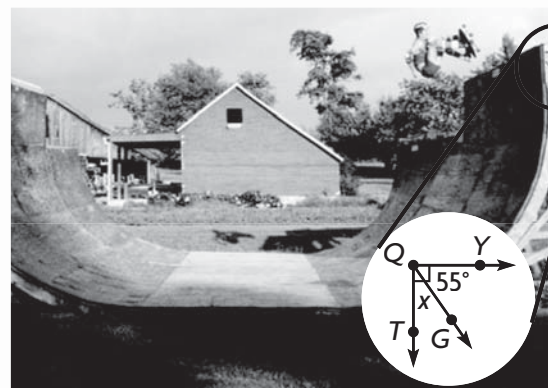
Find the measure of the unknown angle.

STEP 1 The sum of the measures of complementary angles is 90°.

$$m\angle TQG + m\angle GQY = 90°$$

STEP 2 Substitute the measures of the angles.

$$x + 55° = 90°$$

STEP 3 Solve the equation by using Properties of Equality.

$$x + 55° - 55° = 90° - 55°$$

Simplify.

$$x = 35°$$

So, the $m\angle TQG$ is 35°.

Math Talk
Mathematical Practices

Explain whether it is possible for two angles to be both congruent and complementary.

Try This! Find $m\angle ABD$.

$\angle ABC$ is a straight angle so it measures _____, $\angle ABD$ and $\angle CBD$ together form $\angle ABC$, so they are _____ angles.
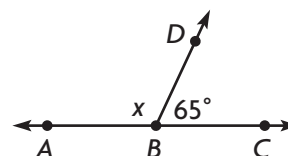
$$m\angle ABD + m\angle CBD = \text{_____}$$

$$x + \text{_____} = \text{_____}$$

$$x + \text{_____} - \text{_____} = \text{_____} - \text{_____}$$

$$x = \text{_____}$$

So, $m\angle ABD = $ _____.

1. Find the unknown angle measure.

STEP 1 The angles are _____.

STEP 2 Substitute the measures of the angles.

STEP 3 Solve the equation by using Properties of Equality.

Subtract _____ from both sides.
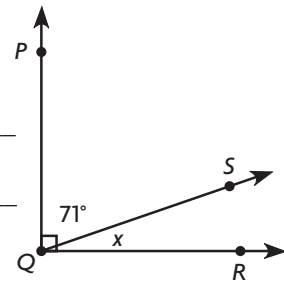
Simplify.

So, m∠SQR is _____.

m∠PQS + m∠SQR = _____

_____ + x = _____

_____ − _____ + x = _____ − _____

x = _____

Find the unknown angle measure.

2.

E

68° x

B   C   D

3.

G

x

H   33°

J

I

4.

G

x   102°

D   E   F

_____

Find the unknown angle measure.

5.

A

D

x

41°

C   B

_____

6.

F   G   H

x   40°

I

_____

7.

K

N

x   66°

M   L

_____

8.

S

x 52°

P   Q   R

_____

9.

T   U

x

18°

W   V

_____

10.

M

60° x

J   K   L

_____
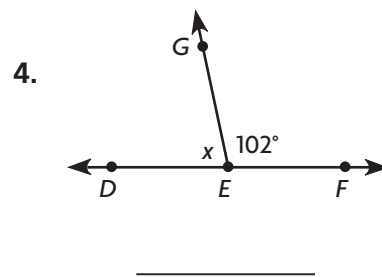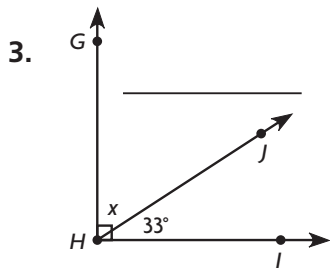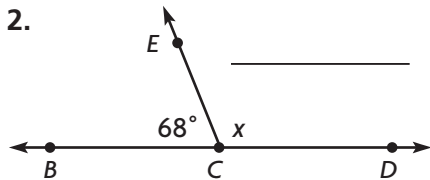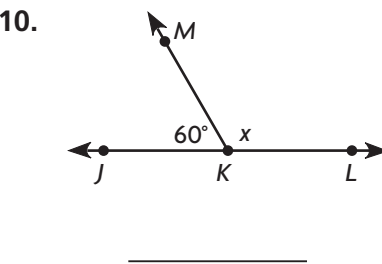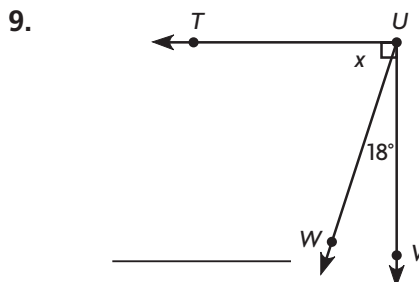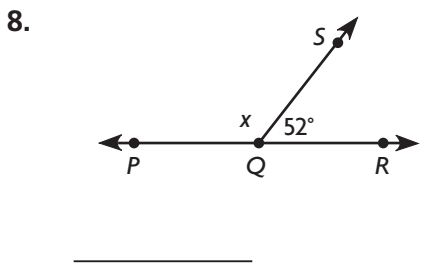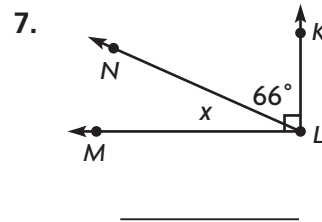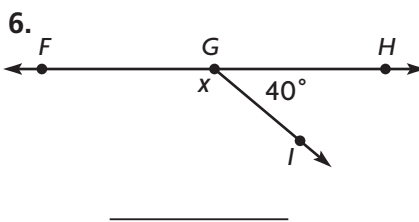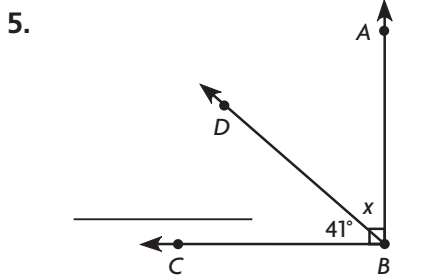
11. The figure shows a ramp meeting a wall. Find m∠CBD.

_____

wall

A

B 160°

x

C   D

floor

GR30
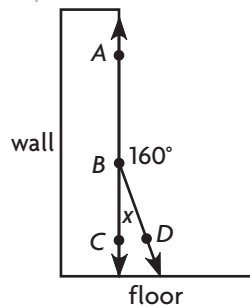
Name _____

Parts of a Circle

Essential Question How can you identify and draw the parts of a circle?

A **circle** is a closed figure made up of points that are the same distance from a point called the **center.** A circle is named by its center point. Other parts of a circle include:

- A **radius** is a line segment with one endpoint at the center of the circle and the other endpoint on the circle.
- A **chord** is a line segment that has both of its endpoints on the circle.
- A **diameter** is a chord that passes through the center of the circle.

⚷ Unlock the Problem (Real World)

Use a compass to draw and label the parts of a circle.

🔑 Activity

Materials ■ compass, straightedge

A **Draw circle *O* with radius $\overline{OP}$ that measures 5 centimeters.**

- Draw and label center point *O.* Place the compass point on it.
- Open the compass to 5 centimeters and draw the circle.
- Label point *P* on the edge of the circle.
- With a straightedge, draw the radius $\overline{OP}$.

B **Draw chord $\overline{QR}$ on circle *O*.**

- Label points *Q* and *R* on the circle.
- Use a straightedge to connect *Q* and *R* to create chord $\overline{QR}$.

C **Draw diameter $\overline{QT}$ on circle *O*.**

- Draw a line segment that includes point *Q* and passes through the center.
- Label point *T* where the line segment meets the other side of the circle.
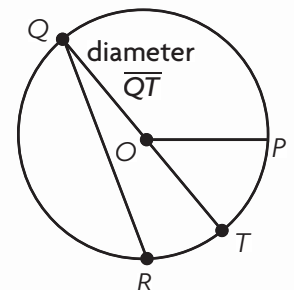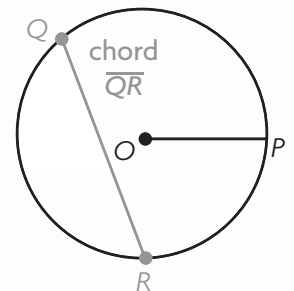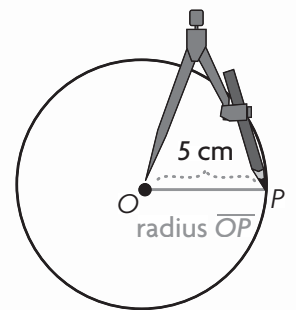
Math Talk **Mathematical Practices**

How is the length of the diameter related to the length of the radius?

Draw and label a circle with a radius of 4 cm. Then draw and label the following.

1. center O

2. radius $\overline{OB}$

3. chord $\overline{CD}$

4. diameter $\overline{BE}$

Identify each part of the circle shown below.

5. the center

6. a radius

_____

_____

7. a chord

8. a diameter

_____

_____
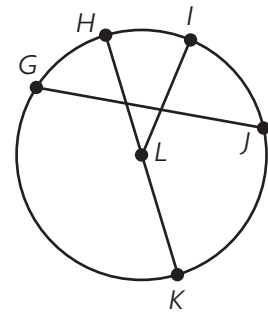
On Your Own

Draw and label a circle with a radius of 6 cm. Then draw and label the following.

9. center A

10. radius $\overline{AY}$

11. chord $\overline{MN}$

12. diameter $\overline{XY}$

Problem Solving Real World

13. Dave needs to buy steel to replace 5 broken spokes on his bicycle wheel. Each spoke is equal to the length of the radius of the wheel. The diameter of the wheel is 24 inches. How many inches of steel does Dave need to make 5 spokes?

_____

Name _____

Estimate Circumference

Essential Question How does the diameter of a circle relate to the circumference?

The **circumference** is the distance around a circle. You can use a ruler and string to estimate the circumference of a circle.

🔑 Unlock the Problem 🌎

🔑 Activity

In this Activity, you will explore the relationship between the diameter and the circumference of a circle.

Math Idea

A diameter is a line segment that passes through the center and has both endpoints on the circle.

Materials: compass, centimeter ruler, string, and calculator

STEP 1 Use a compass to draw a circle. Mark the center of the circle. Use a ruler to draw a diameter through the center of the circle.

STEP 2 Measure the diameter of the circle to the nearest millimeter. Record your measurement.

STEP 3 Lay the string around the circle. Mark the string where it meets itself.

STEP 4 Use the ruler to measure the string from its end to the mark you made. Measure to the nearest millimeter. Record your measurement.

STEP 5 Use a calculator to divide the circumference of your circle by the diameter. Record your result.

STEP 6 Display your results on the board with those of other students in the class by making a table like the one below.

Circumference (*C*)	Diameter (*d*)	*C* ÷ *d*

- Compare your results with those of other students. What appears to be the approximate ratio $\frac{C}{d}$ for any circle?

_____

Share and Show

MATH BOARD

Use a compass to draw a circle with radius 4 cm. Use the circle to answer Exercises 1–4.

1. What is the diameter of this circle? _____

2. Use a string to measure the circumference of the circle, as you did in the activity. What is an estimate of the circumference? _____

3. What is an estimate for the ratio of the circumference to the diameter of the circle? _____

4. If you know the diameter of a circle, how can you use the ratio you found to estimate the circumference? _____

Estimate the circumference of the circle.

5. radius = 8 cm

   _____

6. 6 yd

   _____

7. 5.4 m

   _____

8. 18 mm

   _____
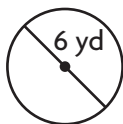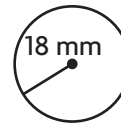
On Your Own

Estimate the circumference of the circle.

9. $9\frac{1}{2}$ ft

   _____

10. 25 in.

    _____

11. $\frac{1}{2}$ in.

    _____

12. 14 in.

    _____

13. 7.2 cm

    _____

14. 9 yd

    _____
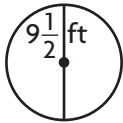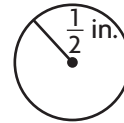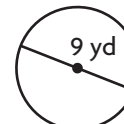
Problem Solving Real World

15. The diameter of the clock on the face of Big Ben in London is 23 feet. Estimate the circumference.

    _____

16. The Cevahir clock at a shopping mall in Turkey may be the world's largest clock. The diameter of its face is 118 feet. A football field is 100 yd long. How does the circumference of the Cevahir clock compare with the length of a football field?

    _____

    _____

GR34

✓ Checkpoint

Concepts and Skills

Solve the equation.

1. $\frac{x}{7} - 8 = 0$

2. $13p + 19 = 97$

3. $3c - 42 = 15$

_____

_____

_____

Solve the inequality.

4. $8y - 55 < 129$

5. $21 + 16k \geq 101$

6. $82 + \frac{p}{4} > 96$

_____

_____

_____

For 7–9, use the drawing to find the measure of the angle.
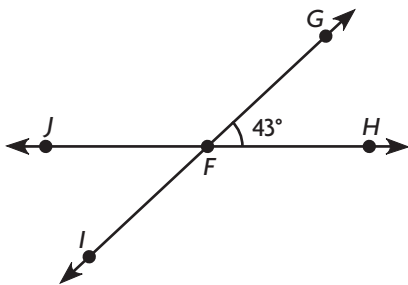
7. m∠JFG

_____

8. m∠JFI

_____

9. m∠IFH

_____

For 10–12, use the drawing to identify the parts of the circle.
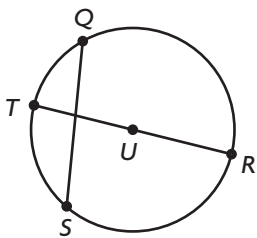
10. the center

_____

11. a radius

_____

12. a chord

_____

Estimate the circumference of the circle.

13. _____

14. _____

15. _____

Choose the letter of the correct answer.

16. On Thursday, Gia took two aerobics classes and walked for an additional 20 minutes on the treadmill. On Friday, she took 1 aerobics class and walked for an additional 35 minutes. Simplify the expression $(2m + 35) + (3m + 15)$, where m represents the length in minutes of each class, to show the total number of minutes Gia exercised on those two days.

Ⓐ $5m + 20$ Ⓒ $6m + 20$

Ⓑ $5m + 50$ Ⓓ $6m + 50$

17. Hank and his friend are drawing a big circle as a boundary for their game. Hank holds one end of a rope and stands at what will be the center of the circle. His friend holds the other end and stands 10 feet away at what will be the edge of the circle. The 10-foot rope is the radius of the circle. What will be its estimated circumference?

Ⓐ 20 ft   Ⓒ 40 ft

Ⓑ 30 ft   Ⓓ 60 ft

18. Kyle drew $\angle KLM$ and has been asked to draw an angle that is complementary. What should be the measure of the new angle?

Ⓐ 64°   Ⓒ 164°

Ⓑ 74°   Ⓓ 174°

19. Carmen has a $30 gift card for her favorite online store. She is hoping to buy 4 bracelets. She knows the bracelets will be priced the same but also knows she needs to allow for the $6 shipping fee. Use the inequality $4p + 6 \leq 30$, where p represents the cost of each bracelet, to find the maximum price she can afford to pay per bracelet.

Ⓐ $p \leq \$5$ Ⓒ $p \leq \$8$

Ⓑ $p \leq \$6$ Ⓓ $p \leq \$9$

20. Emma looked at the way one of the painted parking lot lines met the curb. She incorrectly described the angles formed as vertical angles. Which term below would describe the two angles correctly?

Ⓐ acute   Ⓒ equal

Ⓑ complementary   Ⓓ supplementary

Name _____

Samples and Surveys

Essential Question How can you learn about a population by taking a sample?

A **survey** is a method of gathering information about a group. Surveys are usually made up of questions or other items that require responses. You can survey a population, which is the entire group of individuals or objects. If the population is large, you can survey a part of the population, called a **sample**.

🔑 Unlock the Problem (Real World)

Ron surveys students at his school about their favorite pizza toppings. He surveys the first 25 students to walk into school on Monday morning. What sampling method does he use?

Math Idea

Underline the sentence that tells you what you are trying to find.

🔑 **Identify the sampling method.**

A **sampling method** is a way to choose a sample of a population. The table summarizes some sampling methods.

Sampling Method	Definition	Example
Random Sampling	Every individual or object has an equal chance of being chosen for the survey.	Assign a number to every student in the school. Then use a computer to randomly select numbers.
Convenience Sampling	Individuals or objects that are easily available are chosen for the survey.	Choose a convenient location, such as the library, and survey students as they enter.
Systematic Sampling	Choose a random individual or object as the starting point and then use a pattern to choose additional individuals or objects.	Randomly choose a name from a list of all students and then choose every 10th name after that.

Describe how Ron chooses the sample for his survey.

_____

So, Ron uses _____ sampling.

Math Talk **Mathematical Practices**

Describe why someone might use a convenience sample rather than a random sample.

Try This!

Meg takes a similar survey. She chooses one name at random from a list of all students at the school. Then she chooses every 15th name after that. What sampling method does she use?

_____

Identify the sampling method.

1. Brianna randomly chooses 20 names from a database of all students at her school.

   Every student has an equal chance of being chosen. So, Brianna's method is

   _____

2. Jorge randomly chooses one name from a phone list of all employees at his company. Then he chooses every 10th name after that.

   _____

On Your Own

Identify the sampling method.

3. Mitchell stands at the exit of a train station and surveys 25 commuters as they leave the station.

   _____

4. Marie wants to survey owners of pet stores in her city. She chooses the name of a pet store from the phone book. Then she chooses every 3rd pet store after that.

   _____

5. A caterer randomly chooses 20 names from a list of clients and surveys them to see if they are satisfied with his service.

   _____

6. Ray wants to know how many books people in his town read each month. He surveys the first 50 people that walk into a grocery store.

   _____

Problem Solving Real World

7. A manager wants to know how many of the light bulbs that a factory produces might be defective. She randomly chooses and tests 30 light bulbs produced at the factory. Identify the sampling method the manager used.

   _____

8. Lashonda wants to know the favorite type of music of teens in her town. She surveys 10 students sitting near her at lunch. Identify the sampling method she used.

   _____

Name _____

Make Predictions from Samples

Essential Question How can you use a sample to make a prediction about a population?

You can use equivalent ratios to make predictions about samples.

🔑 Unlock the Problem 〔Real World〕

There are 90 sixth graders at Webb Middle School. In a randomly selected sample of 25 sixth graders at the school, 20 said that they spend more than 3 hours per week exercising. Based on the sample, predict how many of the sixth graders at Webb Middle School spend more than 3 hours exercising per week.

Find equivalent ratios by using a unit rate.

STEP 1

Write ratios that compare the number of students that exercise more than 3 hours per week to total number of students.

$$\frac{20}{25} = \frac{\square}{90}$$

STEP 2

90 is not a multiple of 25.
Write the known ratio as a unit rate.

$$\frac{20 \div \square}{25 \div 25} = \frac{\square}{90}$$

$$\frac{\square}{1} = \frac{\square}{90}$$

STEP 3

Write an equivalent rate by multiplying the _____
and the _____ by the same value.

Think: Multiply 1 by _____ to get 90.

So, multiply the numerator by _____ also.

$$\frac{0.8 \cdot \square}{1 \cdot \square} = \frac{\square}{90}$$

So, based on the sample, _____ students out of the 90 sixth graders at Webb Middle School are predicted to spend more than 3 hours per week exercising.

〔**Math Talk**〕 **Mathematical Practices**

Explain how you know that your prediction is reasonable.

Share and Show

1. There are 80 children registered for a swimming contest. In a randomly selected sample of 15 children, 3 were over the age of 12. Based on the sample, predict how many contestants are over the age of 12.

$$\frac{3}{15} = \frac{\boxed{}}{80}$$

$$\frac{3 \div \boxed{}}{15 \div 15} = \frac{\boxed{}}{80}$$

$$\frac{\boxed{}}{1} = \frac{\boxed{}}{80}$$

$$\frac{0.2 \cdot \boxed{}}{1 \cdot \boxed{}} = \frac{\boxed{}}{80}$$

2. A comic book store carries 80 different titles. In a randomly selected sample of 10 titles, 2 had been published in the last year. Based on the sample, predict how many titles in the store had been published in the last year.

3. Annita has 300 songs on her computer. In a randomly selected sample of 12 songs, 4 songs were rock. Base on the sample, predict how many rock songs Annita has.

On Your Own

4. A car dealership has 200 cars in the parking lot. In a randomly selected sample of 25 cars, 6 cars were white. Based on the sample, predict how many cars at the dealership are white.

5. There are 480 pages in a sixth grade math book. In a randomly selected sample of 40 pages, 15 had color pictures on them. Based on the sample, predict how many pages in the entire book have color pictures.

Problem Solving Real World

6. There are 170 students at Riverdale Middle School. In a randomly selected sample of 30 students, 12 said that they would attend the play. Based on the sample, predict how many students at Riverdale Middle School will attend the play.

7. The Widget Factory produces 500 widgets in one hour. In a randomly selected sample of 20 widgets, 2 were found to be defective. Based on the sample, predict how many widgets produced in one hour are defective.

Name _____

Probability and Likelihood

Essential Question How can you use probability to describe the likelihood of an event?

An **experiment** is an activity involving chance where the results are observed or measured, such as spinning a spinner. A possible result of an experiment is an **outcome**.

The **sample space** of an experiment is the set of all possible outcomes.

An **event** is a set of one or more outcomes. The **probability** of an event measures the likelihood that the event will occur. Probabilities range from 0 (the event is impossible) to 1 (the event is certain).

impossible		as likely as not		certain
	unlikely		likely	

0, or 0% $\frac{1}{2}$, 0.5, or 50%   1, or 100%
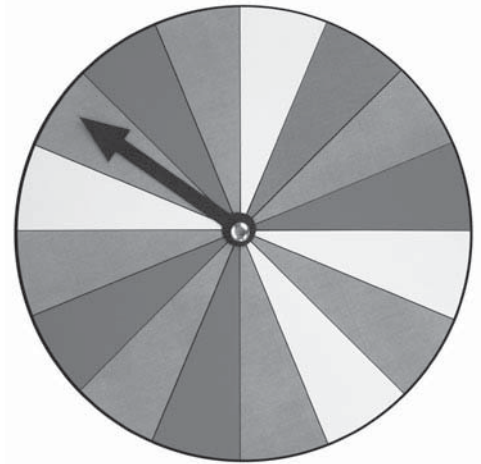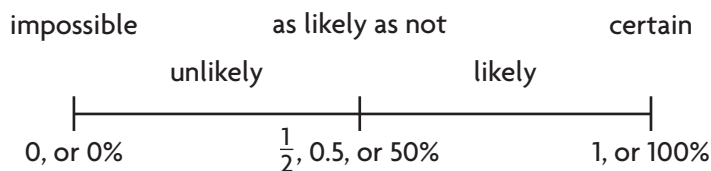
🔑 Unlock the Problem (Real World)

A number cube used in a board game has faces labeled 1 to 6. Describe each event as *impossible, unlikely, as likely as not, likely,* or *certain.*

A. Evelyn rolls the number cube and gets a 6.

_____

B. James rolls the number cube and gets an even number.

_____

C. Stefan rolls the number cube and gets a number

greater than 0. _____

Math Idea
When rolling a six-sided number cube, an impossible event would be rolling a 9.

Math Talk **Mathematical Practices**
Give an example of an event that is certain to happen.

Try This! **According to a weather forecast, the probability that it will rain on Monday is 70%. Describe the likelihood of rain on Monday as impossible, unlikely, as likely as not, likely, or certain.**

_____

MATH BOARD

A spinner has 10 equal-sized sections that are numbered 1–10. Describe each event as *impossible, unlikely, as likely as not, likely,* **or** *certain.*

1. spinning a 4 or 5

_____

2. spinning a number greater than 2

_____

3. spinning an even number

_____

4. spinning a number less than 9

_____

On Your Own

A jar contains 10 marbles. There are 1 green, 1 red, and 8 blues. A marble is picked at random. Describe each event as *impossible, unlikely, as likely as not, likely,* **or** *certain.*

5. picking a blue

_____

6. picking a yellow

_____

Describe each event as *impossible, unlikely, as likely as not, likely,* **or** *certain.*

7. The probability that Jack throws a paper ball into a wastebasket is 16%.

_____

8. There is a 50% chance of snow on Tuesday.

_____

Problem Solving

Real World

9. The probability that Marguerite will win a game is 20%. Describe the event of Marguerite winning as impossible, unlikely, as likely as not, or certain.

_____

10. A spinner has 5 equal-sized sections. The sections are numbered 1–5. If Jeff spins the spinner, describe the likelihood that he spins a number less than 5.

_____

Name _____

Write Probabilities

Essential Question How can you write the probability of an event?

You can write the probability of an event as a fraction, decimal, or percent.

🔑 Unlock the Problem (Real World)

A restaurant is having a prize giveaway. The probability that a customer will win a prize is $\frac{1}{8}$. Write this probability as a decimal and as a percent.

STEP 1 Write the probability as a decimal. Divide the numerator by the denominator.

$$\frac{1}{8} = 1 \div 8$$

$$\begin{array}{r} \boxed{} \\ 8\overline{)1.000} \\ \underline{8} \\ 20 \\ 16 \\ 40 \\ \underline{40} \end{array}$$

$$\frac{1}{8} = 1 \div 8 = \underline{}$$

STEP 2 Write the probability as a percent. Multiply the decimal by 100, and include a percent sign.

$$0.125 = \underline{}$$

STEP 3 Check that 12.5% is correct.

$$12.5\% = \frac{12.5}{100} = \frac{125}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

Math Talk — **Mathematical Practices**

Describe the likelihood that a customer will win a prize.

Try This! **Write the probability in two different ways.**

A. The probability of rain on Wednesday is 85%.

Fraction: $\frac{85}{100} = \dfrac{\boxed{}}{\boxed{}}$

Decimal: _____

B. The probability of scoring a point is 0.625.

Fraction: $\dfrac{\boxed{}}{\boxed{}}$ Percent: _____

MATH BOARD

Write the probability in two different ways.

1. The probability of a light bulb being defective is 15%.

Fraction: $\dfrac{\boxed{}}{100} = \dfrac{\boxed{}}{\boxed{}}$

Decimal: _____

2. The probability of a thunderstorm occurring today is 0.66.

Fraction: $\dfrac{\boxed{}}{100} = \dfrac{\boxed{}}{\boxed{}}$

Percent: _____

On Your Own

Write the probability in two different ways.

3. Sarah randomly chooses a cookie from a jar. The probability that the cookie is peanut butter is $\frac{3}{5}$.

Decimal: _____

Percent: _____

4. The probability that a player wins a prize at a carnival is 5%.

Decimal: _____

Fraction: _____

5. The probability that Jan makes a free throw is 0.94.

Percent: _____

Fraction: _____

6. The probability that Max wins a competition is $\frac{9}{20}$.

Decimal: _____

Percent: _____

Problem Solving Real World

Write the probability in two different ways.

7. A teacher will randomly choose a student to help with decorations for an upcoming dance. The probability that Raymond will be chosen is 0.08.

8. Melvin tries to throw a ball into a cup. The probability that he makes the shot is $\frac{27}{40}$.

Name _____

Experimental Probability

Essential Question How can you calculate the experimental probability of an event?

A **trial** is one performance of an experiment. The **experimental probability** of an event is the ratio of the number of times the event occurs to the total number of trials of the experiment.

Each time an experiment is performed is called a **trial**.

Experimental Probability

$P(\text{event}) = \dfrac{\text{number of times that the event occurs}}{\text{total number of trials}}$

Unlock the Problem (Real World)

A spinner has 16 sections that are red, orange, yellow, or green. Cara spins the pointer 20 times and records her results in the table. Write each probability as a fraction, decimal, and percent. What color is the most likely result of a spin?

Color	Frequency
Red	2
Orange	11
Yellow	4
Green	3

🔑 **Write each probability as a fraction, decimal, and percent.**

$P(\text{red}) = \dfrac{2}{20} = \dfrac{1}{10}$ or 0.1 or 10%

$P(\text{orange}) = \dfrac{11}{20}$ or _____ or _____

$P(\text{yellow}) = \dfrac{\boxed{}}{20} = \dfrac{1}{\boxed{}}$ or _____ or _____

$P(\text{green}) = \dfrac{\boxed{}}{\boxed{}}$ or _____ or _____

So, _____ is the most likely result of a spin because this color has the greatest experimental probability.

Math Talk **Mathematical Practices**

Explain how you compared the experimental probabilities.

Try This! **Amirah and Scott each roll a number cube at the same time, and they record the sum. They performed 50 trials, and rolled a sum of 5 seven different times. Find the experimental probability of rolling a sum of 5.**

$P(5) =$ _____ out of _____, or _____%

Dylan randomly selects a marble from a bag and replaces it. He does this a total of 40 times and records his results in the table. Use the table to find the experimental probability. Write the probability as a fraction, decimal, and percent.

Color	Red	Blue	Green
Frequency	12	20	8

1. P(red)

$$= \frac{\square}{40} = \frac{\square}{\square}$$

$$= \square \, \underline{\qquad}$$

2. P(blue)

3. P(green)

On Your Own

A spinner has 10 sections, labeled 1–10. Trey spins the spinner and records his results each time. Use the results in the table to find the experimental probability. Write the probability as a percent.

4. spinning a 4

_____

5. spinning a 1

_____

6. spinning a 7

_____

7. spinning a 9 or 10

_____

9	2	3	10	8
3	2	6	5	8
1	4	9	3	4
1	10	2	1	6

Problem Solving Real World

8. Ling tossed two coins, at the same time, 5 times. Her results were TT, TH, TH, HH, and HT. What is her experimental probability of flipping two tails? Express your answer as a decimal, fraction, and percent.

_____

9. The letters R, A, N, D, O, and M are written on cards and placed in a bag. Jack randomly chooses and replaces a card several times. Find the experimental probability of picking an N. Express your answer as a percent.

Letter	R	A	N	D	O	M
Frequency	0	2	3	1	2	4

_____

Name _____

<p align="center">✓ **Checkpoint**</p>

Concepts and Skills

Identify the sampling method.

1. Sam wants to know which genre of movie is the favorite among his classmates. He randomly chooses 15 names from a list of the students in his class.

_____

2. Shaelun is interested in finding the number of students in her school who like math. She asks the people on her team in gym class.

_____

Write the probability in two different ways.

3. The probability of precipitation is 30%.

Fraction: _____   Decimal: _____

4. The probability of picking a diamond from a standard deck of cards is $\frac{1}{4}$.

Decimal: _____   Percent: _____

5. The probability that Jiho will select a blue marble from a bag is 0.19.

Fraction: _____   Percent: _____

6. The probability that the pointer will land on red when Yvette spins a prize wheel is $\frac{1}{20}$.

Decimal: _____   Percent: _____

Problem Solving Real World

Valerie places some cards in a bag. Each card shows a color. She randomly chooses and replaces a card 20 times. Use the results in the table to find the experimental probability indicated. Express your answer as a percent.

Color	Red	Yellow	Blue	Green	Orange	Purple	Black	Brown
Frequency	2	5	3	0	2	2	3	3

7. What is the experimental probability of choosing an orange card?

_____

8. What is the experimental probability of choosing a yellow card?

_____

Choose the letter of the correct answer.

9. The probability that Jordan will make a free throw shot in his basketball game is 50%. Which term describes the likelihood of Jordan making a free throw?

   (A) impossible   (C) as likely as not

   (B) unlikely   (D) certain

10. There are 25 students in Winnie's class. In a randomly selected sample of 10 classmates, 4 have dogs. Based on the sample, predict how many students in Winnie's class who have dogs.

    (A) 4   (B) 8   (C) 10   (D) 12

11. There are 140 students enrolled at Madame LaComtesse's Dance School. In a random sample of 30 students, 6 said they would be interested in the new hip-hop class. Based on the sample, how many students out of 140 are interested in the hip-hop class?

    (A) 6   (B) 18   (C) 24   (D) 28

12. Kyran has 6 socks in a bag. There are 4 white socks and 2 blue socks. Which term describes the event of picking a white sock from the bag?

    (A) certain   (C) as likely as not

    (B) likely   (D) unlikely

13. The probability that Yvette wins a game is 15%. Which of the following shows this probability written in two different ways?

    (A) 0.15 and $\frac{3}{10}$ (C) 0.15 and $\frac{3}{20}$

    (B) 1.5 and $\frac{3}{10}$ (D) 15.0 and $\frac{3}{20}$

14. Nika has a bag of cards, each showing a shape. She randomly chooses and replaces a card 12 times. Use the results in the table to find the experimental probability that Nika will choose a card showing a heart.

    (A) 25%   (C) 0.20

    (B) $\frac{3}{10}$ (D) $\frac{1}{3}$

15. Use the results in the table to find the experimental probability that Nika will choose a card that does NOT show a triangle.

    (A) $\frac{1}{6}$ (C) $\frac{7}{12}$

    (B) $\frac{5}{12}$ (D) $\frac{5}{6}$

Shape	Frequency
Square	2
Heart	3
Diamond	5
Triangle	2