LWHS AP Calculus BC Summer Assignment

Name ______________________________

AP Calculus BC builds on the concepts of limits, derivatives, and integrals mastered in Calculus AB. You will learn advanced integration techniques and new concepts in polar, parametric, vectors and infinite series. In order to focus on the new topics, please review the limit and derivative concepts learned in Calc AB. Your work should be written on a separate piece of paper and is due the 2nd day of school. *A calculator should not be used except for arithmetic.*

Part I: PreCalculus Flashback

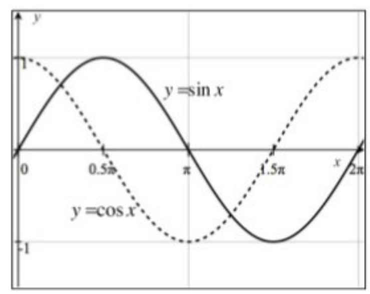
1. Consider the circle of radius 5 centered at (0, 0). Find an equation of the line tangent to the circle at the point (3, 4) in slope intercept form.

2. Graph the function shown below. Also indicate any key points and state the domain and

$$f(x)=\begin{cases}4-x^2, & x<1\\ \frac{3}{2}x+\frac{3}{2}, & 1\le x\le 3\\ x+3, & x>3\end{cases}$$

3. Complete the table for each trig function in Quadrant I.

Trig Function	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin θ					
Cos θ					
Tan θ					



Part II: Unlimited and Continuous!

For #1-4 below, find the limits, if they exist.

1) $\lim\limits_{x\to 4}\dfrac{2x^3-7x^2-4x}{x-4}$

2) $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{9-x}$

3) $\lim\limits_{x\to 1}\dfrac{x^2-2x-5}{x+1}$

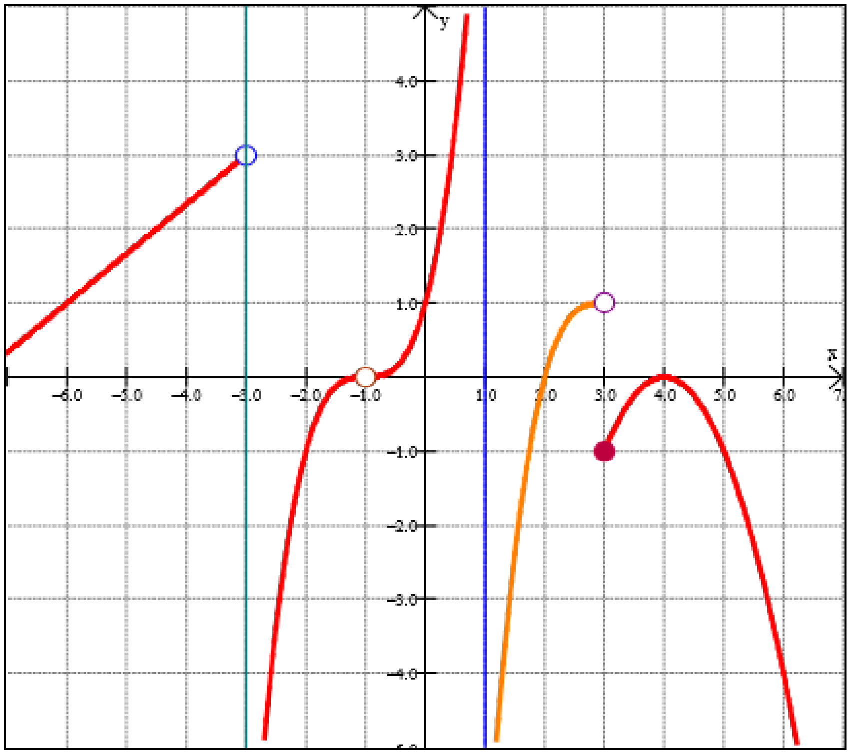
For #5-7, explain why each function is discontinuous and determine if the discontinuity is removable or nonremovable.

5) $g(x)=\begin{cases}2x-3, & x<3\\ -x+5, & x\ge 3\end{cases}$

6) $b(x)=\dfrac{x(3x+1)}{3x^2-5x-2}$

7) $h(x)=\dfrac{\sqrt{x^2-10x+25}}{x-5}$

For #8-13, determine if the following limits exist, based on the graph below of p(x). If the limits exist, state their value. Note that x = -3 and x = 1 are vertical asymptotes.



8) $\lim_{x \to 1^-} p(x)$

9) $\lim_{x \to -3^-} p(x)$

10) $\lim_{x \to 2} p(x)$

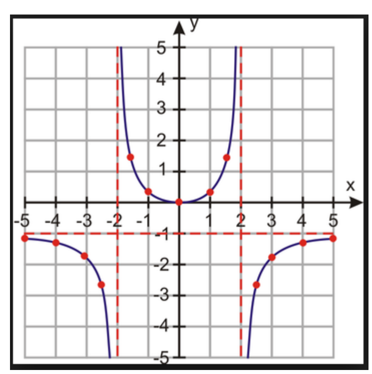
11) $\lim_{x \to 3^-} p(x)$

12) $\lim_{x \to 3^+} p(x)$

13) $\lim_{x \to -1} p(x)$

14) Consider the function $f(x) = \begin{cases} x^2 + kx & x \le 5 \\ 5\sin\left(\frac{\pi}{2}x\right) & x > 5 \end{cases}$,

In order for the function to be continuous at x = 5, the value of k must be



16) For what value of a is $\lim_{x \to a} f(x)$ nonexistent?

17) $\lim_{x \to \infty} f(x) =$

18) $\lim_{x \to -\infty} f(x) =$

Part III: Designated Deriving!

Find the derivative function, $f'(x)$, for each of the following using the limit definition.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(a) $f(x) = 2x^2 + 3x - 4$ (b) $f(x) = \frac{3}{x-1}$ (c) $f(x) = \sqrt{x-2}$

For #3-8, find the derivative.

3) $y = \ln(1 + e^x)$

4) $y = \csc(1 + \sqrt{x})$

5) $y = (\tan^2 x)(3\pi x - e^{2x})$

6) $y = \sqrt[7]{x^3 - 4x^2}$

7) $f(x) = (x+1)e^{3x}$

8) $f(x) = \frac{e^{x/2}}{\sqrt{x}}$

9) Consider the function $f(x) = \sqrt{x-2}$. On what intervals are the hypotheses of the Mean Value Theorem satisfied?

10) If $xy^2 - y^3 = x^2 - 5$, then $\frac{dy}{dx} =$

11) The distance of a particle from its initial position is given by $s(t) = t - 5 + \frac{9}{(t+1)}$, where s is feet and t is minutes. Find the velocity at t = 1 minute in appropriate units.

Use the table below for #12-13.

X	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	4	2	5	$\frac{1}{2}$
3	7	-4	$\frac{3}{2}$	-1

12) The value of $\frac{d}{dx}(f \cdot g)$ at x = 3 is

13) The value of $\frac{d}{dx}\left(\frac{f}{g}\right)$ at x = 1 is

In #14-15, use the table below to find the value of the first derivative of the given functions for the given value of x.

X	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	3	2	0	3/4
2	7	-4	$\frac{1}{3}$	-1

14) $[f(x)]^2$ at x = 2 is

15) $f(g(x))$ at x = 1 is

16) Let f be the function defined by $f(x) = \frac{x + \sin x}{\cos x}$ *for* $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(a) State whether *f* is an even function or an odd function. Justify your answer..
(b) Find $f'(x)$.
(c) Write an equation for the line tangent to the graph of *f* at the point $(0, f(0))$.

Part IV: Derived and Applied!

For #1-3, find all critical values, intervals of increasing and decreasing, any local extrema, points of inflection, and all intervals where the graph is concave up and concave down.

1) $f(x) = \frac{5 - 4x + 4x^2 - x^3}{x - 2}$

2) $y = 3x^3 - 2x^2 + 6x - 2$

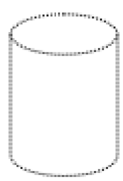
3) $f'(x) = 5x^3 - 15x + 7$

4) The graph of the function $y = x^5 - x^2 + \sin x$ changes concavity at x =

5) Find the equation of the line tangent to the function $y = \sqrt[4]{x^7}$ at $x = 16$.

6) For what value of x is the slope of the tangent line to $y = x^7 + \frac{3}{x}$ undefined?

7)



The balloon shown is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute. (The volume of a cylinder with radius *r* and height *h* is $\pi r^2 h$, and the volume of a sphere with radius *r* is $\frac{4}{3}\pi r^3$.)

(a) At this instant, what is the height of the cylinder?
(b) At this instant, how fast is the height of the cylinder increasing?