

## TO: Algebra II Students

Summer is finally here . . .

In anticipation of the next school year, we want to do everything possible to ensure that it will be successful for you. It is important that the foundation for Algebra II is thoroughly understood. Therefore, we have designed a summer packet that reviews these essential skills. You are required to review this material and complete all of the exercises. These problems can be completed in the packet itself or you may complete the problems on loose leaf paper showing all your work. **You will be tested on this material with-in the first few weeks of class!** The grade for this quiz will be included in your first marking period average.

During the first week of the semester, we will offer review sessions during activity periods for those who feel they need some extra help. Attendance is optional, so if you are comfortable with the work there is no need to attend. However, we will be glad to give you a refresher if you've had some trouble completing the packet. Please come prepared with questions when you attend review sessions.

Graphing calculators will be used throughout the course. **This year you will be required to have your own graphing calculator.** We will be using a TI-83+ or TI 84 calculator in the classroom so this is the type of calculator you are required to have.

We hope you have an enjoyable and relaxing summer. We are looking forward to seeing you next year. Bring your completed summer packet with you the first day of class so we can immediately begin a fun and challenging semester of Algebra II!

Sincerely,

The Algebra II Teachers  
Notre Dame High School

# Order of Operations

**Evaluate Algebraic Expressions** Algebraic expressions may contain more than one operation. Algebraic expressions can be evaluated if the values of the variables are known. First, replace the variables with their values. Then use the order of operations to calculate the value of the resulting numerical expression.

**Example: Evaluate  $x^3 + 5(y - 3)$  if  $x = 2$  and  $y = 12$ .**

$$\begin{aligned}x^3 + 5(y - 3) &= 2^3 + 5(12 - 3) && \text{Replace } x \text{ with } 2 \text{ and } y \text{ with } 12. \\ &= 8 + 5(12 - 3) && \text{Evaluate } 2^3. \\ &= 8 + 5(9) && \text{Subtract } 3 \text{ from } 12. \\ &= 8 + 45 && \text{Multiply } 5 \text{ and } 9. \\ &= 53 && \text{Add } 8 \text{ and } 45.\end{aligned}$$

The solution is 53.

## Exercises

Evaluate each expression if  $x = 2$ ,  $y = 3$ ,  $z = 4$ ,  $a = \frac{4}{5}$ , and  $b = \frac{3}{5}$ .

1.  $x + 7$

2.  $x^3 + y + z^2$

3.  $23 - (a + b)$

4.  $x(2y + 3z)$

5.  $(10x)^2 + 100a$

6.  $\frac{(z - y)^2}{x}$

7.  $\frac{5a^2b}{y}$

8.  $(z \div x)^2 + ax$

# Solving Linear Equations

**Properties of Equality** To solve equations, we can use properties of equality.

<b>Addition and Subtraction Properties of Equality</b>	For any real numbers $a$ , $b$ , and $c$ , if $a = b$ , then $a + c = b + c$ and $a - c = b - c$ .
<b>Multiplication and Division Properties of Equality</b>	For any real numbers $a$ , $b$ , and $c$ , if $a = b$ , then $a \cdot c = b \cdot c$ and, if $c \neq 0$ , $\frac{a}{c} = \frac{b}{c}$ .

**Example 1: Solve  $10 - 8x = 50$ .**

$$\begin{aligned}10 - 8x &= 50 && \text{Original equation} \\10 - 8x - 10 &= 50 - 10 && \text{Subtract 10 from both sides.} \\-8x &= 40 && \text{Simplify.} \\x &= -5 && \text{Divide both sides by } -8.\end{aligned}$$

**Example 2: Solve  $4x + 5y = 100$  for  $y$ .**

$$\begin{aligned}4x + 5y &= 100 && \text{Original equation} \\4x + 5y - 4x &= 100 - 4x && \text{Subtract } 4x \text{ from both sides.} \\5y &= 100 - 4x && \text{Simplify.} \\y &= \frac{1}{5}(100 - 4x) && \text{Divide both sides by 5.} \\y &= 20 - \frac{4}{5}x && \text{Apply the distributive property.}\end{aligned}$$

## Exercises

**Solve each equation. Check your solution.**

1.  $3s = 45$

2.  $17 = 9 - a$

3.  $5t - 1 = 6t - 5$

4.  $\frac{2}{3}m = \frac{1}{2}$

5.  $-8 = -2(z + 7)$

6.  $3x + 17 = 5x - 13$

7.  $5(4 - k) = -10$

8.  $120 - \frac{3}{4}y = 60$

9.  $4n + 20 = 53 - 2n$

**Solve each equation or formula for the specified variable.**

10.  $a = 3b - c$ , for  $b$

11.  $\frac{s}{2t} = 10$ , for  $t$

12.  $\frac{3pq}{r} = 12$ , for  $p$

13.  $2xy = x + 7$ , for  $x$

14.  $\frac{d}{2} + \frac{f}{4} = 6$ , for  $f$

15.  $3(2j - k) = 108$ , for  $j$

16.  $\frac{m}{n} + 5m = 20$ , for  $m$

17.  $4x - 3y = 10$ , for  $y$

# Solving Linear Inequalities

**Example 1:** Solve  $2x + 4 > 36$ .

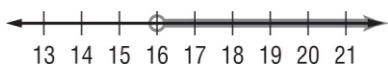
Graph the solution set on a number line.

$$2x + 4 - 4 > 36 - 4$$

$$2x > 32$$

$$x > 16$$

The solution set is  $\{x \mid x > 16\}$ .



**Example 2:** Solve  $17 - 3w \geq 35$ .

Graph the solution set on a number line.

$$17 - 3w \geq 35$$

$$17 - 3w - 17 \geq 35 - 17$$

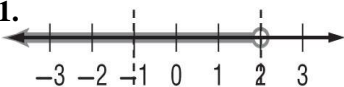
$$-3w \geq 18$$

$$w \leq -6$$

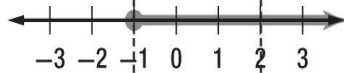
The solution set is  $\{w \mid w \leq -6\}$ .



**Example 3:** Graph the solution set of  $x < 2$  and  $x \geq -1$ .



Graph  $x < 2$ .



Graph  $x \geq -1$ .



Find the intersection.

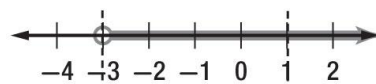
The solution set is  $\{x \mid -1 \leq x < 2\}$ .

**Example 4:** Solve  $-1 < x + 2 < 3$ . Then graph the solution set.

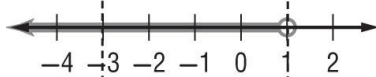
$$\begin{aligned} -1 < x + 2 \\ -1 - 2 < x + 2 - 2 \\ -3 < x \end{aligned}$$

and

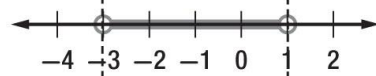
$$\begin{aligned} x + 2 < 3 \\ x + 2 - 2 < 3 - 2 \\ x < 1 \end{aligned}$$



Graph  $x > -3$ .



Graph  $x < 1$ .



Find the intersection.

The solution set is  $\{x \mid -3 < x < 1\}$ .

**Solve each inequality. Then graph the solution set on a number line.**

1.  $7(7a - 9) \leq 84$

2.  $3(9z + 4) > 35z - 4$

3.  $5(12 - 3n) < 165$

4.  $18 - 4k < 2(k + 21)$

5.  $2 + 3(m + 5) \geq 4(m + 3)$

6.  $4 < w + 3 \leq 5$

7.  $-4 < 2x + 2 \leq -2$

8.  $3 < 3w$  or  $3w \geq 9$

9.  $2x + 4 \leq 6$  or  $x \geq 2x - 4$

# Solving Equations Involving Absolute Value

**Absolute Value Expressions** Expressions with absolute values define an upper and lower range in which a value must lie. Expressions involving absolute value can be evaluated using the given value for the variable

**Example 1: Solve  $|x + 4| = 1$ .**

Write  $|x + 4| = 1$  as  $x + 4 = 1$  or  $x + 4 = -1$ .

$$x + 4 = 1 \quad \text{or} \quad x + 4 = -1$$

$$x + 4 - 4 = 1 - 4 \quad x + 4 - 4 = -1 - 4$$

$$x = -3 \quad x = -5$$

The solution set is  $\{-5, -3\}$ .

**Example: Solve  $|3a + 4| < 10$ . Then graph the solution set.**

Write  $|3a + 4| < 10$  as  $3a + 4 < 10$  and  $3a + 4 > -10$ .

$$3a + 4 < 10 \quad \text{and} \quad 3a + 4 > -10$$

$$3a + 4 - 4 < 10 - 4 \quad 3a + 4 - 4 > -10 - 4$$

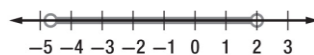
$$3a < 6 \quad 3a > -14$$

$$\frac{3a}{3} < \frac{6}{3} \quad \frac{3a}{3} > \frac{-14}{3}$$

$$a < 2 \quad a > -4\frac{2}{3}$$

The solution set is  $\{a \mid -4\frac{2}{3} < a < 2\}$ .

Now graph the solution set.



## Exercises

**Solve each equation.**

1.  $|b + 2| = 3$

2.  $|x - 4| = 4$

3.  $|y + 3| = 2$

4.  $|2x| = 8$

5.  $|5y - 2| = 7$

6.  $|2x - 1| = 11$

**Solve each inequality. Then graph the solution set.**

7.  $|x - 4| < 4$

8.  $|y + 3| \leq 2$

9.  $|2x| \leq 8$

# Graphing Linear Equations

## Slope-Intercept Form

**Slope-Intercept Form**

$y = mx + b$ , where  $m$  is the slope and  $b$  is the  $y$ -intercept

### Example: Graph $3x - 4y = 8$ .

$$3x - 4y = 8$$

$$-4y = -3x + 8$$

$$\frac{-4y}{-4} = \frac{-3x + 8}{-4}$$

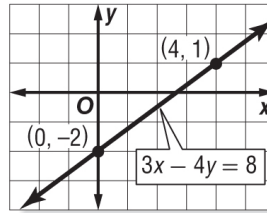
$$y = \frac{3}{4}x - 2$$

Original equation

Subtract  $3x$  from each side.

Divide each side by  $-4$ .

Simplify.

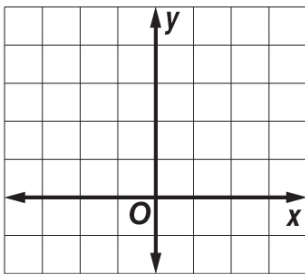


The  $y$ -intercept of  $y = \frac{3}{4}x - 2$  is  $-2$  and the slope is  $\frac{3}{4}$ . So graph the point  $(0, -2)$ . From this point, move up 3 units and right 4 units. Draw a line passing through both points.

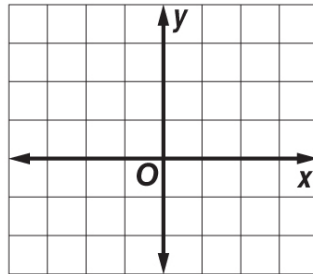
## Exercises

Graph each equation.

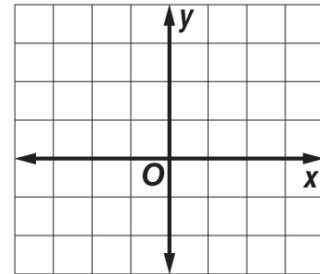
1.  $y = 2x + 1$



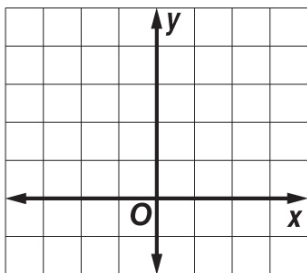
2.  $y = -3x + 2$



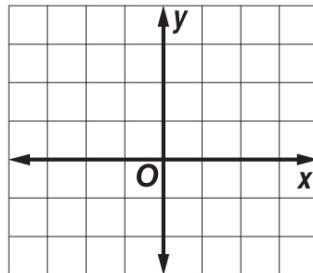
3.  $y = -x - 1$



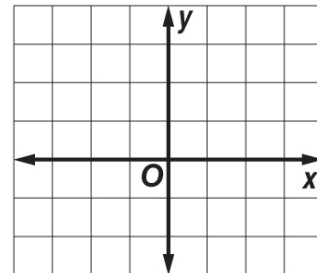
4.  $y = 2$



5.  $x = -3$



6.  $y = \frac{1}{3}x - 2$



# Slope

## Slope $m$ of a Line

For points  $(x_1, y_1)$  and  $(x_2, y_2)$ , where  $x_1 \neq x_2$ ,  $m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$

**Example 1:** Find the slope of the line that passes through  $(2, -1)$  and  $(-4, 5)$ .

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\ &= \frac{5 - (-1)}{-4 - 2} && (x_1, y_1) = (2, -1), (x_2, y_2) = (-4, 5) \\ &= \frac{6}{-6} = -1 && \text{Simplify.} \end{aligned}$$

The slope of the line is  $-1$ .

## Exercises

**Find the slope of the line that passes through each pair of points. Express as a fraction in simplest form.**

1.  $(4, 7)$  and  $(6, 13)$

2.  $(6, 4)$  and  $(3, 4)$

3.  $(5, 1)$  and  $(7, -3)$

4.  $(5, -3)$  and  $(-4, 3)$

5.  $(5, 10)$  and  $(-1, -2)$

6.  $(-1, -4)$  and  $(-13, 2)$

## Writing Equations in Slope-Intercept Form

**Example 1:** Write an equation in slope-intercept form for the line with a slope of  $-4$  and a  $y$ -intercept of  $3$ .

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form} \\ y &= -4x + 3 && \text{Replace } m \text{ with } -4 \text{ and } b \text{ with } 3. \end{aligned}$$

## Exercises

**Write an equation for the line containing the given point with the given slope.**

1.  $(2, 1)$ ,  $m = 4$

2.  $(-7, 2)$ ,  $m = 6$

3.  $(8, 3)$ ,  $m = 1$

## Writing Equations Continued

**Example 2:** Write an equation in point-slope form for the line that passes through (6, 1) with a slope of  $-\frac{5}{2}$ .

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$
$$y - 1 = -\frac{5}{2}(x - 6) \quad m = -\frac{5}{2}; (x_1, y_1) = (6, 1)$$

Therefore, the equation is  $y - 1 = -\frac{5}{2}(x - 6)$ .

**Example 3:** Write an equation in slope-intercept form for the line that passes through (-1, 6) and is parallel to the graph of  $y = 2x + 12$ .

A line parallel to  $y = 2x + 12$  has the same slope, 2. Replace  $m$  with 2 and  $(x_1, y_1)$  with (-1, 6) in the point-slope form.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$
$$y - 6 = 2(x - (-1)) \quad m = 2; (x_1, y_1) = (-1, 6)$$
$$y - 6 = 2(x + 1) \quad \text{Simplify.}$$
$$y - 6 = 2x + 2 \quad \text{Distributive Property}$$
$$y = 2x + 8 \quad \text{Slope-intercept form}$$

Therefore, the equation is  $y = 2x + 8$ .

**Example 4:** Write an equation in slope-intercept form for the line that passes through (-4, 2) and is perpendicular to the graph of  $y = \frac{2}{3}x - 3$ .

Use the point-slope form to find the equation.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$
$$y - 2 = -\frac{3}{2}(x - (-4)) \quad m = -\frac{3}{2}; (x_1, y_1) = (-4, 2)$$
$$y - 2 = -\frac{3}{2}(x + 4) \quad \text{Simplify.}$$
$$y - 2 = -\frac{3}{2}x - 6 \quad \text{Distributive Property}$$
$$y = -\frac{3}{2}x - 4 \quad \text{Slope-intercept form}$$

### Exercises

Write an equation of the line that passes through each pair of points.

4. (-1, 6), (7, -10)

5. (0, 2), (1, 7)

6. (6, -25), (-1, 3)

7. Write an equation of the line that passes through the point (-2, 2) and is parallel to  $y = 4x - 2$ .

8. Write an equation of the line that passes through the point (6, 4) and is parallel to  $y = \frac{1}{3}x + 1$ .

9. Write an equation of the line that passes through the point (4, 2) and is perpendicular to  $y = \frac{1}{2}x + 1$ .



## Adding and Subtracting Polynomials

**Example:** Find  $(3x^2 + 2x - 6) - (2x + x^2 + 3)$ .

### Horizontal Method

Use additive inverses to rewrite as addition.  
Then group like terms.

$$\begin{aligned}(3x^2 + 2x - 6) - (2x + x^2 + 3) &= (3x^2 + 2x - 6) + [(-2x) + (-x^2) + (-3)] \\ &= [3x^2 + (-x^2)] + [2x + (-2x)] + [-6 + (-3)] \\ &= 2x^2 + (-9) \\ &= 2x^2 - 9\end{aligned}$$

The difference is  $2x^2 - 9$ .

### Vertical Method

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{r} 3x^2 + 2x - 6 \\ (-) \quad x^2 + 2x + 3 \\ \hline 3x^2 + 2x - 6 \\ (+) \quad -x^2 - 2x - 3 \\ \hline 2x^2 \quad \quad -9\end{array}$$

The difference is  $2x^2 - 9$ .

### Exercises

Find each sum or difference.

1.  $(4a - 5) + (3a + 6)$

2.  $(6xy + 2y + 6x) + (4xy - x)$

3.  $(x^2 + y^2) + (-x^2 + y^2)$

4.  $(8x^2 - 4x - 3) - (-2x - x^2 + 5)$

5.  $(3x^2 - 2x) - (3x^2 + 5x - 1)$

6.  $(4x^2 + 6xy + 2y^2) - (-x^2 + 2xy - 5y^2)$

## Multiplying Polynomials

**Example 1:** Find  $(x - 2)(x + 5)$  using the FOIL method.

$$\begin{aligned}(x - 2)(x + 5) &\quad \begin{array}{cccc} \text{First} & \text{Outer} & \text{Inner} & \text{Last} \end{array} \\ &= (x)(x) + (x)(5) + (-2)(x) + (-2)(5) \\ &= x^2 + 5x + (-2x) - 10 \\ &= x^2 + 3x - 10\end{aligned}$$

The product is  $x^2 + 3x - 10$ .

**Example:** Find  $(3x + 2)(2x^2 - 4x + 5)$ .

$$\begin{aligned}(3x + 2)(2x^2 - 4x + 5) &= 3x(2x^2 - 4x + 5) + 2(2x^2 - 4x + 5) \\ &\quad \text{Distributive Property} \\ &= 6x^3 - 12x^2 + 15x + 4x^2 - 8x + 10 \\ &\quad \text{Distributive Property} \\ &= 6x^3 - 8x^2 + 7x + 10 \\ &\quad \text{Combine like terms.}\end{aligned}$$

The product is  $6x^3 - 8x^2 + 7x + 10$ .

### Exercises

Find each product.

1.  $(x + 2)(x + 3)$

2.  $(x - 4)(x + 1)$

3.  $(x - 6)(x - 2)$

4.  $(x + 2)(x^2 - 2x + 1)$

5.  $(x + 3)(2x^2 + x - 3)$

6.  $(2x - 1)(x^2 - x + 2)$