

Summer Assignment

Fractional and Negative Exponents

Simplify, using positive exponents.

$$1. 3 \left(\frac{3}{3-x} \right) \left[\frac{-3}{(3-x)^2} \right]^{-3}$$

$$2. \frac{\sqrt{9x-81}}{\sqrt[4]{(x-9)^2}}$$

$$3. \frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$$

$$4. \left(\frac{1}{x^{-2}} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^{-2}} \right)^{\frac{1}{2}}$$

Domain

Find the domain of the following functions.

$$5. y = \frac{x^2-5x+6}{x^2-3x-18}$$

$$6. f(x) = \frac{\sqrt{3x-4}}{3x+4}$$

$$7. g(x) = \sqrt{x^2-5x-14}$$

$$8. y = \ln(3x+15)$$

Asymptotes

For each function, find the equations of both the vertical and horizontal asymptote(s), if they exist.

$$9. y = \frac{x+3}{x^2+2}$$

$$10. f(x) = \frac{x^2-9}{x^3+3x^2-18x}$$

$$11. g(x) = \frac{3x^3}{x^3-27}$$

Complex Fractions

Simplify.

$$12. \frac{\frac{2}{x} \frac{5}{y}}{\frac{5}{x} \frac{2}{y}}$$

$$14. \frac{1-\frac{3}{4x}}{x-\frac{9}{16x}}$$

$$16. \frac{\frac{4}{x-5} + \frac{2}{x+2}}{\frac{2x}{x^2-3x-10} + 3}$$

$$13. \frac{x^{-4}-x^{-1}}{x^{-3}-1}$$

$$15. \frac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}}$$

$$17. \frac{\frac{4x}{x^2-4} + \frac{-2}{x+2}}{x-2}$$

Rationalizing:

Remove the sum or difference from the denominator by multiplying the numerator and denominator by the conjugate of the denominator.

$$18. \frac{1}{1-\cos x}$$

$$19. \frac{x}{1-\sqrt{x^2+1}}$$

$$20. \frac{2}{x+\sqrt{x^2+1}}$$

Factoring

Monomial Factoring: Factor out the greatest common factor or as indicated.

$$1. 3x^4 + 4x^3 - x^2$$

$$2. 2\sqrt{x} + 6x^{3/2}$$

$$3. e^{-x} - xe^{-x} + 2x^2e^{-x}$$

$$4. x^{-1} - 2 + x = x^{-1} (\quad)$$

$$5. \frac{x}{2} - 6x^2 = \frac{x}{2} (\quad)$$

$$6. \sin x + \tan x = \sin x (\quad)$$

$$7. \frac{1}{2x^2+4x} = \frac{1}{2x} (\quad)$$

Factor completely, or state that the polynomial is prime.

8. $x^2 - 9$

9. $3x^3 - 3x$

10. $5x^3 - 45x$

11. $4x^2 - 4x - 24$

12. $6x^2 - 18x - 30$

13. $2x^4 - 162$

14. $x^3 + 2x^2 - 9x - 18$

15. $2x^2 - 2x - 112$

16. $x^3 - 4x$

17. $x^2 + 64$

18. $x^3 + 2x^2 - 4x - 8$

19. $y^5 - 81y$

20. $20y^4 - 45y^2$

21. $e^{2x} + 2 + e^{-2x}$

22. $x^4 - 7x^2 + 12$

23. $1 - \sin^2 x$

24. $x^2 - 12x + 36 - 49y^2$

25. $9b^2x - 16y - 16x + 9b^2y$

26. $x^2y - 16y + 32 - 2x^2$

27. $2x^3 - 8a^2x + 24x^2 + 72x$

Factor and simplify each algebraic expression.

28. $x^{3/2} - x^{1/2}$

29. $4x^{-2/3} + 8x^{1/3}$

30. $(x+3)^{1/2} + (x+3)^{3/2}$

31. $(x+5)^{-1/2} + (x+5)^{-3/2}$

32. $(4x-1)^{1/2} - \frac{1}{3}(4x-1)^{3/2}$

Simplify each of the following:

33. $\frac{3x+9}{6x}$

34. $\frac{x^2}{x^{1/2}}$

35. $y = \frac{x^3-1}{x+1}$

36. $y = \frac{x^6-1}{x+1}$

37. $y = \frac{x^3+1}{x^2-x+1}$

38. $y = \frac{\sqrt{x+7}}{\sqrt{(x+7)^3}}$

39. $y = \frac{3x(x^3+3x)^{-3/2}}{(x^3+3x)^{1/2}}$

40. $y = \frac{1}{2}(x^4+8x)^{-1/2}(4x^3+8)$

41. $y = \frac{1}{2} \frac{3^x}{\sqrt{x}} + 3^x \sqrt{x}$

42. $y = \frac{1}{x^2+x} - \frac{2}{x^2-1}$

43. $y = \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(e^x-e^{-x})}{(e^x-e^{-x})^2}$

44. $\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}$

45. $\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

46. $\frac{x^{1/2}-x^{1/3}}{x^{1/6}}$

47. $\frac{\sqrt{x-1}+(x-1)^{3/2}}{\sqrt{x-1}}$

48. $\frac{1-(\sin x + \cos x)^2}{2 \sin x}$

Rewrite each of the following in simplest form.

$$49. \frac{(x-1)(x+3)-(x+1)^2}{x+1}$$

$$50. \frac{\sqrt{x^2+1} - \frac{1}{\sqrt{x^2+1}}}{x^2+1}$$

$$51. \frac{x^2-5x+6}{x^2-4x+4}$$

$$52. \frac{1}{x+1} - \frac{1}{x-1} - \frac{2}{x^2-1}$$

$$53. \frac{x(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}}$$

Expanding Logarithmic Expressions

For $A > 0$ and $B > 0$, $b > 0$, $b \neq 1$, and p any real number.

1. Product Rule $\log_b(AB) = \log_b A + \log_b B$

2. Quotient Rule $\log_b\left(\frac{A}{B}\right) = \log_b A - \log_b B$

3. Power Rule $\log_b A^p = p \log_b A$

Expand using properties:

$$54. \log \sqrt[3]{\frac{x}{y}}$$

$$55. \log \sqrt[5]{\frac{x}{y}}$$

$$56. \log_b \left(\frac{\sqrt{xy^3}}{x^3} \right)$$

$$57. \log_b \left(\frac{\sqrt[3]{xy^4}}{x^5} \right)$$

$$58. \log_5 \sqrt[3]{\frac{x^2 y}{25}}$$

$$59. \log_2 \sqrt[5]{\frac{xy^4}{16}}$$

$$60. \ln \left[\frac{x^3 \sqrt{x^2+1}}{(x+1)^4} \right]$$

$$61. \ln \left[\frac{x^4 \sqrt{x^2+3}}{(x+3)^5} \right]$$

$$62. \log \left[\frac{10x^2 \sqrt{1-x}}{7(x+1)^2} \right]$$

$$63. \log \left[\frac{100x^3 \sqrt[3]{5-x}}{3(x+7)^2} \right]$$

Geometry. Trigonometry.

Evaluate each trigonometric function for the value or angle given.

64. $\csc \frac{\pi}{2}$

65. $\cos^{-1}(1)$

66. $\sec \frac{-2\pi}{3}$

67. $\cot \frac{11\pi}{6}$

68. $\arcsin \frac{-\sqrt{2}}{2}$

69. $\arctan(1)$

Determine each of the following:

70. $\sin(\arctan \sqrt{3})$

71. $\cos(\arcsin(-\frac{1}{2}))$

72. $\sin(\arccos -\frac{\sqrt{3}}{2})$

73. $\tan(\arcsin \frac{\sqrt{2}}{2})$