Prerequisite Skills for Calculus

- You should be able to do these problems without using a calculator.
- If you have questions, you can search the *italicized phrase(s)* in khanacademy.org.
- See Additional Problems on the following pages for more practice of each type.

Simplify with positive exponents.

$$\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$$

Keywords: *simplifying positive exponents*
Additional Problems

Write the domain in interval notation.

$$y=\frac{\sqrt{2x+14}}{x^2-49}$$

Keywords: *domain of radical and rational functions*
Additional Problems

Find the equation of the line going through the following points in point-slope form $(3,-2)$ and $(1,5)$.

Keywords: *point-slope form*
Additional Problems

Find the horizontal and vertical asymptotes for the following:

$$y=\frac{x^2-1}{x^2+5x-6}$$

Keywords: *horizontal and vertical asymptotes*
Additional Problems

Simplify the following:

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$$

Keywords: *simplifying rational expressions*
Additional Problems

Referring to the following table, find $(f\circ g)(2)$ and $f(f(3))$.

x	1	2	3	4	5	6
$f(x)$	2	3	5	1	6	3
$g(x)$	3	5	6	2	1	4

Keywords: *composition of functions*
Additional Problems

Find k such that the lines $3x-5y=9$ and $2x+ky=11$ are

a) parallel
b) perpendicular

Keywords: *parallel and perpendicular lines*
Additional Problems

Solve for x without a calculator, and solve for y with a calculator.

Keywords: *special triangles and right triangle trigonometry*
Additional Problems

Solve the following equation on the interval $[0,2\pi)$: $2\cos x+\sqrt{3}=0$

Keywords: *solving trigonometric equations*
Additional Problems

Solve for y.

$$3xy+3y-2x=7x^2$$

Keywords: *isolate a variable*
Additional Problems

Graph the following functions:

a) $y=(x-2)^2+3$

b) $y=5-\sqrt{x+2}$

c) $y=e^x+2$

Keywords: *graphing quadratic, radical, and exponential functions*
Additional Problems

Solve.

a) $\log_2 16=x$ (without calculator)

b) $4^{x+2}=14$ (with calculator)

Expand.

b) $\ln\frac{\sqrt{x-2}}{x^3}$

Keywords: *solving logarithmic and exponential functions*
Additional Problems

Key

Simplify with positive exponents.

$$\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$$

$\frac{2}{3}\cdot\frac{1}{2}(2x+5)^{-3/2}$

$= \frac{1}{3(2x+5)^{3/2}}$

Write the domain in interval notation.

$$y=\frac{\sqrt{2x+14}}{x^2-49} = \frac{\sqrt{2(x+7)}}{(x+7)(x-7)}$$

Domain: all Real Numbers except $x=-7$ and 7

Find the equation of the line going through the following points in point-slope form $(3,-2)$ and $(1,5)$.

slope $= \frac{5-(-2)}{1-3} = \frac{7}{-2}$

$y+2 = -\frac{7}{2}(x-3)$ OR

$y-5 = -\frac{7}{2}(x-1)$

Find the horizontal and vertical asymptotes for the following:

$$y=\frac{x^2-1}{x^2+5x-6}$$

$y = \frac{(x+1)(x-1)}{(x-1)(x+6)}$

Vertical Asymptote: $x=-6$

Horizontal Asymptote: $y=1$

Hole at $x=1$

Simplify the following:

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}} = \frac{\frac{x}{x}+\frac{1}{x}}{\frac{x}{x}-\frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{x-1}{x}}$$

$\frac{x+1}{x}\cdot\frac{x}{x-1} = \frac{x+1}{x-1}$

Referring to the following table, find $(f\circ g)(2)$ and $f(f(3))$.

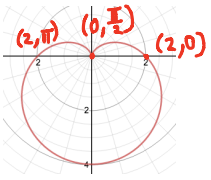
x	1	2	3	4	5	6
$f(x)$	2	3	5	1	6	3
$g(x)$	3	5	6	2	1	4

$f(g(2)) = f(5) = 6$

$f(f(3)) = f(5) = 6$

***Graph and label 3 points on the polar curve**

$r=2-2\sin\theta$



$(2,\pi)\left(0,\frac{\pi}{2}\right)(2,0)$

Solve for x without a calculator, and solve for y with a calculator.

$x=10$ $y=12.799$

$\tan 52 = \frac{y}{10}$

$10(\tan 52) = y$

Solve the following equation on the interval $[0,2\pi)$: $2\cos x+\sqrt{3}=0$

$2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$

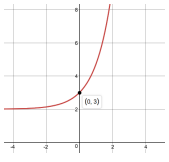
***Find the sum without using a calculator**

$$\sum_{n=1}^{\infty} 4\left(\frac{1}{2}\right)^{n-1}$$

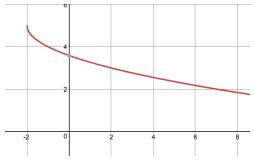
$\frac{a_1}{1-r} = \frac{4}{1-\frac{1}{2}} = 8$

Graph the following functions:

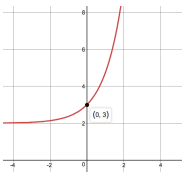
a) $y=(x-2)^2+3$



b) $y=5-\sqrt{x+2}$



c) $y=e^x+2$



a) $\log_2 16 = x$ (without calculator)

$2^x = 16$ $x=4$

b) $4^{x+2}=14$ (with calculator)

$\log_4(x+2) = \log_4 14$

$x = (\log_4 14) - 2$

Expand.

a) $\ln\frac{\sqrt{x-2}}{x^3}$

$= \frac{1}{2}\ln(x-2) - 3\ln x$

Simplify using only positive exponents

1) $\left(n^{8}\right)^{\frac{1}{2}}$

2) $\left(81k^{6}\right)^{\frac{1}{2}}$

3) $\left(\sqrt[5]{3n}\right)^{7}$

4) $\sqrt[4]{m}$

5) $\dfrac{nm^{-\frac{5}{4}} \cdot m^{4}n^{\frac{4}{3}}}{\left(m^{-2}\right)^{\frac{5}{4}}}$

6) $\left(\dfrac{y^{2}}{x^{\frac{1}{2}}y^{\frac{3}{4}} \cdot y^{-3}}\right)^{\frac{3}{2}}$

7) $\left(\dfrac{a^{-\frac{5}{4}}b^{-1}}{a^{\frac{1}{2}}b^{2} \cdot a^{-\frac{3}{2}}}\right)^{\frac{1}{2}}$

8) $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^{3}}}$

9) $\left(\dfrac{1}{x^{-2}} + \dfrac{4}{x^{-1}y^{-1}} + \dfrac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

Answers to

1) n^4

2) $9k^3$

3) $(3n)^{\frac{7}{5}}$

4) $m^{\frac{1}{4}}$

5) $m^{\frac{21}{4}} n^{\frac{7}{3}}$

6) $\frac{x^{\frac{1}{4}} y^{\frac{51}{8}}}{x}$

7) $\frac{a^{\frac{7}{8}} b^{\frac{1}{2}}}{ab^2}$

8) $\frac{2}{\sqrt[4]{x-4}}$

9) $\frac{1}{\left(x^2 + 4xy + y^2\right)^{\frac{1}{2}}}$

Identify the domain of each function.

1) $f(x) = \log_4 (2x + 2) + 2$

2) $y = \frac{1}{2}\sqrt{x} - 2$

3) $y = \sqrt{x + 4} - 5$

4) $y = \sqrt{x^2 - 5x - 14}$

5) $f(x) = \frac{x^3 - x^2 - 2x}{-4x^2 + 12x + 16}$

6) $f(x) = \frac{x^2 + 3x + 2}{x^2 - 3x}$

7) $f(x) = \frac{x^3 - x^2 - 12x}{3x^2 - 12}$

8) $f(x) = \ln (3x - 8) - 3$

9) $y = \frac{x}{\cos x}$

10) $y = \sqrt[3]{x} - 2$

Answers to

1) Domain: $x > -1$ 2) Domain: $x \geq 0$ 3) Domain: $x \geq -4$

4) Domain: $\{ (-\infty,-2), (7,\infty) \}$ 5) Domain: All reals except 4, −1

6) Domain: All reals except 0, 3 7) Domain: All reals except 2, −2

8) Domain: $x > \frac{8}{3}$ 9) Domain: All reals excepts $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

10) Domain: { All real numbers. }

Write the equation of the line through the given point with the given slope.

1) through: $(0, -1)$, slope = undefined

2) through: $(-3, 1)$, slope = $-\frac{5}{3}$

Write the equation of the line through the given points.

3) through: $(-3, -5)$ and $(0, 3)$

4) through: $(4, -1)$ and $(0, 2)$

Write the equation of the line described.

5) through: $(-2, 0)$, parallel to $y = \frac{3}{2}x - 4$

6) through: $(-3, 1)$, parallel to $y = -x + 2$

7) through: $(3, -2)$, perp. to $y = 2x - 4$

8) through: $(-5, 0)$, perp. to $x = 0$

9) Line containing (4, −2) and

a) parallel to the line containing (−1, 4) and (2, 3)

b) perpendicular to the line containing (−1, 4) and (2, 3).

10) Find k such that the lines $3x - 5y = 9$ and $2x + ky = 11$ are

a) parallel

b) perpendicular

Answers to

1) $x = 0$

2) $y = -\frac{5}{3}x - 4$

3) $y = \frac{8}{3}x + 3$

4) $y = -\frac{3}{4}x + 2$

5) $y = \frac{3}{2}x + 3$

6) $y = -x - 2$

7) $y + 2 = -\frac{1}{2}(x - 3)$

8) $y = 0$

9) a) $y + 2 = -\frac{1}{3}(x - 4)$

b) $y + 2 = 3(x - 4)$

10) a) $k = -\frac{10}{3}$

b) $k = \frac{6}{5}$

Identify the holes, vertical asymptotes, and horizontal asymptote of each.

1) $f(x) = \dfrac{x^2 - 4x + 3}{x^2 + x - 2}$

2) $f(x) = \dfrac{x^2 + 3x}{x^2 - 4}$

3) $f(x) = \dfrac{2x^2 - 4x - 16}{x^2 - x - 12}$

4) $f(x) = \dfrac{x}{3x - 3}$

5) $f(x) = \dfrac{-3x - 9}{x + 2}$

6) $f(x) = -\dfrac{x}{x - 3}$

7) $f(x) = -\dfrac{1}{x^2 + 2x - 3}$

8) $f(x) = \dfrac{x^3 - 2x^2 - 8x}{3x^2 - 15x + 12}$

9) $f(x) = \dfrac{1}{x^2 + 3x}$

10) $f(x) = \dfrac{x^2 - x - 6}{x^2 + 5x + 6}$

Answers to

1) Vertical Asym.: $x = -2$
   Holes: $x = 1$
   Horz. Asym.: $y = 1$

2) Vertical Asym.: $x = 2$, $x = -2$
   Holes: None
   Horz. Asym.: $y = 1$

3) Vertical Asym.: $x = -3$
   Holes: $x = 4$
   Horz. Asym.: $y = 2$

4) Vertical Asym.: $x = 1$
   Holes: None
   Horz. Asym.: $y = \frac{1}{3}$

5) Vertical Asym.: $x = -2$
   Holes: None
   Horz. Asym.: $y = -3$

6) Vertical Asym.: $x = 3$
   Holes: None
   Horz. Asym.: $y = -1$

7) Vertical Asym.: $x = 1$, $x = -3$
   Holes: None
   Horz. Asym.: $y = 0$

8) Vertical Asym.: $x = 1$
   Holes: $x = 4$
   Horz. Asym.: None

9) Vertical Asym.: $x = 0$, $x = -3$
   Holes: None
   Horz. Asym.: $y = 0$

10) Vertical Asym.: $x = -3$
    Holes: $x = -2$
    Horz. Asym.: $y = 1$

Simplify each expression.

1) $\dfrac{\frac{4}{3}}{\frac{x^2}{4}}$

2) $\dfrac{\frac{m}{25}}{\frac{m^2}{3}}$

3) $\dfrac{\frac{1}{2}}{\frac{4}{x^2} - \frac{1}{x}}$

4) $\dfrac{\frac{2}{5} - \frac{2}{x^2}}{\frac{25}{2}}$

5) $\dfrac{\frac{4}{m^2} + \frac{16}{m}}{\frac{1}{2}}$

6) $\dfrac{4}{\frac{2}{u} - \frac{u}{2}}$

7) $\dfrac{u^2}{\frac{u^2}{16} + \frac{16}{v^2}}$

8) $\dfrac{\frac{x}{2}}{\frac{4}{y^2} - \frac{2}{x^2}}$

9) $\dfrac{ab}{\frac{b}{25} + \frac{a^2}{b^2}}$

10) $\dfrac{\frac{y}{x^2} - \frac{x^2}{y}}{x^2}$

Answers to

1) $\frac{16}{3x^2}$

2) $\frac{3}{25m}$

3) $\frac{x^2}{8-2x}$

4) $\frac{4x^2-20}{125x^2}$

5) $\frac{8+32m}{m^2}$

6) $\frac{8u}{4-u^2}$

7) $\frac{16v^2u^2}{u^2v^2+256}$

8) $\frac{x^3y^2}{8x^2-4y^2}$

9) $\frac{25b^3a}{b^3+25a^2}$

10) $\frac{y^2-x^4}{x^4y}$

Perform the indicated operation.

1) $g(n) = 3n - 1$
$h(n) = n^2 + 5$
Find $g(h(-5))$

2) $f(n) = 2n + 5$
$g(n) = -2n^2 - 2n$
Find $(f \circ g)(5)$

3) $f(x) = 2x + 3$
$g(x) = -3x + 1$
Find $f(g(8))$

4) $h(n) = 2n$
$g(n) = 3n - 3$
Find $h(g(10))$

5) $f(n) = 3n - 2$
$g(n) = 3n + 5$
Find $(f \circ g)(-8)$

6) $h(n) = 3n$
$g(n) = -3n + 3$
Find $h(g(10))$

7) $f(a) = 3a + 5$
$g(a) = a^2 - 2a$
Find $(f \circ g)(5)$

8) $g(n) = n^2 - 2$
$f(n) = 4n - 3$
Find $(g \circ f)(n^2)$

9) $g(t) = t^3 - 3t^2$
$f(t) = 4t$
Find $(g \circ f)\left(\frac{t}{3}\right)$

10) $g(a) = -a$
$h(a) = a - 4$
Find $g(h(a^2))$

Answers to

1) 89
2) −115
3) −43
4) 54
5) −59
6) −81
7) 50
8) $16n^4 - 24n^2 + 7$
9) $\frac{64t^3 - 144t^2}{27}$
10) $-a^2 + 4$

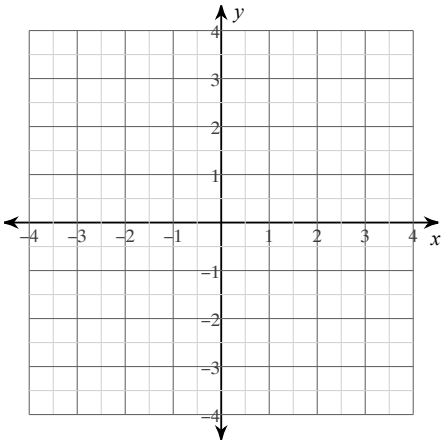
Write each pair of parametric equations in rectangular form. Then sketch the curve.

1) $x = 3\cos t,\ y = 3\sin t$

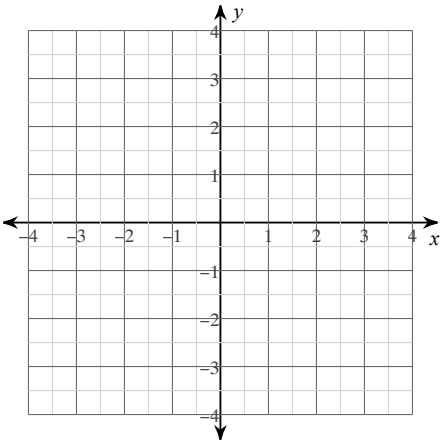
2) $x = t,\ y = \frac{t^2}{6}$

Convert each pair of polar coordinates to rectangular coordinates.

3) (1, 45°)

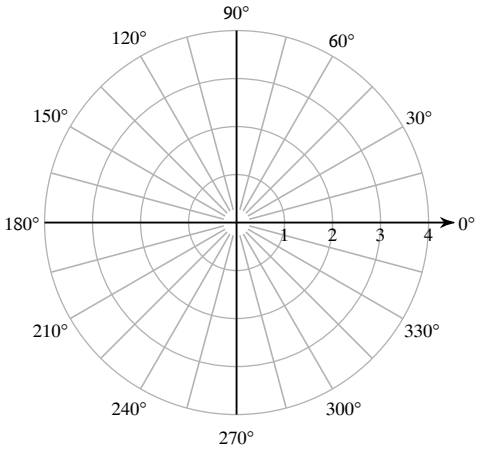


4) (3, 150°)

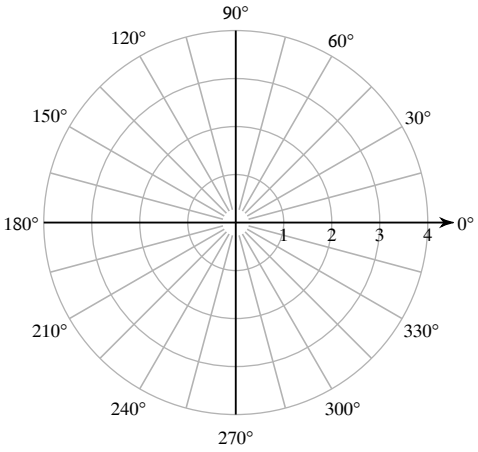


Find all pairs of polar coordinates that describe the same point as the provided polar coordinates.

5) $(2, 75°)$

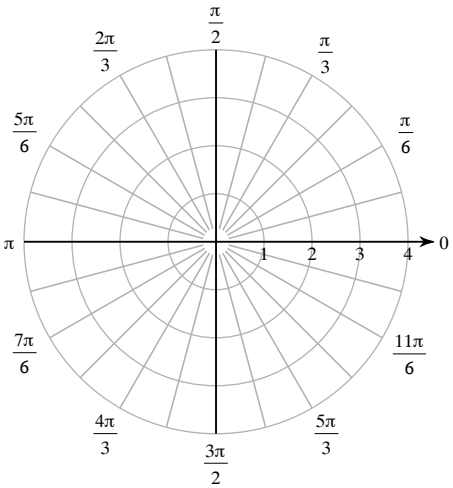


6) $(4, 180°)$

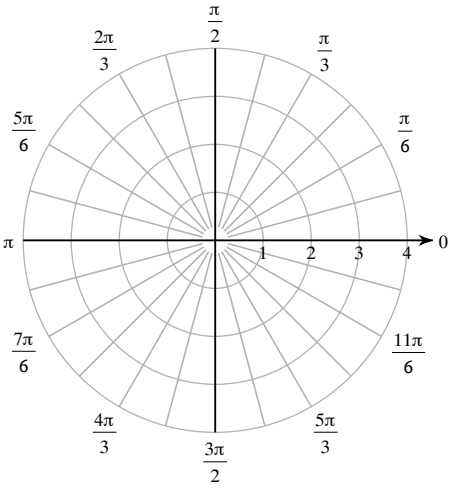


Convert each pair of rectangular coordinates to polar coordinates where $r > 0$ and $0 \le \theta < 2\pi$.

7) $\left(-2\sqrt{2}, -2\sqrt{2}\right)$

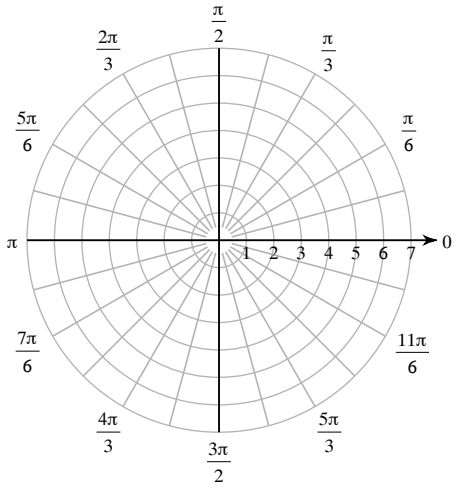


8) $\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

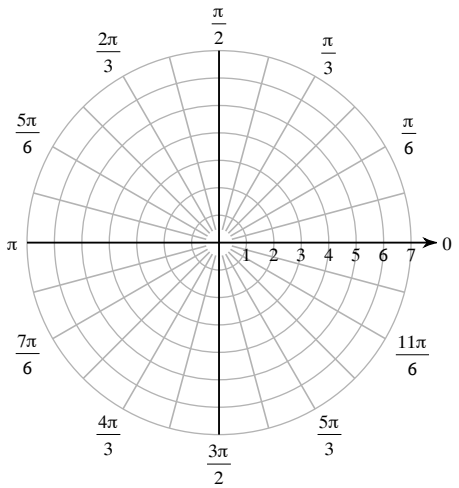


Consider each polar equation over the given interval. Classify the curve; find the values of θ where r is zero; find the maximum $|r|$ value and the values of θ where this occurs; and sketch the graph.

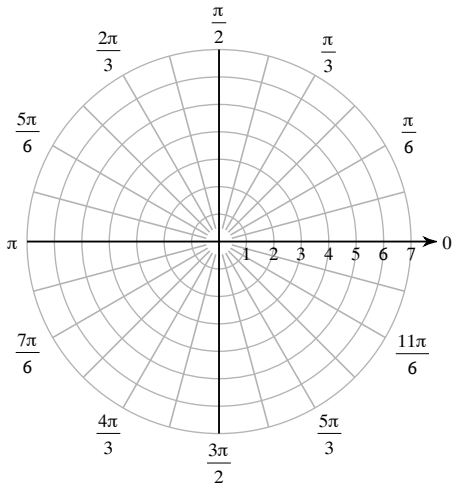
9) $r = 5 + 2\cos\theta,\ 0 \le \theta < 2\pi$



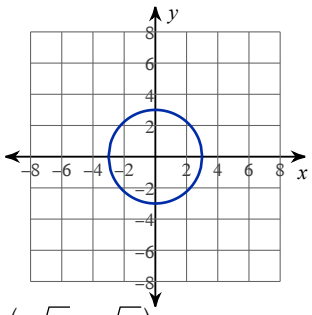
10) $r = 3 - \cos\theta,\ 0 \le \theta < 2\pi$



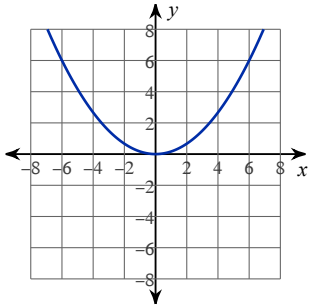
11) $r = 1 + \cos\theta, 0 \leq \theta < 2\pi$



Answers to

1) 

$\frac{x^2}{9} + \frac{y^2}{9} = 1$

2) 

$y = \frac{x^2}{6}$

3) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

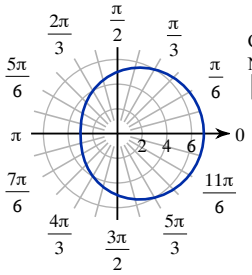
4) $\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

5) $(2, 75° + 360n°)$ and $(-2, 255° + 360n°)$ where n is an integer

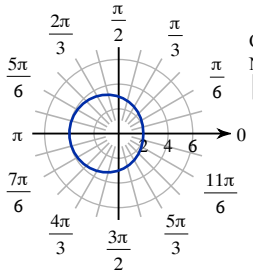
6) $(4, 180° + 360n°)$ and $(-4, 0° + 360n°)$ where n is an integer

7) $\left(4, \frac{5\pi}{4}\right)$

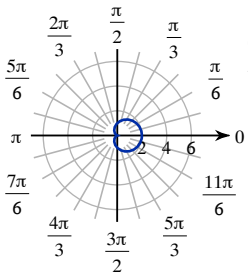
8) $\left(3, \frac{5\pi}{3}\right)$

9) 

Convex limaçon
No values of θ where $r = 0$
$|r| = 7$ when $\theta = \{0\}$

10) 

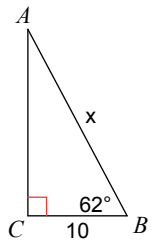
Convex limaçon
No values of θ where $r = 0$
$|r| = 4$ when $\theta = \{\pi\}$

11) 

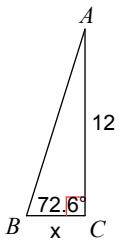
Cardioid (Limaçon)
$r = 0$ when $\theta = \{\pi\}$
$|r| = 2$ when $\theta = \{0\}$

Find the measure of each side indicated.

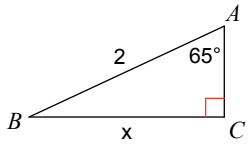
1)



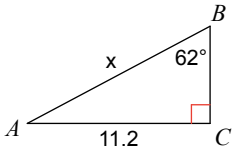
2)



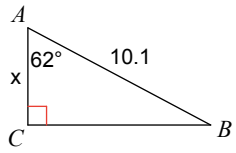
3)



4)



5)



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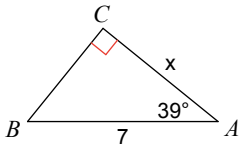
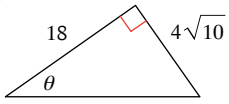
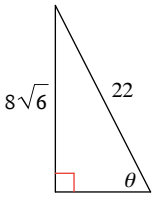


Find the value of the trig function indicated.

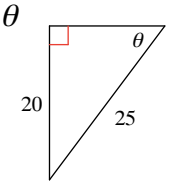
7) csc θ



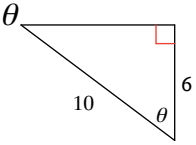
8) tan θ



9) cos θ



10) tan θ



Answers to

1) 21.3
2) 3.8
3) 1.8
4) 12.7
5) 4.7
6) 5.4
7) $\frac{11\sqrt{10}}{20}$
8) $\frac{4\sqrt{6}}{5}$
9) $\frac{3}{5}$
10) $\frac{4}{3}$

Find the exact value of each trigonometric function.

1) $\sin \frac{\pi}{4}$

2) $\cos \frac{2\pi}{3}$

3) $\sin -\pi$

4) $\tan \frac{11\pi}{6}$

Solve each equation for $0 \le \theta < 2\pi$**.**

5) $-1 = \sin \theta$

6) $-1 + \sin \theta = -2$

7) $-3 + \tan \theta = -4$

8) $-2 + \tan \theta = \frac{-6 + \sqrt{3}}{3}$

9) $0 = -2\sin \theta$

10) $3 + \tan \theta = 3$

11) $3 + 2\tan \frac{\theta}{4} = 5$

12) $-5 - 2\cos -3\theta = -7$

Answers to

1) $\frac{\sqrt{2}}{2}$

2) $-\frac{1}{2}$

3) 0

4) $-\frac{\sqrt{3}}{3}$

5) $\left\{\frac{3\pi}{2}\right\}$

6) $\left\{\frac{3\pi}{2}\right\}$

7) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

8) $\left\{\frac{\pi}{6}, \frac{7\pi}{6}\right\}$

9) $\{0, \pi\}$

10) $\{0, \pi\}$

11) $\{\pi\}$

12) $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

Evaluate each infinite geometric series described.

1) $\sum_{k=1}^{\infty} 8.6 \cdot 0.8^{k-1}$

2) $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^{n-1}$

3) $\sum_{k=1}^{\infty} 324 \cdot \left(\frac{1}{3}\right)^{k-1}$

4) $\sum_{k=1}^{\infty} 135 \cdot \left(\frac{1}{3}\right)^{k-1}$

Rewrite each series using sigma notation.

5) $1 + 4 + 9 + 16 + 25$

6) $4 + 8 + 12 + 16 + 20$

7) $2 + 4 + 8 + 16 + 32 + 64$

8) $401 + 402 + 403 + 404 + 405 + 406$

Evaluate each series.

9) $\sum_{m=1}^{6} (m + 500)$

10) $\sum_{n=1}^{5} (5n^2 + 3)$

Answers to

1) 43

2) 4

3) 486

4) $\frac{405}{2}$

5) $\sum_{a=1}^{5} a^2$

6) $\sum_{k=1}^{5} 4k$

7) $\sum_{m=1}^{6} 2^m$

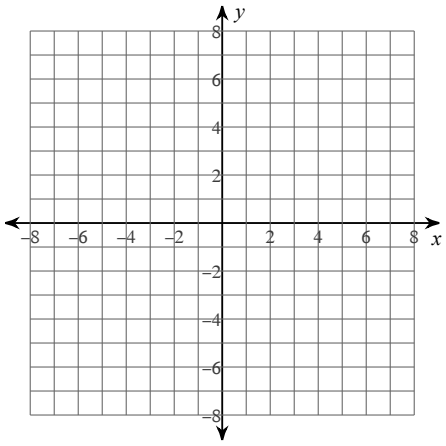
8) $\sum_{k=1}^{6} (k + 400)$

9) 3021

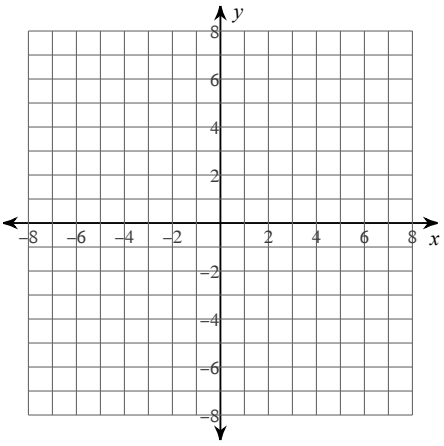
10) 290

Sketch the graph of each function.

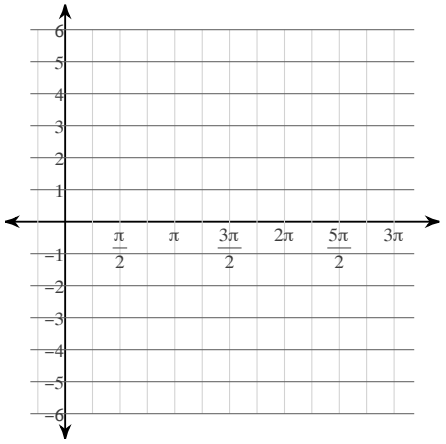
1) $y = \ln(x - 1) - 5$



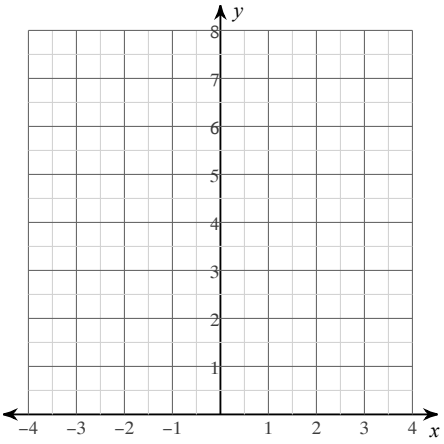
2) $f(x) = x^3 + 2x^2 - 4x - 5$



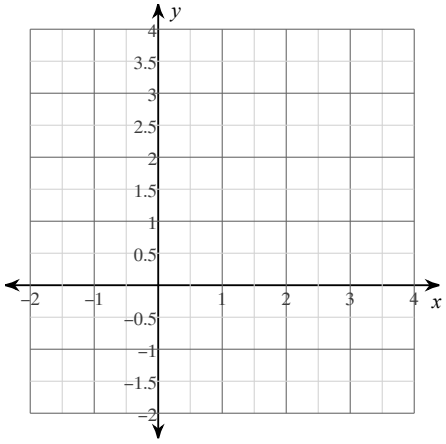
3) $y = 4\cos\theta$



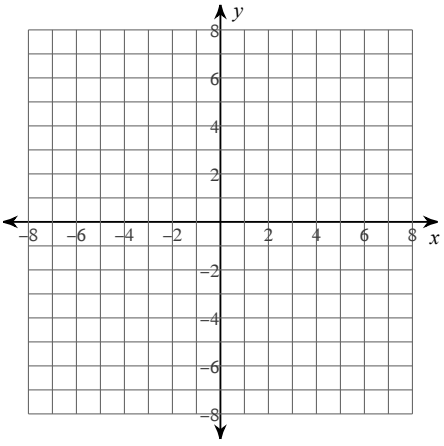
4) $y = (x - 2)^2 + 3$



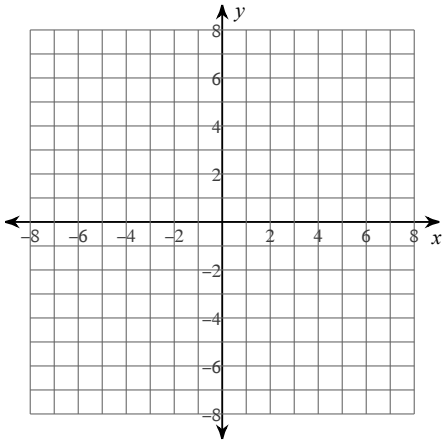
5) $y = (x - 1)^2 - 1$



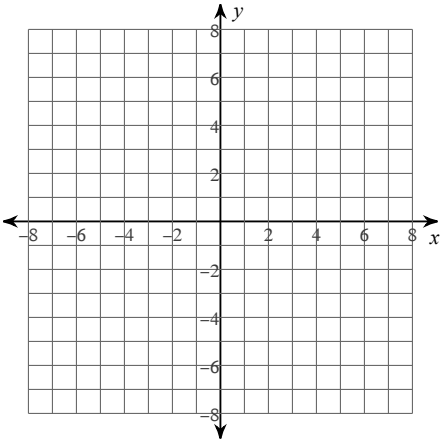
6) $y = \sqrt{x} - 3$



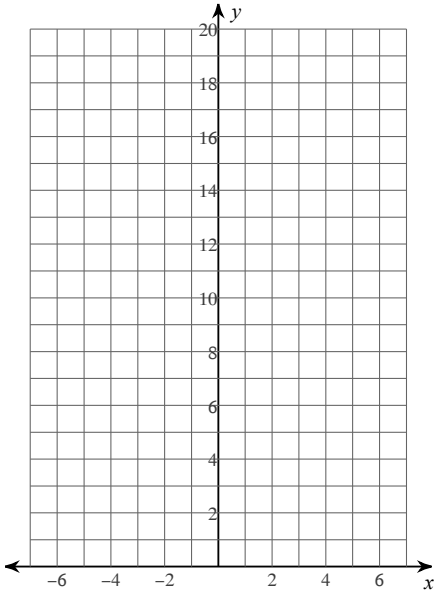
7) $(x-2)^2+(y-1)^2=4$



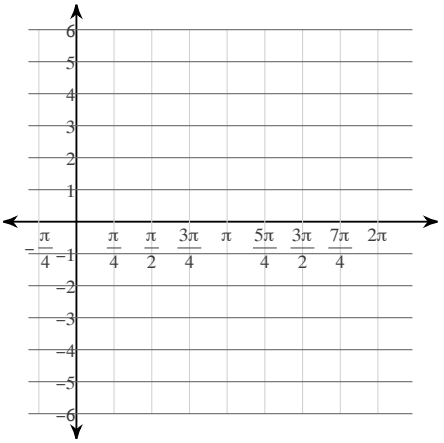
8) $f(x)=-\dfrac{1}{x+3}-3$



9) $y=3^x$

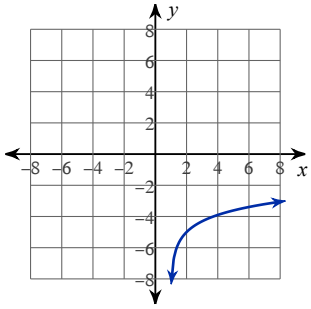


10) $y=\dfrac{1}{2}\cdot\sin 2\theta$



Answers to

1)



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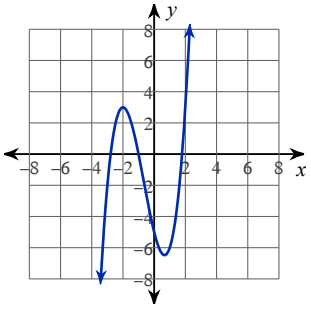

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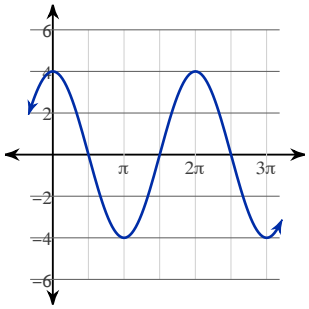

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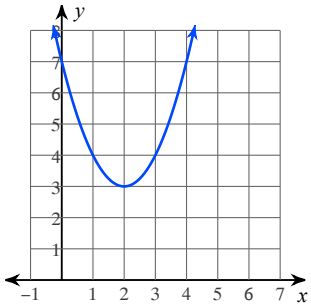

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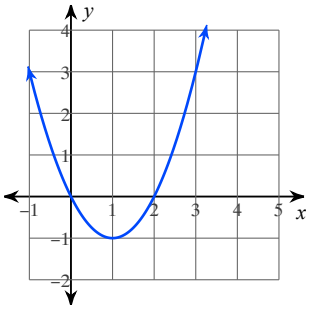

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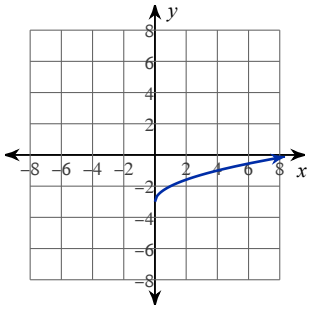

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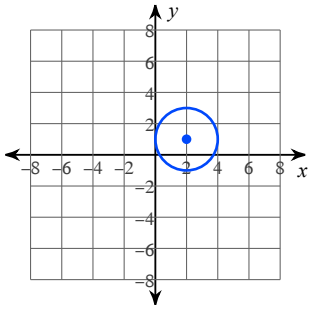

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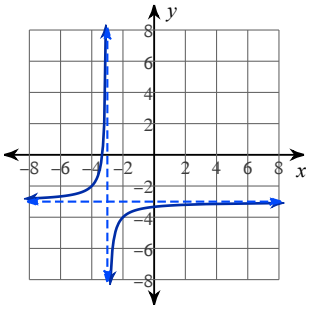

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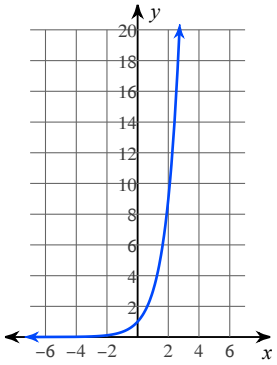

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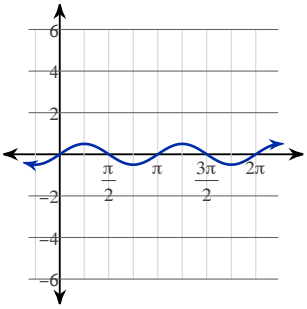

Expand each logarithm.

1) $\log_6 \left(\frac{x^4}{y}\right)^6$

2) $\log_2 \left(3 \cdot 8^2\right)^2$

Condense each expression to a single logarithm.

3) $\log_9 6 + \frac{\log_9 5}{3} + \frac{\log_9 11}{3}$

4) $3\log_3 7 - 15\log_3 5$

Solve each equation without a calculator

5) $243^{2v} = 81$

6) $2^{-2r} = 2^{3r}$

Solve each equation. Round your answers to the nearest ten-thousandth.

7) $3^{4x} - 6 = 17$

8) $-6 \cdot 4^{b+5} = -43$

9) $7 \cdot 18^{10m} = 42$

10) $6^{x-1} - 2 = 62$

Answers to

1) $24\log_6 x - 6\log_6 y$

2) $2\log_2 3 + 12$

3) $\log_9 \left(6\sqrt[3]{55}\right)$

4) $\log_3 \dfrac{7^3}{5^{15}}$

5) $\left\{\dfrac{2}{5}\right\}$

6) $\{0\}$

7) 0.7135

8) –3.5793

9) 0.062

10) 3.3211