



## Honors Pre-Calculus Summer Packet

Welcome to the challenging world of higher-level mathematics. Included in this packet is a description of the skills that you are expected to bring with you into the Honors Pre-Calculus course. These concepts were learned in Algebra and Geometry. Please refresh your memory by doing all the exercises completely. Please read all the enclosed material and do all the problems, showing your work. Bring the completed packet with you on the first day of class for your first homework grade. There will also be a quiz on this material at some time during the first week of classes. We will spend a little bit of class time going over the packet and Activity Period will be available to you for extra help prior to this quiz.

If you are doubling up this year and have Honor's Algebra II first semester, then you will need to wait until you have finished most of this course before completing this packet. Any student who has the course second semester is more than welcome to come to any Activity Period during the first semester to receive help on the packet.

Enjoy your summer and we look forward to working with you next school year!

## Section 1

## Exponents & Logarithms

### PROPERTIES OF EXPONENTS

Let  $a$  and  $b$  be real numbers, variables, or algebraic expressions, and let  $m$  and  $n$  be integers. (Assume all denominators and bases are nonzero.)

Property	Example
1. $a^m a^n = a^{m+n}$	$3^2 \cdot 3^4 = 3^{2+4} = 3^6$
2. $\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^7}{x^4} = x^{7-4} = x^3$
3. $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$	$y^{-4} = \frac{1}{y^4} = \left(\frac{1}{y}\right)^4$
4. $a^0 = 1, a \neq 0$	$(x^2 + 1)^0 = 1$
5. $(ab)^m = a^m b^m$	$(5x)^3 = 5^3 x^3 = 125x^3$
6. $(a^m)^n = a^{mn}$	$(y^3)^{-4} = y^{3(-4)} = y^{-12} = \frac{1}{y^{12}}$
7. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{2}{x}\right)^3 = \frac{2^3}{x^3} = \frac{8}{x^3}$
8. $ a^2  =  a ^2 = a^2$	$ (-2)^2  =  -2 ^2 = (-2)^2 = 4$

The numerator of a rational exponent denotes the *power* to which the base is raised, and the denominator denotes the *index* or the *root* to be taken, as shown below.

$$b^{m/n} = \left( \sqrt[n]{b} \right)^m = \sqrt[n]{b^m}$$

### PROPERTIES OF RADICALS

Let  $a$  and  $b$  be real numbers, variables, or algebraic expressions such that the indicated roots are real numbers, and let  $m$  and  $n$  be positive integers.

Property	Example
1. $\sqrt[m]{a^n} = (\sqrt[m]{a})^n$	$\sqrt[3]{8^2} = (\sqrt[3]{8})^2 = (2)^2 = 4$
2. $\sqrt[m]{a} \cdot \sqrt[m]{b} = \sqrt[m]{ab}$	$\sqrt{3} \cdot \sqrt{12} = \sqrt{3 \cdot 12} = \sqrt{36} = 6$
3. $\frac{\sqrt[m]{a}}{\sqrt[m]{b}} = \sqrt[m]{\frac{a}{b}}, b \neq 0$	$\frac{\sqrt[4]{27}}{\sqrt[4]{9}} = \sqrt[4]{\frac{27}{9}} = \sqrt[4]{3}$
4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt[3]{\sqrt{10}} = \sqrt[6]{10}$
5. $(\sqrt[n]{a})^m = \sqrt[nm]{a}$	$(\sqrt{3})^2 = \sqrt[4]{3}$
6. For $n$ even, $\sqrt[n]{a^2} =  a $ .	$\sqrt{(-12)^2} =  -12  = 12$
For $n$ odd, $\sqrt[n]{a^n} = a$ .	$\sqrt[3]{(-12)^3} = -12$

## Section 1

Simplify each expression. Write answers in positive exponents only.

1.  $(-a)^6(2a^4)$

2.  $5x^0 + (5x)^0$

3.  $8x^3(2x^5)^2$

4.  $\frac{15b^3}{3b^6}$

5.  $(x^{-5}y^6)(x^2y^3)$

6.  $\frac{c^2 \cdot c^{4x}}{c^x \cdot c^3}$

7.  $\sqrt[3]{8x^3y^4}$

8.  $\sqrt[6]{x^6}$

9.  $\sqrt{6xy^2}\sqrt{2x}$

10.  $\frac{\sqrt[4]{16b^5}}{\sqrt[4]{b}}$

11.  $\frac{9x}{\sqrt[3]{(3x)^2}}$

12.  $\sqrt{75y} - \sqrt{3y}$

13.  $\sqrt[3]{250} + 2\sqrt[3]{16}$

14.  $(3x^5)(-x)^4$

15.  $(6a)^0 + 3a^0$

16.  $2y^5(3y^3)^3$

17.  $\frac{25c^5}{35c^2}$

18.  $(a^7b^2)(a^3b^{-6})$

19.  $\frac{n^{2x} \cdot n^4}{n^2 \cdot n^{5x}}$

20.  $\sqrt[4]{16m^6n^5}$

21.  $\sqrt[2]{9^9}$

22.  $\sqrt{6s^2t}\sqrt{3t}$

23.  $\frac{\sqrt[3]{8c^{10}}}{\sqrt[3]{c}}$

24.  $\frac{6y}{\sqrt[4]{(2y)^3}}$

25. Write the expression as a single radical

$$\sqrt{18}\sqrt[3]{2}$$

**SPECIAL PRODUCTS**

Let  $u$  and  $v$  be real numbers, variables, or algebraic expressions.

<i>Special Product</i>	<i>Example</i>
<b>Sum and Difference of Two Terms</b> $(u + v)(u - v) = u^2 - v^2$	$(x + 4)(x - 4) = x^2 - 4^2$ $= x^2 - 16$
<b>Square of Binomial</b> $(u + v)^2 = u^2 + 2uv + v^2$ $(u - v)^2 = u^2 - 2uv + v^2$	$(x + 3)^2 = x^2 + 2(x)(3) + 3^2$ $= x^2 + 6x + 9$ $(3x - 2)^2$ $= (3x)^2 - 2(3x)(2) + 2^2$ $= 9x^2 - 12x + 4$
<b>Cube of a Binomial</b> $(u + v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$ $(u - v)^3 = u^3 - 3u^2v + 3uv^2 - v^3$	$(x + 2)^3$ $= x^3 + 3x^2(2) + 3x(2^2) + 2^3$ $= x^3 + 6x^2 + 12x + 8$ $(x - 1)^3$ $= x^3 - 3x^2(1) + 3x(1^2) - 1^3$ $= x^3 - 3x^2 + 3x - 1$

**GUIDELINES FOR FACTORING POLYNOMIALS**

- Factor out any common factors.
- Factor according to one of the special polynomial forms.
- Factor  $ax^2 + bx + c$  using the FOIL method.
- Factor by grouping.

**FACTORIZING SPECIAL POLYNOMIAL FORMS**

<i>Factored Form</i>	<i>Example</i>
<b>Difference of Two Squares</b> $u^2 - v^2 = (u + v)(u - v)$	$9x^2 - 4 = (3x)^2 - 2^2$ $= (3x + 2)(3x - 2)$
<b>Perfect Square Trinomial</b> $u^2 + 2uv + v^2 = (u + v)^2$ $u^2 - 2uv + v^2 = (u - v)^2$	$x^2 + 6x + 9$ $= x^2 + 2(x)(3) + 3^2$ $= (x + 3)^2$ $x^2 - 6x + 9$ $= x^2 - 2(x)(3) + 3^2$ $= (x - 3)^2$
<b>Sum or Difference of Two Cubes</b> $u^3 + v^3 = (u + v)(u^2 - uv + v^2)$ $u^3 - v^3 = (u - v)(u^2 + uv + v^2)$	$x^3 + 8 = x^3 + 2^3$ $= (x + 2)(x^2 - 2x + 4)$ $27x^3 - 1$ $= (3x)^3 - 1^3$ $= (3x - 1)(9x^2 + 3x + 1)$

**Factoring by Grouping**

$$\begin{aligned} x^3 - 2x^2 - 3x + 6 &= (x^3 - 2x^2) - (3x - 6) \\ &= x^2(x - 2) - 3(x - 2) \\ &= (x - 2)(x^2 - 3) \end{aligned}$$

**Examples** Factoring Polynomials and Solving Polynomial Equations

- a.  $16x^3 - 80x^2 + 100x = 4x(4x^2 - 20x + 25)$  *Common monomial*  
 $= 4x(2x - 5)^2$  *Perfect-square trinomial*
- b.  $3x^3 + 21x = 24x^2$  *Original equation*  
 $3x^3 - 24x^2 + 21x = 0$  *Collect terms on the left side.*  
 $3x(x^2 - 8x + 7) = 0$  *Factor out common monomial factor.*  
 $3x(x - 7)(x - 1) = 0$  *Factor trinomial.*  
 $3x = 0 \Rightarrow x = 0$  *Set first factor equal to 0.*  
 $x - 7 = 0 \Rightarrow x = 7$  *Set second factor equal to 0.*  
 $x - 1 = 0 \Rightarrow x = 1$  *Set third factor equal to 0.*
- c.  $x^3 + x^2 - 2x = 2$  *Original equation*  
 $x^3 + x^2 - 2x - 2 = 0$  *Collect terms on the left side.*  
 $(x^3 + x^2) - (2x + 2) = 0$  *Group terms (to factor by grouping).*  
 $x^2(x + 1) - 2(x + 1) = 0$  *Factor out common monomial factors.*  
 $(x + 1)(x^2 - 2) = 0$  *Factor out common binomial factor.*  
 $x + 1 = 0 \Rightarrow x = -1$  *Set first factor equal to 0.*  
 $x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{2}$  *Set second factor equal to 0.*

**Examples** Solving a Quadratic Equation by the Quadratic Formula

- a.  $x^2 - 2x + 1 = 3x + 5$  *Original equation*  
 $x^2 - 5x - 4 = 0$  *Write in standard form:  $a = 1$ ,  $b = -5$ , and  $c = -4$ .*  
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-4)}}{2(1)}$$
 *Quadratic formula*  
$$x = \frac{5 \pm \sqrt{41}}{2}$$
 *Simplify.*  
The solutions are  $x = \frac{5 + \sqrt{41}}{2}$  and  $x = \frac{5 - \sqrt{41}}{2}$ .
- b.  $8x^2 + 28x = 12$  *Original equation*  
 $8x^2 + 28x - 12 = 0$  *Write the equation in standard form.*  
 $2x^2 + 7x - 3 = 0$  *Divide by the common multiple 4.*  
$$x = \frac{-7 \pm \sqrt{7^2 - 4(2)(-3)}}{2(2)}$$
 *Substitute 2 for  $a$ , 7 for  $b$ , and  $-3$  for  $c$  into quadratic formula.*  
$$x = \frac{-7 \pm \sqrt{73}}{4}$$
 *Simplify.*  
The solutions are  $x = \frac{-7 + \sqrt{73}}{4}$  and  $x = \frac{-7 - \sqrt{73}}{4}$ .

## Section 2

Perform the indicated operations and write the result in standard form.

1.  $(3x-2)-(8x-6)$

2.  $(x-2)(x^2-5)$

3.  $(3x-7y)^2$

4.  $(x-4)^3$

5.  $5a(4a-6)-a(3a+2)$

6.  $(y-z)(y+z)+3(y-z)$

7.  $[3-(a+b)][3+(a+b)]$

8.  $(a+2b)^3$

9.  $3y^2-[2y^2-(5y^2+4)]$

10.  $[(x+2)-y]^2$

Factor completely.

11.  $6y^3 - 4y^2 + 2y$

12.  $8x^3 - 1$

13.  $x^3 + 6x^2 - 5x - 30$

14.  $2r^3 + 2$

15.  $x^2 - \frac{9}{16}$

16.  $9t^2 - 30t + 25$

17.  $4x^2 + 5x + 1$

18.  $y^5 + 3y^3 + y^2 + 3$

19.  $27a^3 + 125b^3$

20.  $4 - (a - b)^2$



Perform the indicated operations and write the result in standard form.

21.  $(11x-12)-(7x-2)$       22.  $(4x-5y)^2$       23.  $(2t-3)^3$

24.  $6b(2b+3)-b(2b+1)$       25.  $[2-(x+y)][2+(x+y)]$

Factor completely.

26.  $c^3 + 7c^2 - 5c - 35$       27.  $2r^3 - 50r$       28.  $4x^2 - 28x + 49$

Solve.

29.  $6x^2 + 5x - 4 = 0$       30.  $4x^2 - 3x - 2 = 0$       31.  $2x^3 - x^2 = 6x - 3$

## Section 3

## Rational Expressions

## Reducing Rational Expressions

$$\begin{aligned}\frac{x^2 - 4x}{x^2 + x - 2} &= \frac{x(x^2 - 4)}{(x+2)(x-1)} \\ &= \frac{x\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-1)} \\ &= \frac{x(x-2)}{x-1}, \quad x \neq -2\end{aligned}$$

$$\begin{aligned}\frac{12 + x - x^2}{2x^2 - 9x + 4} &= \frac{(4-x)(3+x)}{(2x-1)(x-4)} \\ &= \frac{-\cancel{(x-4)}(3+x)}{(2x-1)\cancel{(x-4)}} \\ &= -\frac{3+x}{2x-1}, \quad x \neq 4\end{aligned}$$

## Multiplying and Dividing Rational Expressions

$$\begin{aligned}\frac{2x^2 + x - 6}{x^2 + 4x - 5} \cdot \frac{x^3 - 3x^2 + 2x}{4x^2 - 6x} \\ &= \frac{\cancel{(2x-3)}(x+2)}{(x+5)\cancel{(x-1)}} \cdot \frac{x(x-2)\cancel{(x-1)}}{2x\cancel{(2x-3)}} \\ &= \frac{(x+2)(x-2)}{2(x+5)}, \quad x \neq 0, x \neq 1, x \neq \frac{3}{2}\end{aligned}$$

$$\begin{aligned}\frac{x^3 - 8}{x^2 - 4} \div \frac{x^2 + 2x + 4}{x^3 + 8} &= \frac{x^3 - 8}{x^2 - 4} \cdot \frac{x^3 + 8}{x^2 + 2x + 4} \\ &= \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x+2)}(x-2)} \cdot \frac{\cancel{(x+2)}(x^2 - 2x + 4)}{x^2 + 2x + 4} \\ &= x^2 - 2x + 4, \quad x \neq \pm 2\end{aligned}$$

## Subtracting Rational Expressions Using the Basic Definition

$$\begin{aligned}\frac{x}{x-3} - \frac{2}{3x+4} &= \frac{x(3x+4) - 2(x-3)}{(x-3)(3x+4)} \\ &= \frac{3x^2 + 4x - 2x + 6}{(x-3)(3x+4)} \\ &= \frac{3x^2 + 2x + 6}{(x-3)(3x+4)}\end{aligned}$$

## Compound Fractions

Fractional expressions with fractions in the numerator or denominator, or both, are called **compound fractions** (or complex fractions).

A compound fraction can be simplified by first simplifying both its numerator and its denominator into single fractions, then inverting the denominator and multiplying.

### Simplifying a Compound Fraction

$$\begin{aligned}\frac{\left(\frac{2}{x} - 3\right)}{\left(1 - \frac{1}{x-1}\right)} &= \frac{\left(\frac{2-3(x)}{x}\right)}{\left(\frac{1(x-1)-1}{x-1}\right)} \\ &= \frac{\left(\frac{2-3x}{x}\right)}{\left(\frac{x-2}{x-1}\right)} \\ &= \frac{2-3x}{x} \cdot \frac{x-1}{x-2} \\ &= \frac{(2-3x)(x-1)}{x(x-2)}, \quad x \neq 1\end{aligned}$$

### Section 3

Write in reduced form.

1.  $\frac{12b^6}{60b^2}$

2.  $\frac{5w^2 - 10w}{15w - 15}$

3.  $\frac{z - 5}{25 - z^2}$

4.  $\frac{x^3 + 2x^2 - 4x - 8}{x^3 - 8}$

Perform the indicated operations and simplify.

5.  $\frac{y^2 - 9}{y^2 + y - 6} \cdot \frac{y^2 - 4y + 4}{y - 3}$

6.  $\left(\frac{a+b}{b}\right) + \left(\frac{a}{b} - \frac{b}{a}\right)$

$$7. \frac{2}{s-4} - \frac{1}{s+3}$$

$$8. \frac{3x}{x^2-6x+8} + \frac{4}{x^2-x-12}$$

$$9. \frac{2 + \frac{2}{x}}{\frac{1}{x+2} - 1}$$

$$10. \frac{\frac{3}{\sqrt{x-1}} - \sqrt{x-1}}{4\sqrt{x-1}}$$

Write in reduced form.

$$11. \frac{25a^3}{70a^5}$$

$$12. \frac{8y^2+8y}{3y+3}$$

$$13. \frac{x^2-49}{7-x}$$

$$14. \frac{t^3-27}{t^3-3t^2-2t+6}$$

$$15. \frac{x^3-3x^2+2x}{x^3-4x}$$

Perform the indicated operations and simplify.

$$16. \frac{b^2-9}{b^2-b-6} \cdot \frac{b^2+8b+9}{b+3}$$

$$17. \frac{x+1}{x^2-1} + \frac{x^2+1}{x-1}$$

$$18. \left(\frac{1}{x+y}\right)\left(\frac{x}{y} + \frac{y}{x}\right)$$

$$19. \frac{2}{c-3} + \frac{1}{c+2}$$

$$20. \frac{3}{x^2-1} - \frac{4}{x^2-3x+2}$$

$$21. \frac{\frac{3}{x}-4}{1+\frac{1}{x-1}}$$

$$22. \frac{\frac{3}{\sqrt{x+2}} - \sqrt{x+2}}{5\sqrt{x+2}}$$

The distance  $d$  between the two points  $(x_1, y_1)$  and  $(x_2, y_2)$  in the coordinate plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

#### Finding the Distance Between Two Points

Find the distance between the points  $(-2, 1)$  and  $(3, 4)$ .

#### SOLUTION

Letting  $(x_1, y_1) = (-2, 1)$  and  $(x_2, y_2) = (3, 4)$ , apply the Distance Formula to obtain

$$\begin{aligned} d &= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} \\ &= \sqrt{5^2 + 3^2} \\ &= \sqrt{25 + 9} \\ &= \sqrt{34} \\ &\approx 5.83. \end{aligned}$$

#### THE MIDPOINT FORMULA

The midpoint of the line segment joining the points  $(x_1, y_1)$  and  $(x_2, y_2)$  in the coordinate plane is

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

#### Finding the Midpoint of a Line Segment

Find the midpoint of the line segment joining the points  $(-5, -3)$  and  $(9, 3)$ .

#### SOLUTION

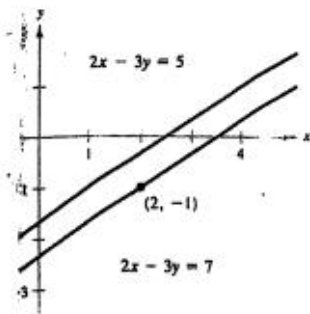
Figure 13 shows the two given points and their midpoint. Using the Midpoint Formula, you can write

$$\begin{aligned} \text{Midpoint} &= \left( \frac{-5 + 9}{2}, \frac{-3 + 3}{2} \right) \\ &= (2, 0). \end{aligned}$$



### Equations of Lines

1. General form:  $Ax + By + C = 0$
2. Vertical line:  $x = a$
3. Horizontal line:  $y = b$
4. Slope-intercept form:  $y = mx + b$
5. Point-slope form:  $y - y_1 = m(x - x_1)$



### Parallel Lines

Two distinct nonvertical lines are parallel if and only if their slopes are equal.

### Perpendicular Lines

Two nonvertical lines are perpendicular if and only if their slopes are related by the equation

$$m_1 = -\frac{1}{m_2}$$

### Equations of Parallel Lines

Find an equation of the line that passes through the point  $(2, -1)$  and is parallel to the line  $2x - 3y = 5$ , as shown in Figure 2.40.

#### SOLUTION

Writing the given equation in slope-intercept form, we have

$$2x - 3y = 5 \qquad \text{Given equation}$$

$$3y = 2x - 5$$

$$y = \frac{2}{3}x - \frac{5}{3} \qquad \text{Slope-intercept form}$$

Therefore, the given line has a slope of  $m = 2/3$ . Since any line parallel to the given line must also have a slope of  $2/3$ , the required line through  $(2, -1)$  has the equation

$$y - (-1) = \frac{2}{3}(x - 2)$$

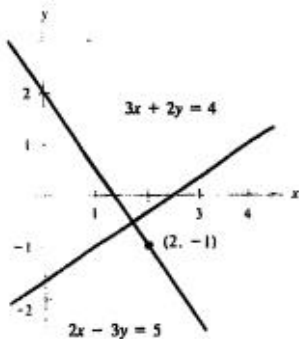
$$3(y + 1) = 2(x - 2)$$

$$3y + 3 = 2x - 4$$

$$-2x + 3y = -7$$

$$2x - 3y = 7$$

Note the similarity to the original equation  $2x - 3y = 5$ .



### Equations of Perpendicular Lines

Find an equation of the line that passes through the point  $(2, -1)$  and is perpendicular to the line  $2x - 3y = 5$ .

#### SOLUTION

From Example 5, the given line has slope  $2/3$ . Hence, any line perpendicular to this line must have a slope of  $-3/2$ . Therefore, the required line through  $(2, -1)$  has the equation

$$y - (-1) = -\frac{3}{2}(x - 2)$$

$$2(y + 1) = -3(x - 2)$$

$$3x + 2y = 4$$

## Section 4

- Find  $y$  so that the distance between  $(3,y)$  and  $(-1,1)$  is 5.
- Determine the quadrant(s) in which  $(x,y)$  is located so that the conditions are satisfied.
  - $x > 0$  and  $y < 0$
  - $y > 2$
- Determine if the lines  $L_1$  and  $L_2$  passing through the given pairs of points are parallel, perpendicular or neither.
  - $L_1 : (0, -1), (5, 9)$   
 $L_2 : (0, 3), (4, 1)$
  - $L_1 : (4, 8), (-4, 2)$   
 $L_2 : (3, -5), (-1, \frac{1}{3})$
- Find the slope and  $y$ -intercept (if possible) of the line specified by the given equation.
  - $5x - y + 3 = 0$
  - $5x - 2 = 0$
- Write an equation of the line through the given point (a) parallel to the given line and (b) perpendicular to the given line.  
  
Point  $(-6, 4)$  Line  $3x + 4y = 7$

6. Show that the points form the vertices of the indicated polygon:

Right Triangle:  $(4, 0)$ ,  $(2, 1)$ ,  $(-1, -5)$

7. Find an equation of the line that passes through the given point and has the indicated slope. Sketch the graph of the line.

a) Point  $(-3, 6)$   $m = -2$

b) Point  $(6, -1)$   $m$  is undefined

8. Determine the quadrant(s) in which  $(x, y)$  is located so that the given conditions are satisfied:

a)  $(x, -y)$  is in the second quadrant

b)  $(-x, y)$  is in the fourth quadrant





