TO: Honors Algebra II Students

Summer is finally here . . .

In anticipation of the next school year, we want to do everything possible to ensure that it will be a successful for you. It is important that the foundation for Algebra II is thoroughly understood. Therefore, we have designed a summer packet that reviews these essential skills. You are required review this material and complete all of the exercises. These problems can be completed in packet, if not complete the problems on loose leaf paper showing all your work. **You will be tested on this material during the first week of class!** The grade for this quiz will be included in your first marking period average.

During the first week of the semester, we will offer review sessions during activity periods for those who feel they need some extra help. Attendance is optional, so if you are comfortable with the work there is no need to attend. However, we will be glad to give you a refresher if you've had some trouble completing the packet. Please come prepared with questions when you attend review sessions.

Graphing calculators will be used throughout the course. **This year you will be required to have your own graphing calculator.** We will be using a TI-83+ or TI 84 calculator in the classroom so this is the type of calculator you are required to have.

We hope you have an enjoyable and relaxing summer. We are looking forward to seeing you next year. Bring your completed summer packet with you the first day of class so we can immediately begin a fun and challenging semester of Honors Algebra II!

Sincerely,

The Honors Algebra II Teachers
Notre Dame High School

Order of Operations

Evaluate Algebraic Expressions Algebraic expressions may contain more than one operation. Algebraic expressions can be evaluated if the values of the variables are known. First, replace the variables with their values. Then use the order of operations to calculate the value of the resulting numerical expression.

Example: Evaluate $x^3 + 5(y - 3)$ if $x = 2$ and $y = 12$.

$x^3 + 5(y - 3) = 2^3 + 5(12 - 3)$	Replace x with 2 and y with 12.
$= 8 + 5(12 - 3)$	Evaluate 2^3.
$= 8 + 5(9)$	Subtract 3 from 12.
$= 8 + 45$	Multiply 5 and 9.
$= 53$	Add 8 and 45.

The solution is 53.

Exercises

Evaluate each expression if $x = 2$, $y = 3$, $z = 4$, $a = \frac{4}{5}$, and $b = \frac{3}{5}$.

1. $x + 7$

2. $x^3 + y + z^2$

3. $23 - (a + b)$

4. $x(2y + 3z)$

5. $(10x)^2 + 100a$

6. $\frac{(z - y)^2}{x}$

7. $\frac{5a^2b}{y}$

8. $(z \div x)^2 + ax$

Solving Linear Equations

Properties of Equality To solve equations, we can use properties of equality.

Addition and Subtraction Properties of Equality	For any real numbers a, b, and c, if $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division Properties of Equality	For any real numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$ and, if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.

Example 1: Solve $10 - 8x = 50$.

$10 - 8x = 50$	Original equation
$10 - 8x - 10 = 50 - 10$	Subtract 10 from both sides.
$-8x = 40$	Simplify.
$x = -5$	Divide both sides by –8.

Example 2: Solve $4x + 5y = 100$ for y.

$4x + 5y = 100$	Original equation
$4x + 5y - 4x = 100 - 4x$	Subtract $4x$ from both sides.
$5y = 100 - 4x$	Simplify.
$y = \frac{1}{5}(100 - 4x)$	Divide both sides by 5.
$y = 20 - \frac{4}{5}x$	Apply the distributive property.

Exercises

Solve each equation. Check your solution.

1. $3s = 45$

2. $17 = 9 - a$

3. $5t - 1 = 6t - 5$

4. $\frac{2}{3}m = \frac{1}{2}$

5. $-8 = -2(z + 7)$

6. $3x + 17 = 5x - 13$

7. $5(4 - k) = -10$

8. $120 - \frac{3}{4}y = 60$

9. $4n + 20 = 53 - 2n$

Solve each equation or formula for the specified variable.

10. $a = 3b - c$, for b

11. $\frac{s}{2t} = 10$, for t

12. $\frac{3pq}{r} = 12$, for p

13. $2xy = x + 7$, for x

14. $\frac{d}{2} + \frac{f}{4} = 6$, for f

15. $3(2j - k) = 108$, for j

16. $\frac{m}{n} + 5m = 20$, for m

17. $4x - 3y = 10$, for y

Solving Linear Inequalities

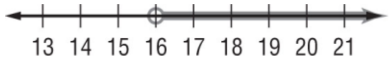
Example 1: Solve $2x + 4 > 36$.
Graph the solution set on a number line.

$2x + 4 - 4 > 36 - 4$

$2x > 32$

$x > 16$

The solution set is $\{x \mid x > 16\}$.



Example 2: Solve $17 - 3w \geq 35$.

Graph the solution set on a number line.

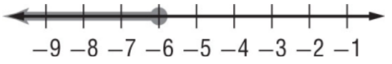
$17 - 3w \geq 35$

$17 - 3w - 17 \geq 35 - 17$

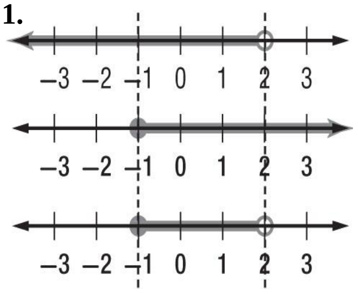
$-3w \geq 18$

$w \leq -6$

The solution set is $\{w \mid w \leq -6\}$.



Example 3: Graph the solution set of $x < 2$ and $x \geq -1$.



Graph $x < 2$.

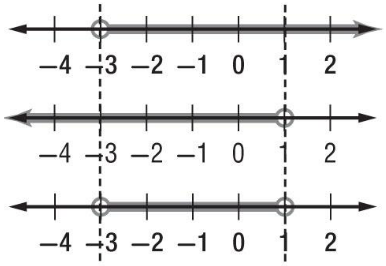
Graph $x \geq -1$.

Find the intersection.

The solution set is $\{x \mid -1 \leq x < 2\}$.

Example 4: Solve $-1 < x + 2 < 3$.
Then graph the solution set.

$-1 < x + 2$	and	$x + 2 < 3$
$-1 - 2 < x + 2 - 2$		$x + 2 - 2 < 3 - 2$
$-3 < x$		$x < 1$



Graph $x > -3$.

Graph $x < 1$.

Find the intersection.

The solution set is $\{x \mid -3 < x < 1\}$.

Solve each inequality. Then graph the solution set on a number line.

1. $7(7a - 9) \leq 84$

2. $3(9z + 4) > 35z - 4$

3. $5(12 - 3n) < 165$

4. $18 - 4k < 2(k + 21)$

5. $2 + 3(m + 5) \geq 4(m + 3)$

6. $4 < w + 3 \leq 5$

7. $-4 < 2x + 2 \leq -2$

8. $3 < 3w$ or $3w \geq 9$

9. $2x + 4 \leq 6$ or $x \geq 2x - 4$

Solving Equations Involving Absolute Value

Absolute Value Expressions Expressions with absolute values define an upper and lower range in which a value must lie. Expressions involving absolute value can be evaluated using the given value for the variable

.

Example 1: Solve $|x + 4| = 1$.

Write $|x + 4| = 1$ as $x + 4 = 1$ or $x + 4 = -1$.

$x + 4 = 1$ or $x + 4 = -1$

$x + 4 - 4 = 1 - 4$ $\quad$ $x + 4 - 4 = -1 - 4$

$x = -3$ $\quad$ $x = -5$

The solution set is $\{-5, -3\}$.

Example: Solve $|3a + 4| < 10$. Then graph the solution set.

Write $|3a + 4| < 10$ as $3a + 4 < 10$ and $3a + 4 > -10$.

$3a + 4 < 10$ and $3a + 4 > -10$

$3a + 4 - 4 < 10 - 4$ $\quad$ $3a + 4 - 4 > -10 - 4$

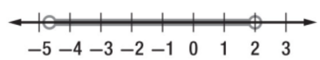
$3a < 6$ $\quad$ $3a > -14$

$\frac{3a}{3} < \frac{6}{3}$ $\quad$ $\frac{3a}{3} > \frac{-14}{3}$

$a < 2$ $\quad$ $a > -4\frac{2}{3}$

The solution set is $\left\{a \mid -4\frac{2}{3} < a < 2\right\}$.

Now graph the solution set.



Exercises

Solve each equation.

1. $|b + 2| = 3$

2. $|x - 4| = 4$

3. $|y + 3| = 2$

4. $|2x| = 8$

5. $|5y - 2| = 7$

6. $|2x - 1| = 11$

Solve each inequality. Then graph the solution set.

7. $|x - 4| < 4$

8. $|y + 3| \leq 2$

9. $|2x| \leq 8$

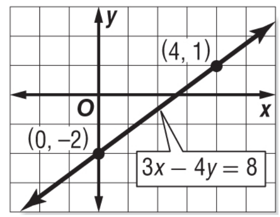
Graphing Linear Equations

Slope-Intercept Form

Slope-Intercept Form	$y = mx + b$, where m is the slope and b is the y-intercept

Example: Graph $3x - 4y = 8$.

$3x - 4y = 8$	Original equation
$-4y = -3x + 8$	Subtract $3x$ from each side.
$\frac{-4y}{-4} = \frac{-3x+8}{-4}$	Divide each side by –4.
$y = \frac{3}{4}x - 2$	Simplify.

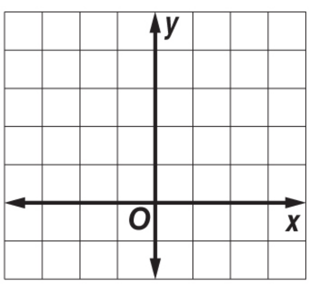


The y-intercept of $y = \frac{3}{4}x - 2$ is –2 and the slope is $\frac{3}{4}$. So graph the point (0, –2). From this point, move up 3 units and right 4 units. Draw a line passing through both points.

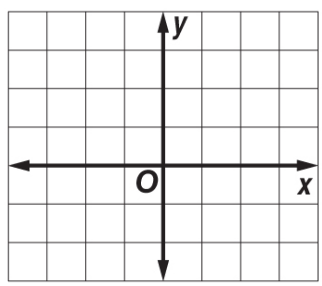
Exercises

Graph each equation.

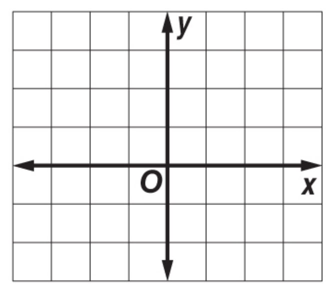
1. $y = 2x + 1$



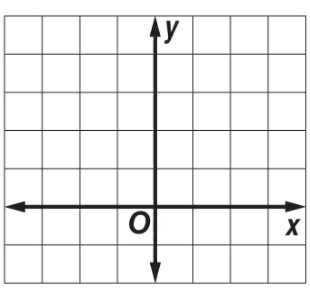
2. $y = -3x + 2$



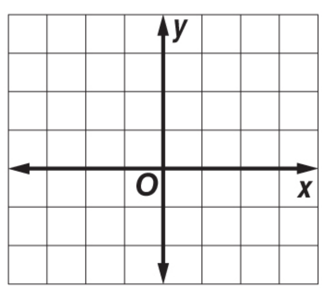
3. $y = -x - 1$



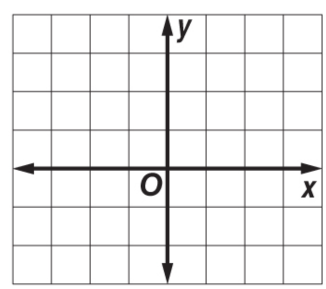
4. $y = 2$



5. $x = -3$



6. $y = \frac{1}{3}x - 2$



Slope

Slope *m* of a Line	For points (x_1, y_1) and (x_2, y_2), where $x_1 \neq x_2$, $m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$

Example 1: Find the slope of the line that passes through (2, –1) and (– 4, 5).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{5-(-1)}{-4-2}$ $(x_1, y_1) = (2, -1)$, $(x_2, y_2) = (-4, 5)$

$= \frac{6}{-6} = -1$ Simplify.

The slope of the line is –1.

Exercises

Find the slope of the line that passes through each pair of points. Express as a fraction in simplest form.

1. (4, 7) and (6, 13) **2.** (6, 4) and (3, 4) **3.** (5, 1) and (7, –3)

4. (5, –3) and (– 4, 3) **5.** (5, 10) and (–1,–2) **6.** (–1, – 4) and (–13, 2)

Writing Equations in Slope-Intercept Form

Example 1: Write an equation in slope-intercept form for the line with a slope of –4 and a *y*-intercept of 3.

$y = mx + b$ Slope-intercept form
$y = -4x + 3$ Replace *m* with –4 and *b* with 3.

Exercises

Write an equation for the line containing the given point with the given slope.

1. (2, 1), $m = 4$ **2.** (–7, 2), $m = 6$ **3.** (8, 3), $m = 1$

Writing Equations Continued

Example 2: Write an equation in point-slope form for the line that passes through (6, 1) with a slope of $-\frac{5}{2}$.

$y - y_1 = m(x - x_1)$ Point-slope form
$y - 1 = -\frac{5}{2}(x - 6)$ $m = -\frac{5}{2}$; $(x_1, y_1) = (6, 1)$
Therefore, the equation is $y - 1 = -\frac{5}{2}(x - 6)$.

Example 3: Write an equation in slope-intercept form for the line that passes through (–1, 6) and is parallel to the graph of $y = 2x + 12$.

A line parallel to $y = 2x + 12$ has the same slope, 2. Replace m with 2 and (x_1, y_1) with (–1, 6) in the point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 6 = 2(x - (-1))$	$m = 2$; $(x_1, y_1) = (-1, 6)$
$y - 6 = 2(x + 1)$	Simplify.
$y - 6 = 2x + 2$	Distributive Property
$y = 2x + 8$	Slope–intercept form

Therefore, the equation is $y = 2x + 8$.

Example 4: Write an equation in slope-intercept form for the line that passes through (–4, 2) and is perpendicular to the graph of $y = \frac{2}{3}x - 3$.

Use the point-slope form to find the equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 2 = -\frac{3}{2}(x - (-4))$	$m = -\frac{3}{2}$; $(x_1, y_1) = (-4, 2)$
$y - 2 = -\frac{3}{2}(x + 4)$	Simplify.
$y - 2 = -\frac{3}{2}x - 6$	Distributive Property
$y = -\frac{3}{2}x - 4$	Slope-intercept form

Exercises

Write an equation of the line that passes through each pair of points.

4. (–1, 6), (7, –10) **5.** (0, 2), (1, 7) **6.** (6, –25), (–1, 3)

7. Write an equation of the line that passes through the point (-2, 2) and is parallel to $y = 4x - 2$.

8. Write an equation of the line that passes through the point (6, 4) and is parallel to $y = \frac{1}{3}x + 1$.

9. Write an equation of the line that passes through the point (4, 2) and is perpendicular to $y = \frac{1}{2}x + 1$

Adding and Subtracting Polynomials

Example: Find $(3x^2 + 2x - 6) - (2x + x^2 + 3)$.

Horizontal Method

Use additive inverses to rewrite as addition.
Then group like terms.

$(3x^2 + 2x - 6) - (2x + x^2 + 3)$

$= (3x^2 + 2x - 6) + [(-2x) + (-x^2) + (-3)]$

$= [3x^2 + (-x^2)] + [2x + (-2x)] + [-6 + (-3)]$

$= 2x^2 + (-9)$

$= 2x^2 - 9$

The difference is $2x^2 - 9$.

Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{lr} & 3x^2 + 2x - 6 \\ \underline{(-)} & \underline{x^2 + 2x + 3} \\ & 3x^2 + 2x - 6 \\ \underline{(+)} & \underline{-x^2 - 2x - 3} \\ & 2x^2 \qquad - 9 \end{array}$$

The difference is $2x^2 - 9$.

Exercises

Find each sum or difference.

1. $(4a - 5) + (3a + 6)$

2. $(6xy + 2y + 6x) + (4xy - x)$

3. $(x^2 + y^2) + (-x^2 + y^2)$

4. $(8x^2 - 4x - 3) - (-2x - x^2 + 5)$

5. $(3x^2 - 2x) - (3x^2 + 5x - 1)$

6. $(4x^2 + 6xy + 2y^2) - (-x^2 + 2xy - 5y^2)$

Multiplying Polynomials

Example 1: Find $(x - 2)(x + 5)$ using the FOIL method.

$(x - 2)(x + 5)$

First Outer Inner Last

$= (x)(x) + (x)(5) + (-2)(x) + (-2)(5)$

$= x^2 + 5x + (-2x) - 10$

$= x^2 + 3x - 10$

The product is $x^2 + 3x - 10$.

Example: Find $(3x + 2)(2x^2 - 4x + 5)$.

$(3x + 2)(2x^2 - 4x + 5)$

$= 3x(2x^2 - 4x + 5) + 2(2x^2 - 4x + 5)$
Distributive Property

$= 6x^3 - 12x^2 + 15x + 4x^2 - 8x + 10$
Distributive Property

$= 6x^3 - 8x^2 + 7x + 10$
Combine like terms.

The product is $6x^3 - 8x^2 + 7x + 10$.

Exercises

Find each product.

1. $(x + 2)(x + 3)$

2. $(x - 4)(x + 1)$

3. $(x - 6)(x - 2)$

4. $(x + 2)(x^2 - 2x + 1)$

5. $(x + 3)(2x^2 + x - 3)$

6. $(2x - 1)(x^2 - x + 2)$

Graphing Linear Inequalities

Graph Linear Inequalities A **linear inequality**, like $y \geq 2x - 1$, resembles a linear equation, but with an inequality sign instead of an equals sign. The graph of the related linear equation separates the coordinate plane into two half-planes. The line is the boundary of each half-plane.

To graph a linear inequality, follow these steps.

Step 1 Graph the boundary; that is, the related linear equation. If the inequality symbol is ≤ or ≥, the boundary is solid. If the inequality symbol is < or >, the boundary is dashed.

Step 2 Choose a point not on the boundary and test it in the inequality. (0, 0) is a good point to choose if the boundary does not pass through the origin.

Step 3 If a true inequality results, shade the half–plane containing your test point. If a false inequality results, shade the other half-plane.

Example: Graph $x + 2y \geq 4$.

The boundary is the graph of $x + 2y = 4$.

Use the slope-intercept form, $y = -\frac{1}{2}x + 2$, to graph the boundary line.

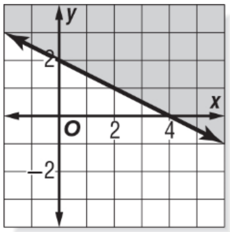
The boundary line should be solid.

Test the point (0, 0).

$0 + 2(0) \overset{?}{\geq} 4$ $(x, y) = (0, 0)$

$0 \geq 4$ false

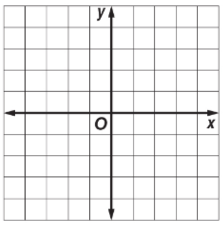
Shade the region that does *not* contain (0, 0).



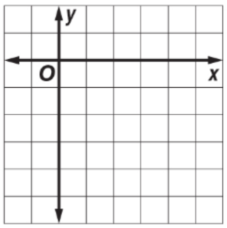
Exercises

Graph each inequality.

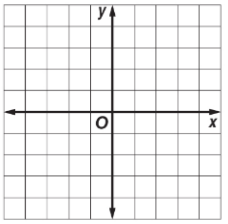
1. $y < 3x + 1$



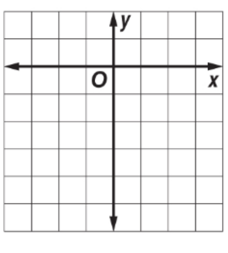
2. $y \geq x - 5$



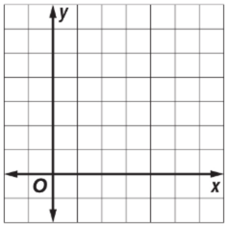
3. $4x + y \leq -1$



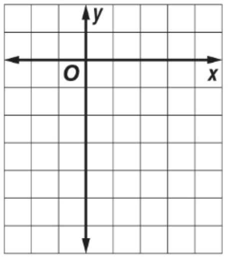
4. $y < \frac{x}{2} - 4$



5. $x + y > 6$



6. $0.5x - 0.25y < 1.5$



Solving Systems of Equations

Solve Systems Algebraically To solve a system of linear equations by **substitution**, first solve for one variable in terms of the other in one of the equations. Then substitute this expression into the other equation and simplify. To solve a system of linear equations by **elimination**, add or subtract the equations to eliminate one of the variables.

Example 1: Use substitution to solve the system of equations.

$$2x - y = 9$$
$$x + 3y = -6$$

Solve the first equation for y in terms of x.

$2x - y = 9$	First equation
$-y = -2x + 9$	Subtract $2x$ from both sides.
$y = 2x - 9$	Multiply both sides by –1.

Substitute the expression $2x - 9$ for y into the second equation and solve for x.

$x + 3y = -6$	Second equation
$x + 3(2x - 9) = -6$	Substitute $2x - 9$ for y.
$x + 6x - 27 = -6$	Distributive Property
$7x - 27 = -6$	Simplify.
$7x = 21$	Add 27 to each side.
$x = 3$	Divide each side by 7.

Now, substitute the value 3 for x in either original equation and solve for y.

$2x - y = 9$	First equation
$2(3) - y = 9$	Replace x with 3.
$6 - y = 9$	Simplify.
$-y = 3$	Subtract 6 from each side.
$y = -3$	Multiply each side by –1.

The solution of the system is (3, –3).

Example 2: Use the elimination method to solve the system of equations.

$$3x - 2y = 4$$
$$5x + 3y = -25$$

Multiply the first equation by 3 and the second equation by 2. Then add the equations to eliminate the y variable.

$3x - 2y = 4$	**Multiply by 3.**	$9x - 6y = 12$
$5x + 3y = -25$	**Multiply by 2.**	$\underline{10x + 6y = -50}$
		$19x = -38$
		$x = -2$

Replace x with –2 and solve for y.

$$3x - 2y = 4$$
$$3(-2) - 2y = 4$$
$$-6 - 2y = 4$$
$$-2y = 10$$
$$y = -5$$

The solution is (–2, –5)

Exercises

Solve each system of equations.

1. $3x + y = 7$
$4x + 2y = 16$

2. $2x + y = 5$
$3x - 3y = 3$

3. $2x + 3y = -3$
$x + 2y = 2$

4. $2x - y = 7$
$6x - 3y = 14$

5. $4x - y = 6$
$2x - \frac{y}{2} = 4$

6. $5x + 2y = 12$
$-6x - 2y = -14$

Solving Systems of Inequalities by Graphing

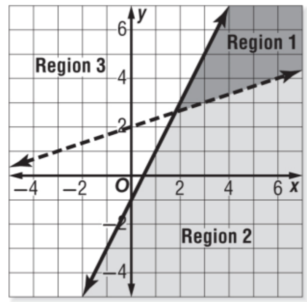
Systems of Inequalities To solve a system of inequalities, graph the inequalities in the same coordinate plane. The solution of the system is the region shaded for all of the inequalities.

Example: Solve the system of inequalities.
$y \le 2x - 1$ and $y > \frac{x}{3} + 2$

The solution of $y \le 2x - 1$ is Regions 1 and 2.

The solution of $y > \frac{x}{3} + 2$ is Regions 1 and 3.

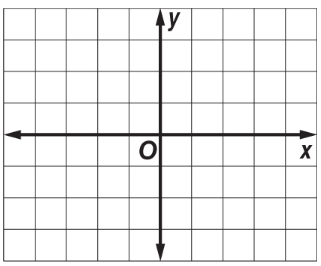
The intersection of these regions is Region 1, which is the solution set of the system of inequalities.



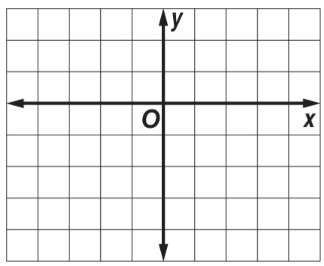
Exercises

Solve each system of inequalities by graphing.

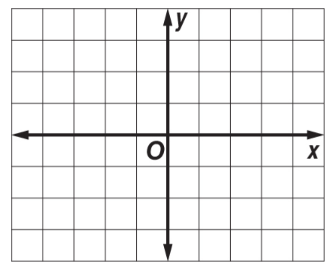
1. $x - y \le 2$
$x + 2y \ge 1$



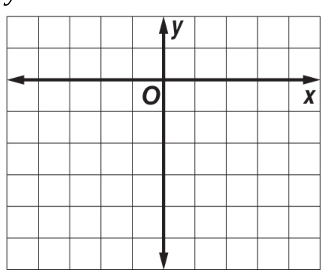
2. $3x - 2y \le -1$
$x + 4y \ge -12$



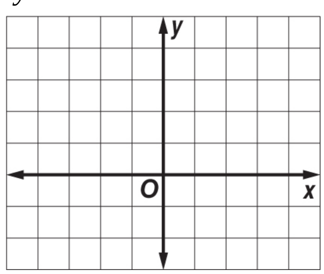
3. $y \le 1$
$x > 2$



4. $y \ge \frac{x}{2} - 3$
$y < 2x$



5. $y < \frac{x}{3} + 2$
$y < -2x + 1$



6. $y \ge -\frac{x}{4} + 1$
$y < 3x - 1$